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Abstract

In this paper, a parallel clustering technique and route construction heuristic have been developed for the vehicle routing problem with split deliveries and pickups. An MILP formulation for determining the exact solution to the problem has also been included. It has been shown through extensive experimentation that the algorithm proposed in this paper statistically produces better results than the only heuristic existing for this class of problems in literature. We also form a basis of comparison between this class of problems and the vehicle routing problem with simultaneous deliveries and pickups. We note that while heuristics for simultaneous deliveries and pickups cannot be applied in situations where customers' delivery or pickup demands exceed the vehicle capacity, heuristics allowing split deliveries and pickups can, in fact, be applied in every situation, even producing superior results under the combined objective of minimization of the fixed charge and mileage associated with vehicle routes. A guideline as to which heuristic could be used under what parametric conditions and objective functions, has also been provided.

Keywords: vehicle routing; backhauling; split deliveries; heuristics; parallel clustering

Introduction

Vehicle Routing Problems (VRP) and Vehicle Routing Problems with Backhauling (VRPB) have been one of the focal points of research in combinatorial optimization for the last few decades. Both problems are NP-hard, and efforts are made to develop exact and heuristic solution procedures to solve gradually larger-sized instances efficiently and effectively. Though the exact methods have so far been limited to solving small-sized instances and rather determining 'tight' lower bounds to the optimal solutions for

medium and large-sized instances, the heuristics available in the literature can now routinely, and efficiently, solve instances with 500 to 600 customer locations. The common approach has been to develop an exact and a heuristic method, compare the performance of the heuristic with respect to the optimal solutions to the small-sized instances, and show improvements in objective values on application of the heuristic with respect to either the lower bounds or the best known solutions to the larger-sized instances. With continuously increasing computing power and developments in meta-heuristics, search and evolutionary algorithms, getting a very good solution to a very large-sized instance today takes very little time.

The focus of the current paper is VRPB. As anyone conversant with research in this area knows, VRP becomes a special case of VRPB when the pickup demands of customers are made equal to zero in VRPB. Recently, research on VRPB has peaked up essentially because pickup demands for packaging and used product returns from customer locations have increased substantially due to environmental and government regulations, and, also, integrating backhauls with linehauls maximally utilizes the vehicle capacity and saves money. The original problem addressed in VRPB is referred to as the classical VRPB in literature where the linehaul and backhaul customers are different, and on a route all the linehaul customers have to be served before visiting the backhaul customers. The rationales cited for this route constraint were the priority of linehaul demands over backhaul demands and the inconvenience caused by 'mixed' loads for rear-loaded vehicles. Exact methods for the classical VRPB were proposed by Toth and Vigo (1997) and Mingozzi et al. (1999). Heuristics have been developed by Deif and Bodin (1984), Casco et al. (1988), Goetschalckx and Jacobs-Blecha (1989), Anily (1996), Potvin et al. (1996), Thangiah et al. (1996), Duhamel et al. (1997), Toth and Vigo (1999), and Brandao (2006). Problems that allow serving backhaul demands before all the linehaul customers are served on a route are referred to as mixed VRPB. The rationales for relaxing the route constraint are that nowadays most vehicles can be rear-loaded as well as side-loaded and allowing mixed loads on a route may reduce cost. Salhi and Nagy (1999) developed several insertion-based heuristics for the single and multiple-depot mixed VRPB. Wade and Salhi (2002) proposed a compromise between the classical and

mixed VRPB by defining a fraction of the delivery demand on a route that must be delivered before a pickup can be made. It was shown that as the fraction was decreased, so did the average route cost.

Both the classical and mixed VRPB require that the linehaul and backhaul customers are different. However, in today's context a customer may have both delivery and pickup demands. Problems that address this situation are referred to as Vehicle Routing Problems with Simultaneous Deliveries and Pickups (VRPSDP). Simultaneous delivery and pickup at a customer location has been considered in the literature primarily based on the assumption that a customer would prefer to have a single stop rather than multiple stops of a vehicle for convenience, though relaxing the constraint might improve the solution to the problem. VRPSDP was introduced by Min (1989). Heuristics to solve the problem were developed by Dethloff (2001), Chen and Wu (2006), and Tang and Galvao (2006). Dethloff (2002) showed that the heuristic developed in Dethloff (2001) for VRPSDP could produce better results than by Salhi and Nagy (1999) when applied for the mixed VRPB. The above are primarily the different variants of VRPB addressed in the literature so far. There may, of course, be additional constraints in a given problem such as time windows, which means every customer has to be served within a given time frame, and route time limitations that allow maximum route times for the vehicles. For a detailed review on the different variants of VRPB, one may refer to Ropke and Pisinger (2006).

The literature mentioned above restricts each customer to be visited exactly once, thereby eliminating the possibility of split deliveries that might improve the solution to the problem. Also, allowing only one visit to a customer limits the delivery or pickup demand to the capacity of the vehicle. However, in many practical situations this may not be so. Consider a manufacturer supplying from a central depot to a number of regional depots where the delivery and/or pickup demands would most probably exceed the capacity of one vehicle. Hence, to meet the demands of the regional depots, the manufacturer has to resort to split deliveries. In VRP, Vehicle Routing Problems with Split Deliveries (VRPSD) were introduced by Dror and Trudeau (1989). Other notable

references are Dror and Trudeau (1990), Dror et al. (1994), Frizzell and Giffin (1992, 1995), Belenguer et al. (2000), Ho and Haugland (2004), and Archetti et al. (2006a, 2006b). However, for Vehicle Routing Problems with Split Deliveries and Pickups (let us refer to this problem as VRPSPDP to distinguish it from VRPSDP), we have so far come across only one reference, i.e., Mitra (2005). In this paper, an MILP formulation to obtain the optimal solution to the problem has been given and a heuristic has been developed. The heuristic first determines the minimum number of vehicles that are required to fulfill the delivery and pickup demands of all the customers, and then sequentially constructs the vehicle routes based on the cheapest insertion criterion. Details of the heuristic may be found in Mitra (2005). A numerical experimentation with 20 locations (even with 20 locations, the CPLEX solver could not solve many instances to optimality) has been conducted which reveals that the heuristic performs very well with respect to the optimal solution or an upper bound to the optimal solution when for every customer the delivery demand is at least equal to the pickup demand, but when the pickup demands of some of the customers exceed their delivery demands, the performance of the heuristic deteriorates.

In this paper, we consider VRPSPDP under the same problem situation as in Mitra (2005). We give an alternative MILP formulation for the problem and develop a parallel clustering technique to arrive at an initial solution to the problem. We test the performance of the method with respect to the same problem sets given in Mitra (2005), and show by a non-parametric test the superiority of the method over the heuristic proposed by Mitra (2005), especially when the pickup loads of the customers exceed their delivery loads. We also try to form a basis of comparison among these heuristics for VRPSPDP and Chen and Wu's heuristic for VRPSDP, and see which heuristic performs better than the other two under what parametric conditions and performance measurement criteria for randomly generated problem sets.

The paper is organized as follows. The problem is defined in the next section. Then the formulation and the proposed clustering technique to arrive at an initial solution to the

problem are presented. The computational results are given in the subsequent section. The final section presents concluding remarks and directions for future research.

Problem definition

The problem definition is the same as in Mitra (2005). Briefly, a depot has to serve the delivery and pickup demands of a number of customers with a fleet of vehicles of the same capacity. Each vehicle has only one tour, i.e., it leaves and returns to the depot only once. Each customer may have both delivery and pickup demands either of which may exceed the vehicle capacity. Split deliveries and pickups are allowed at each location, which implies each customer may be visited by more than one vehicle and more than once by the same vehicle. A customer may prefer a single stop by a vehicle for convenience. However, splitting of loads and multiple stops by the same vehicle may increase efficiency, and under a centralized control system such as a central depot supplying to the regional warehouses, this distribution policy may be followed without any trouble. The delivery and pickup at a location may be in any sequence. There are no time windows and no restrictions on the maximum route times. The route cost between any pair of locations is fixed and independent of the vehicle load. The objective of the problem is to determine the minimum number of vehicles required to fulfill the delivery and pickup demands of all the customers, and then schedule the vehicle routes so as to minimize the total route cost.

Model formulation

In this section, we first give the MILP formulation of the problem, and then we describe the parallel clustering technique and the formation of vehicle routes within these clusters.

MILP formulation

Consider $n+1$ locations, $0, 1, 2, \dots, n$, where 0 represents the depot and $1, 2, \dots, n$ represent n customers. The minimum number of vehicles, which is an input data to the formulation, is calculated as follows:

$$\left\lceil \frac{\max(\text{cumulative delivery demand}, \text{cumulative pickup demand})}{\text{vehicle capacity}} \right\rceil$$

We realize that if we relax this constraint and allow an unlimited number of vehicles, it may be possible to reduce the total route cost. Almost all the exact and heuristic methods in literature allow the same, and consider the only objective to minimize the total route cost. In the process, the fixed costs associated with owning/hiring vehicles are not taken into consideration explicitly. It may, of course, be possible to reduce the total route cost, including fixed costs, by allowing an unlimited number of vehicles under certain circumstances. However, normally in practice, overhead costs of owning or fixed charges for hiring vehicles are much higher than the variable costs or mileage, leading managers to utilize a minimum number of vehicles to fulfill delivery and pickup demands of customers. In this paper, we work under the assumption that the fixed cost per vehicle clearly dominates all conceivable variable costs (or “routing costs”), and consequently arrive at a solution with the minimum number of vehicles. The minimum number of vehicles ensures the least fixed costs of owning/hiring vehicles; the objective then remains to minimize the variable costs or mileage. Hence, essentially this is a sequential procedure - first the fixed costs and then the variable costs are minimized. However, we reiterate the fact that allowing an unlimited number of vehicles would have given the “true” optimal total route cost, including fixed costs, and acknowledge the assumption on the deployment of the minimum number of vehicles as a limitation of the procedure. We, however, note that the “true” optimal total route cost, including fixed costs, can be obtained through the MILP formulation, given below, by adding a fixed cost component to the objective function and gradually incrementing the number of vehicles deployed from the minimum requirement. Only the problem has to be solved a number of times

until the “true” optimal solution is obtained. On the other hand, the clustering algorithm/route construction heuristic, described later, is designed in such a way that it works with the minimum number of vehicles given as an input. Efforts will be made in future to make the heuristic more generalizable allowing it to take any number of vehicles as an input. We will discuss more about the objectives of the problem when we compare among the heuristics for VRPSPDP and Chen and Wu’s (2006) heuristic for VRPSPDP in the subsequent section on computational results.

The following notations have been used in the formulation.

Index

i, j set of depot and customers: $\{0, 1, 2, \dots, n\}$

k set of vehicles: $\{1, 2, \dots, m\}$

Data

m number of vehicles

v vehicle capacity

D_j delivery demand at customer j

R_j pickup demand at customer j

c_{ij} route cost between customer i and customer j ($c_{ii} = \infty, \forall i$)

Decision variables

x_{ijk} number of times vehicle k runs from customer i to customer j

y_{ij} amount of delivery load moved from customer i to customer j

z_{ij} amount of pickup load moved from customer i to customer j

The formulation is given below followed by explanations.

$$\text{Min} \quad \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^m c_{ij} x_{ij}$$

$$\text{S.T.} \quad \sum_{i=0}^n y_{ij} - \sum_{i=0}^n y_{ji} = D_j, \quad \forall j \quad (1)$$

$$\sum_{i=0}^n z_{ji} - \sum_{i=0}^n z_{ij} = R_j, \quad \forall j \quad (2)$$

$$\sum_{i=0}^n y_{i0} = 0 \quad (3)$$

$$\sum_{j=0}^n z_{0j} = 0 \quad (4)$$

$$\sum_{i=0}^n x_{ijk} - \sum_{i=0}^n x_{jik} = 0, \quad \forall j, k \quad (5)$$

$$\sum_{j=0}^n x_{0jk} = 1, \quad \forall k \quad (6)$$

$$y_{ij} + z_{ij} \leq v \sum_{k=1}^m x_{ijk}, \quad \forall i, j \quad (7)$$

$$y_{ij}, z_{ij} \geq 0, \quad \forall i, j$$

$$x_{ijk} \geq 0, \text{ integer} \quad \forall i, j, k$$

The objective is to minimize the total route cost. Constraints (1) and (2) ensure that the delivery and pickup demands at customer j are satisfied. Constraints (3) and (4) ensure that there are no delivery loads directed to and no pickup loads coming from the depot. Constraints (5) state that a vehicle visiting a customer must exit it the same number of times it visits the customer. Constraints (6) state that each vehicle comes out of the depot only once, and constraint (7) checks that the vehicle capacity on a route is not violated. It is to be noted that for the depot the delivery and pickup demands, respectively, are represented by the negative of cumulative delivery and pickup demands of all the customers.

Cluster formation

In this subsection, we describe the parallel clustering technique, and in the subsequent subsection, we outline the construction of vehicle routes as part of the heuristic proposed

in this paper. The heuristic may be categorized as a cluster-first-route-second heuristic. The cluster formation steps are given below followed by explanations.

Step 1: Identify the customer located farthest from the depot. This customer becomes the second element of the first cluster, the first element of every cluster being the depot itself.

Step 2: For each of the remaining clusters, assign the second element as the customer for which the sum of the distances from the second elements already assigned to the previous clusters is the maximum. Repeat until the second elements for all the clusters are assigned.

Step 3: For each cluster, identify the next element as the customer for which the sum of the distances from the already assigned elements to the cluster is the minimum. Repeat for each cluster sequentially until the cumulative delivery and pickup loads assigned to a cluster separately are about to violate the vehicle capacity constraint, and finally all customers have been assigned to the different clusters.

The number of clusters is known a priori and is equal to the minimum number of vehicles required to fulfill the demands of customers the expression for which has been given in the previous subsection. Each cluster is served by one vehicle. The first two steps of cluster formation identify the second elements of the clusters, which are farthest from each other, and are likely to belong to different clusters. The third step assigns one element at a time to each cluster for which the sum of the distances among the elements in a cluster is minimized. The objective of this step is to identify closely-knit clusters within each of which the travel distances are likely to be minimized. Please note this step ensures that the clusters are formed in parallel, and not as a whole at one go, which might lead to better assignments of customers to different clusters. A cluster is grown as long as its delivery or pickup load does not exceed the vehicle capacity, and the process is terminated when all the customers are assigned to different clusters. Please also note since split deliveries are allowed and delivery or pickup loads at customers may exceed

the vehicle capacity, a customer may be included into a cluster for which the delivery or pickup load may have to be split. In this situation, the remaining delivery and pickup load at the customer location are updated for consideration for assignment to a different cluster. This may lead to the assignment of a customer to two or more clusters. To give an example for illustration, suppose a cluster being formed can further accommodate at most 5 units of delivery load based on the restriction of vehicle capacity, and the next customer added to the cluster has a delivery demand of 10 units. In that case, 5 units of the customer's delivery demand will be assigned to the cluster, and the remaining 5 units of delivery demand will be assigned to one or more other clusters. Hence, the customer in question becomes part of more than one cluster, i.e. it is visited by more than one vehicle. In other words, the customer's delivery demand is split. The same example can be cited with respect to the customer's pickup demand. Once a customer is included to a cluster, its delivery and pickup demands are adjusted based on the remaining delivery and pickup capacities of the cluster.

Once clusters are formed, the job remaining is to schedule the vehicle route within each cluster independently of other clusters. If the number of customers in a cluster is reasonable, the problem can even be solved optimally by using the formulation given in the previous subsection. However, we present below a route construction heuristic to arrive at an initial solution to the problem.

Route construction

The heuristic presented in this subsection schedules the vehicle routes for all the clusters. The following notations are used to present the heuristic in addition to those described in the preceding subsection.

<i>no_of_customers</i>	number of customers assigned to a cluster
<i>DL</i>	delivery load carried by a vehicle
<i>PL</i>	pickup load carried by a vehicle
<i>total_cost</i>	total route cost

Please note DL and PL are variable quantities that get updated every time a vehicle visits a customer. By definition, the initial value of DL is equal to the cumulative delivery load of a cluster when the vehicle leaves the depot and its final value is zero when the vehicle returns to the depot. On the other hand, PL is zero when the vehicle leaves the depot and is equal to the cumulative pickup load of the cluster when the vehicle returns to the depot.

The pseudo-code of the heuristic is given below followed by explanations.

1. For $k = 1$ to $k \leq m$ step 1
2. $i = 0$ (set the current location of the vehicle as the depot)
3. Do
4. For $j = 1$ to $j \leq no_of_customers$ step 1
5. If $D_j > 0$ or ($R_j > 0$ and $DL + PL < v$) then
6. If $c_{ij} < minm_cost$ (minimum cost obtained so far from i to a customer)
7. then $next_dest = j$ and $minm_cost = c_{ij}$ (set j as the next destination and c_{ij} as the new minimum cost)
8. Otherwise if $c_{ij} = minm_cost$ then
9. If $DL + PL + R_{next_dest} - D_{next_dest} > v$ and $DL + PL + R_j - D_j \leq v$
10. then $next_dest = j$
11. Next j
12. $total_cost = total_cost + c_{i next_dest}$
13. $DL = DL - D_{next_dest}, D_{next_dest} = 0$
14. If $R_{next_dest} \leq v - DL - PL$ then
15. $PL = PL + R_{next_dest}, R_{next_dest} = 0$
16. Otherwise $R_{next_dest} = R_{next_dest} - (v - DL - PL), PL = v - DL$
17. $i = next_dest$ (set i as the current customer location of the vehicle)
18. While $DL > 0$ or $PL < cum_pickup$ (cumulative pickup load assigned to the cluster)
19. $total_cost = total_cost + c_{i0}$ (the vehicle returns to the depot and the route cost is added to the total cost)
20. Next k

21. Print the routes of the vehicles and the total cost.

Lines 4-11: This portion of the code identifies the next destination on a route. A vehicle will visit a customer if its delivery demand is positive or its pickup demand is positive and there is empty space in the vehicle to carry a part or the whole of the pickup load. The selection of the next destination on a route is based on the cheapest cost of addition to the current location. In the case of a tie, the customer from which the vehicle can carry the entire pickup load while it is unable to do so for the other customer is selected as the next destination. Otherwise, anyone may be selected.

Lines 12-16: Once the next destination is selected, the route cost is added to the total cost. Next, the DL, PL, and delivery and pickup demands at the next destination have to be adjusted based on their current values.

Lines 3-18: The Do-While loop is repeated until the delivery load on the vehicle is zero or the pickup load equals the cumulative pickup load of the cluster.

The heuristic presented in this section is intended to provide a good initial solution, which, of course, may be improved upon by applying the various intra- and inter-route exchange/improvement algorithms available in the literature. However, the challenge has always been to derive a good starting solution that may quickly be improved toward optimality. Hence, we stick to this heuristic alone, and measure its performance in the numerical experimentation in the next section.

Computational results

In this section, first we compare the performance of the proposed heuristic with that of Mitra (2005), and then form a rationale for comparison between these heuristics for VRPSPDP and Chen and Wu's (2006) heuristic for VRPSPDP. For convenience, let SM,

NH and CW represent the heuristic developed by Mitra (2005), the new heuristic proposed in this paper, and the heuristic developed by Chen and Wu (2006), respectively.

Comparison between SM and NH

To compare between SM and NH, we used the same 110-problem set given in Mitra (2005). For each instance, the CPLEX solver incorporating the formulation given in the section on model formulation was run for 30 minutes (maximum) to obtain either the optimal solution or an upper bound to it, and then the heuristics, programmed in Turbo C, were run. The results are shown in Table A1 in the appendix. The two cases are reproduced below for readability.

Case 1: All route costs are equal.

$$c_{ij} = 10, \forall i, j \text{ such that } j > i \text{ and } c_{ii} = \infty, \forall i$$

Case 2: All route costs are not equal.

$$c_{ij} = 9 + j - i, \forall i, j \text{ such that } j > i \text{ and } c_{ii} = \infty, \forall i$$

For both the cases, the route costs are symmetric.

We conducted a paired-sample Wilcoxon signed-rank test to ascertain if there is any significant difference between SM and NH. Out of 110 instances, for 29 the results were found to be the same. Hence, the effective sample size for the test was 81. For each of these 81 instances, the cost obtained by NH was subtracted from that obtained by SM to get the 81 samples. The null hypothesis was stated as θ (median of the differences) = 0 against the alternative hypothesis $\theta > 0$. We computed $T^+ = 2409.5$, $T^- = 911.5$, $E(T) = 1660.5$ and $\text{Var}(T) = 45110.25$. The one-sided level of significance, α , was taken as 0.05, and $T_{0.05}$ was found to equal 1311.12. Since $T_{0.05} > T^-$, the null hypothesis must be rejected at 0.05 level of significance. In other words, there is a significant difference between SM and NH at 0.05 level of significance. For details on the test, please refer to Goon et al. (1993, pp. 585-589).

NH was found to perform statistically better than SM for the test problems. Especially for instances where the pickup loads exceeded the delivery loads, NH significantly outperformed SM, which was highlighted as a shortcoming of SM in Mitra (2005).

Comparison among SM, NH and CW

As already noted, SM and NH belong to the class VRPSPDP while CW has been developed for VRPSDP. For the purpose of comparison, it is assumed that both simultaneous and split deliveries and pickups are permissible. While simultaneous deliveries and pickups are preferred for convenience, splitting of loads may be preferred for efficiency. Splitting of delivery and pickup demands may improve the solution to the problem obtained under the constraint of simultaneous deliveries and pickups, as demonstrated by the following example with 8 customers. Since CW stands for simultaneous deliveries and pickups, it limits the delivery and pickup demands of each customer, separately, to the vehicle capacity for the examples presented in this subsection. Table 1 and Table 2 below present the delivery and pickup demands of the customers, expressed as fractions of the vehicle capacity, and the costs of travelling between each pair of customer locations, expressed in some arbitrary monetary unit. The data for this example problem have been randomly generated. Without loss of generality, it has been assumed that all the route costs are symmetric.

Insert Table 1 and Table 2 here

Next, we apply SM, NH and CW to the problem, and get the corresponding routes, cumulative delivery loads of routes, and total route costs, as shown in Table 3 below. Please note for CW we have used the initial solution and not considered the subsequent improvement/exchange algorithms proposed in Chen and Wu (2006) for comparison purpose. The objective has been to compare among the initial solutions given by the different heuristics. As we can see from Table 3, splitting of loads may produce better

results, but it is not necessary that it will always produce better results. While NH has produced the best result for the example problem, SM's performance has been the worst.

Insert Table 3 here

To compare among SM, NH and CW, we note that while CW is constrained by the requirement of simultaneous deliveries and pickups, SM and NH are also constrained by the availability of the minimum number of vehicles. We already noted earlier that it may be possible to improve the solution if the constraint on the number of vehicles is relaxed. Hence, we see there are two aspects of the problem on which the two classes of heuristics alternately prevail over each other. CW as well as most of the algorithms on VRPB consider only one objective function, i.e., cost or total distance travelled assuming unlimited availability of vehicles, and in the process the fixed costs associated with owning/hiring vehicles are not taken into consideration explicitly. In practice, hiring a vehicle involves charges in two parts: fixed, independent of the distance travelled, and mileage, which is variable. The algorithms take care of the second part, i.e., mileage; however, the first part, i.e., the fixed charge, which may be substantial, is not taken into consideration. We create a hypothetical situation where the fixed charge and mileage for hiring a vehicle are taken as \$100 and \$0.10, respectively, which, we feel, is a very practical estimate given the current market rates. We create six problem sets with two sets of customer populations and three sets of combinations of delivery and pickup demands. Customers are assumed to be populated within a 100-mile radius from the depot. For each problem set, we randomly generate 30 instances, and compute the number of vehicles, distance travelled and route cost (fixed cost + mileage) by each of the heuristics. The setup of the experiment and the corresponding tables of results given in the appendix are represented by Table 4 below.

Insert Table 4 here

Please note here also the delivery and pickup loads are expressed as fractions of the vehicle capacity. In each instance, the delivery and pickup demands of a customer are randomly generated from the corresponding sets for that problem set. In Tables A2-A7, the minimum distance travelled and the minimum cost obtained for each instance are highlighted. From the tables, we can infer that with respect to the distance travelled, CW performs the best for problem sets 1 and 4, SM performs the best for problem sets 2 (confirmed by the paired-sample Wilcoxon signed-rank test with NH) and 5, and NH performs the best for problem sets 3 (confirmed by the paired-sample Wilcoxon signed-rank test with CW) and 6. With respect to the route cost, CW's performance is the worst in all the 6 problem sets, SM performs the best for problem sets 2 and 5, and in rest of the problem sets, NH's performance is the best. Figures 1 and 2 below show the average distance travelled and average route cost, respectively, for each problem set given by each heuristic.

Insert Fig.1 and Fig. 2 here

If we refer to the delivery/pickup demand by 'high' ('low') when it is more than (less than or equal to) half of the vehicle capacity, we may draw some inferences from the results obtained, which are summarized in Table 5 below as to which heuristic will outperform the others under what parametric conditions and objective functions.

Insert Table 5 here

Please note that for 'low'/'low' combinations of delivery/pickup demands, the computation has not been reported, but it was found that for both distance and cost, CW performed the best. This is because for low demands, simultaneous deliveries and pickups will produce shorter distances travelled than split deliveries and pickups, and SM and NH by design try to split from the very beginning of the route irrespective of the

pattern of demands. Also, there is no significant difference with respect to the number of vehicles needed for the heuristics.

It is worthwhile to mention here that the comparison of the heuristics would have been “fairer” had CW been able to incorporate fixed costs with variable costs or mileage in arriving at an initial solution to the problem. However, CW, like other route construction heuristics, seeks to minimize the variable costs assuming unlimited availability of vehicles, and the route construction process is sequential with no provision for backtracking. As a result, it may not be possible to incorporate fixed costs in CW as it is, and hence fixed costs have to be added “ex post” to variable costs or mileage for the purpose of comparison. We note in this context that a heuristic designed to use the minimum number of vehicles (assuming relatively high fixed costs per vehicle compared to variable costs) can be expected to perform better than a heuristic aiming to minimize the variable costs with later-added fixed costs to the solution.

We could not test the performance of the heuristics for more than 80 customers as the compiler available would not be able to handle problems of a bigger size. However, we expect that the inferences drawn in this paper would prevail. Also, the computational times of the problems run on an Intel Centrino 1.6 GHz, 256 MB RAM laptop with Windows XP operating system were negligible for all the heuristics, and hence were not reported.

Conclusions and directions for future research

This paper develops a parallel clustering technique and route construction heuristic that produces statistically better initial solutions than the only one existing heuristic for the class of VRPSPDP. From the numerical experimentation it appears that when delivery demands at all customer locations exceed their corresponding pickup demands, SM still produces marginally better results than NH; however, under more general conditions where at some customer locations pickup demands may exceed the corresponding delivery demands, NH produces far more superior results than SM. This result is

particularly significant in the light of growing pickup demands vis-à-vis delivery demands. Hence, we may conclude that SM and NH could complement each other for the class of VRPSPDP.

VRPSDP and VRPSPDP should not ideally be compared with each other because they refer to two different classes of VRPB. Simultaneous deliveries and pickups may be appropriate under a decentralized control system where customers would prefer one stop of the vehicle for convenience; however, under a centralized control system where, for example, a central depot distributes to a number of regional depots, split deliveries and pickups may be allowed. Also, simultaneous deliveries and pickups are feasible only when the delivery and pickup demands at every customer location separately do not exceed the vehicle capacity. We formed a rationale for comparing among SM, NH and CW (which so far produces the best results for the class of VRPSDP), and showed that when the fixed cost associated with a route is taken into account, which most of the current algorithms including CW do not consider, SM or NH is capable of producing better results than CW even with the minimum number of vehicles. A guideline has been provided as to which heuristic could be used under what demand conditions and objective functions.

As far as directions for future research are concerned, NH can be improved upon as to decide when to split and when not to split. As it stands now, NH tries to split demands at the first opportunity since it has been designed keeping in mind demands on the higher side with respect to the vehicle capacity. Therefore, it splits demands even for lower demands when it could have been economical to have simultaneous deliveries and pickups. The heuristic has been tested under a symmetric cost structure. It may be observed how it performs under an asymmetric cost structure. Also, only one vehicle type has been used. It may be explored how the heuristic has to be modified to take into account the availability of more than one vehicle type. Moreover, in this paper only one tour per vehicle has been assumed. However, in most realistic settings when demands exceed the vehicle capacity to a large extent, it may be possible, favorable and necessary to assume multiple-use of vehicles, i.e. more than one tour per vehicle. In this case, the

whole aspect of fixed costs has to be treated differently, and scheduling aspects (route duration, assignment of tours to vehicles etc.) have to be taken into consideration because the number of vehicles (i.e. fixed costs) is not identical to the number of routes.

In literature on VRPB, we find different heuristics that perform the best for different classes of the problem. Though difficult, it may be explored to develop a single heuristic that performs well for all problem classes. One such step in this direction has already been taken by Ropke and Pisinger (2006). Also, the problems considered so far are static in nature, i.e., demands and travel times are deterministic and are known in advance. Future research on VRP/VRPB may focus on dynamic/stochastic demands and travel times (e.g. see Potvin et al., 2006).

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Appendix

Table A1: Route costs obtained from the formulation, and the SM and NH heuristics (D_j : Delivery demand of customer j , R_j : Pickup demand of customer j , C_{opt} : Optimal cost, C_{ub} : Upper bound to the optimal cost, C_{sm} : SM heuristic cost, C_{nh} : New heuristic cost)

The optimal costs are identified by (Opt.) beside the cost figures.

Problem set #	D_j	R_j	No. of vehicles	Case 1			Case 2		
				C_{opt}/C_{ub}	C_{sm}	C_{nh}	C_{opt}/C_{ub}	C_{sm}	C_{nh}
1	1	1	2	240	210	210	566	247	263
	5	5	10	290	290	290	554	479	515
	10	10	19	380 (Opt.)	380	380	722 (Opt.)	722	722
	15	15	29	790	670	710	1303	1201	1303
	20	20	38	760 (Opt.)	760	760	1444 (Opt.)	1444	1444
	5	1	10	320	290	290	545	479	515
	10	1	19	380 (Opt.)	380	380	722 (Opt.)	722	722
	10	5	19	380 (Opt.)	380	380	722 (Opt.)	722	722
	15	1	29	740	670	750	1353	1201	1365
	15	5	29	740	670	750	1296	1201	1365
	15	10	29	710	670	710	1283	1201	1321
	20	1	38	760 (Opt.)	760	760	1444 (Opt.)	1444	1444
	20	5	38	760 (Opt.)	760	760	1444 (Opt.)	1444	1444
	20	10	38	760 (Opt.)	760	760	1444 (Opt.)	1444	1444
	20	15	38	760 (Opt.)	760	760	1444 (Opt.)	1444	1444
	1	5	10	290	290	290	543	479	515
	1	10	19	380 (Opt.)	380	380	722 (Opt.)	722	722
	1	15	29	730	920	750	1364	1538	1383
	1	20	38	760 (Opt.)	1110	760	1444 (Opt.)	1939	1444
	5	10	19	380 (Opt.)	380	380	722 (Opt.)	722	722
	5	15	29	740	1040	750	1379	1612	1383
	5	20	38	760 (Opt.)	1270	760	1444 (Opt.)	2013	1444
	10	15	29	710	1030	710	1357	1557	1303
10	20	38	760 (Opt.)	760	760	1444 (Opt.)	1444	1444	
15	20	38	760 (Opt.)	1310	760	1444 (Opt.)	1995	1444	
2	$D_1 = 5,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	1	27	730	690	710	1648	1295	1379
		5	27	740	690	710	1341	1295	1379
		10	27	800	760	700	1491	1349	1442
	$D_1 = 10,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	5	37	990	890	890	1814	1685	1773
		10	37	950	890	870	1731	1685	1719
		15	37	920	940	900	1794	1739	1818
	$D_1 = 15,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	10	46	1090	1070	1080	2107	2017	2119
		15	46	1170	1070	1080	2423	2017	2146
		20	46	1130	1150	1070	2948	2071	2191
	1	$R_1 = 5,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	27	810	830	710	1526	1445	1379
	5			830	910	710	1646	1501	1397
	10			800	860	700	1920	1500	1397

Problem set #	D_j	R_j	No. of vehicles	Case 1			Case 2		
				C_{opt}/C_{ub}	C_{sm}	C_{nh}	C_{opt}/C_{ub}	C_{sm}	C_{nh}
3	5	$R_1 = 10,$	37	910	1160	890	1834	1940	1791
	10	$R_j = R_{j-1}+1,$		900	1140	870	1879	1928	1719
	15	$\forall j \geq 2$		980	1090	900	1857	1899	1746
	10	$R_1 = 15,$	46	1190	1260	1080	2519	2212	2119
	15	$R_j = R_{j-1}+1,$		1170	1360	1080	2136	2294	2155
	20	$\forall j \geq 2$		1090	1310	1070	2326	2289	2137
4	$D_1 = 5,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	$R_1 = 1,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	27	750	690	720	1871	1295	1436
		$R_1 = 5,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	27	860	690	700	1528	1295	1370
		$R_1 = 10,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	37	900	1350	930	1777	2177	1859
	$D_1 = 10,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	$R_1 = 5,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	37	970	890	930	1737	1685	1823
		$R_1 = 10,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	37	1010	890	870	2560	1685	1710
		$R_1 = 15,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	46	1200	1590	1090	3425	2525	2165
	$D_1 = 15,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	$R_1 = 10,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	46	1200	1070	1090	2295	2017	2165
		$R_1 = 15,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	46	1260	1070	1070	2565	2017	2110
		$R_1 = 20,$ $R_j = R_{j-1}+1,$ $\forall j \geq 2$	56	1340	1830	1310	2573	2979	2561
		$D_1 = 20,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	$R_1 = 20,$ $R_j = R_{j-1}-1,$ $\forall j \geq 2$	56	1280	1270	1280	2592	2407
5	$D_1 = 25,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	$R_1 = 25,$ $R_j = R_{j-1}-1,$ $\forall j \geq 2$	65	1480	1450	1470	4077	2739	2856
	$D_1 = 30,$ $D_j = D_{j-1}+1,$ $\forall j \geq 2$	$R_1 = 30,$ $R_j = R_{j-1}-1,$ $\forall j \geq 2$	75	1920	1650	1650	3563	3129	3213

Table A2: Number of vehicles, total distance travelled and total route cost obtained by the SM, NH and CW heuristics for the Problem Set # 1

Instance #	No. of vehicles		Total distance			Total route cost		
	SM/NH	CW	SM	NH	CW	SM	NH	CW
1	41	50	7621	7407	7068	4862.1	4840.7	5706.8
2	41	50	8232	7582	7054	4923.2	4858.2	5705.4
3	40	50	7436	7587	7236	4743.6	4758.7	5723.6
4	42	50	8449	7937	7318	5044.9	4993.7	5731.8
5	41	50	7441	7121	6956	4844.1	4812.1	5695.6
6	41	50	8334	7486	7468	4933.4	4848.6	5746.8
7	41	50	7600	7283	6742	4860	4828.3	5674.2
8	41	50	7770	7036	7088	4877	4803.6	5708.8
9	40	50	7819	6890	7148	4781.9	4689	5714.8
10	40	50	7849	6976	6958	4784.9	4697.6	5695.8
11	41	50	7717	7244	6998	4871.7	4824.4	5699.8
12	42	50	8296	8320	7564	5029.6	5032	5756.4
13	39	50	7246	7041	6818	4624.6	4604.1	5681.8
14	42	50	7588	7533	7136	4958.8	4953.3	5713.6
15	41	50	7604	6869	6820	4860.4	4786.9	5682
16	42	50	7873	7482	7068	4987.3	4948.2	5706.8
17	42	50	7899	7353	6706	4989.9	4935.3	5670.6
18	40	50	7426	7699	7158	4742.6	4769.9	5715.8
19	40	50	7648	7038	6858	4764.8	4703.8	5685.8
20	42	50	7721	7124	6924	4972.1	4912.4	5692.4
21	41	50	7879	7307	7186	4887.9	4830.7	5718.6
22	41	50	7418	7135	6778	4841.8	4813.5	5677.8
23	40	50	7676	7424	7538	4767.6	4742.4	5753.8
24	42	50	7907	7731	7438	4990.7	4973.1	5743.8
25	40	50	7881	7789	7376	4788.1	4778.9	5737.6
26	42	50	7597	7493	7058	4959.7	4949.3	5705.8
27	41	50	7767	8395	7522	4876.7	4939.5	5752.2
28	41	50	7783	7827	7154	4878.3	4882.7	5715.4
29	41	50	7604	7702	7128	4860.4	4870.2	5712.8
30	41	50	7585	7145	6928	4858.5	4814.5	5692.8

Table A3: Number of vehicles, total distance travelled and total route cost obtained by the SM, NH and CW heuristics for the Problem Set # 2

Instance #	No. of vehicles		Total distance			Total route cost		
	SM/NH	CW	SM	NH	CW	SM	NH	CW
1	40	50	6833	7019	7206	4683.3	4701.9	5720.6
2	41	50	7163	7498	7422	4816.3	4849.8	5742.2
3	42	50	6582	6772	6676	4858.2	4877.2	5667.6
4	42	50	6897	6896	6954	4889.7	4889.6	5695.4
5	41	50	6944	7203	7078	4794.4	4820.3	5707.8
6	40	50	6390	6546	6590	4639	4654.6	5659
7	40	50	6843	7296	7204	4684.3	4729.6	5720.4
8	39	50	6703	6659	7092	4570.3	4565.9	5709.2
9	41	50	7312	7253	7474	4831.2	4825.3	5747.4
10	41	50	7007	6776	7198	4800.7	4777.6	5719.8
11	39	50	6407	6739	6634	4540.7	4573.9	5663.4
12	39	50	6551	6728	6982	4555.1	4572.8	5698.2
13	43	50	7180	6844	6768	5018	4984.4	5676.8
14	42	50	7390	6941	7132	4939	4894.1	5713.2
15	39	50	6498	7108	7026	4549.8	4610.8	5702.6
16	40	50	6902	6799	7264	4690.2	4679.9	5726.4
17	39	50	6456	6894	6926	4545.6	4589.4	5692.6
18	42	50	7565	7157	7220	4956.5	4915.7	5722
19	42	50	7132	7362	7250	4913.2	4936.2	5725
20	39	50	6696	7100	7142	4569.6	4610	5714.2
21	41	50	6485	6644	6752	4748.5	4764.4	5675.2
22	40	50	7003	7653	7488	4700.3	4765.3	5748.8
23	41	50	6472	6452	6368	4747.2	4745.2	5636.8
24	40	50	7081	6974	7126	4708.1	4697.4	5712.6
25	41	50	7466	8170	7772	4846.6	4917	5777.2
26	42	50	7074	7032	7122	4907.4	4903.2	5712.2
27	41	50	6667	6663	6868	4766.7	4766.3	5686.8
28	39	50	6228	6744	6844	4522.8	4574.4	5684.4
29	40	50	6595	6708	6824	4659.5	4670.8	5682.4
30	41	50	6731	6969	7026	4773.1	4796.9	5702.6

Table A4: Number of vehicles, total distance travelled and total route cost obtained by the SM, NH and CW heuristics for the Problem Set # 3

Instance #	No. of vehicles		Total distance			Total route cost		
	SM/NH	CW	SM	NH	CW	SM	NH	CW
1	42	50	8432	6797	6682	5043.2	4879.7	5668.2
2	40	50	7887	7475	7390	4788.7	4747.5	5739
3	40	50	8096	7142	7162	4809.6	4714.2	5716.2
4	40	50	8700	6956	7302	4870	4695.6	5730.2
5	39	50	7763	6834	7188	4676.3	4583.4	5718.8
6	41	50	7790	6952	6722	4879	4795.2	5672.2
7	40	50	7547	6711	6698	4754.7	4671.1	5669.8
8	41	50	8548	7577	7614	4954.8	4857.7	5761.4
9	40	50	7294	6483	6736	4729.4	4648.3	5673.6
10	40	50	7738	6843	7244	4773.8	4684.3	5724.4
11	42	50	8198	6760	6872	5019.8	4876	5687.2
12	41	50	7589	6479	6580	4858.9	4747.9	5658
13	37	50	7058	6523	6982	4405.8	4352.3	5698.2
14	41	50	7599	6236	6512	4859.9	4723.6	5651.2
15	40	50	7845	6981	6974	4784.5	4698.1	5697.4
16	40	50	8106	7430	7390	4810.6	4743	5739
17	38	50	8310	7527	7286	4631	4552.7	5728.6
18	41	50	7508	6946	6950	4850.8	4794.6	5695
19	41	50	8010	6665	6658	4901	4766.5	5665.8
20	40	50	8491	7394	7392	4849.1	4739.4	5739.2
21	40	50	8546	6744	6908	4854.6	4674.4	5690.8
22	42	50	7231	5856	5916	4923.1	4785.6	5591.6
23	40	50	7620	7069	7254	4762	4706.9	5725.4
24	40	50	7702	6709	6840	4770.2	4670.9	5684
25	39	50	7673	6774	7086	4667.3	4577.4	5708.6
26	41	50	8090	6401	6362	4909	4740.1	5636.2
27	40	50	7652	6301	6566	4765.2	4630.1	5656.6
28	40	50	7622	6888	7286	4762.2	4688.8	5728.6
29	41	50	7388	7383	7200	4838.8	4838.3	5720
30	39	50	8540	6948	6992	4754	4594.8	5699.2

Table A5: Number of vehicles, total distance travelled and total route cost obtained by the SM, NH and CW heuristics for the Problem Set # 4

Instance #	No. of vehicles		Total distance			Total route cost		
	SM/NH	CW	SM	NH	CW	SM	NH	CW
1	65	80	11922	11973	11716	7692.2	7697.3	9171.6
2	67	80	11934	11912	11628	7893.4	7891.2	9162.8
3	64	80	12089	12011	11704	7608.9	7601.1	9170.4
4	65	80	12556	11906	12268	7755.6	7690.6	9226.8
5	66	80	11554	11507	11270	7755.4	7750.7	9127
6	67	80	12335	12527	11826	7933.5	7952.7	9182.6
7	64	80	11430	11273	10892	7543	7527.3	9089.2
8	65	80	11654	11974	11520	7665.4	7697.4	9152
9	65	80	11945	11910	11598	7694.5	7691	9159.8
10	65	80	11161	10594	10690	7616.1	7559.4	9069
11	69	80	12135	12471	11794	8113.5	8147.1	9179.4
12	65	80	11438	11855	11090	7643.8	7685.5	9109
13	65	80	11654	12222	11868	7665.4	7722.2	9186.8
14	66	80	11916	11438	11592	7791.6	7743.8	9159.2
15	65	80	11400	11797	11064	7640	7679.7	9106.4
16	67	80	12295	12147	11720	7929.5	7914.7	9172
17	66	80	11494	12101	11566	7749.4	7810.1	9156.6
18	68	80	11774	11114	11044	7977.4	7911.4	9104.4
19	65	80	11956	11989	11526	7695.6	7698.9	9152.6
20	65	80	11514	11444	11356	7651.4	7644.4	9135.6
21	66	80	11603	11664	11162	7760.3	7766.4	9116.2
22	65	80	11737	11445	11712	7673.7	7644.5	9171.2
23	68	80	11313	10818	10232	7931.3	7881.8	9023.2
24	65	80	11913	11487	11238	7691.3	7648.7	9123.8
25	64	80	11444	11332	11420	7544.4	7533.2	9142
26	65	80	11259	11197	11168	7625.9	7619.7	9116.8
27	66	80	12006	11911	11154	7800.6	7791.1	9115.4
28	65	80	11913	11858	11520	7691.3	7685.8	9152
29	64	80	11274	11184	10932	7527.4	7518.4	9093.2
30	66	80	11887	12256	11448	7788.7	7825.6	9144.8

Table A6: Number of vehicles, total distance travelled and total route cost obtained by the SM, NH and CW heuristics for the Problem Set # 5

Instance #	No. of vehicles		Total distance			Total route cost		
	SM/NH	CW	SM	NH	CW	SM	NH	CW
1	66	80	11119	11884	11706	7711.9	7788.4	9170.6
2	66	80	10843	11579	11392	7684.3	7757.9	9139.2
3	66	80	10944	11376	11792	7694.4	7737.6	9179.2
4	66	80	10840	11518	11570	7684	7751.8	9157
5	65	80	10349	10761	11018	7534.9	7576.1	9101.8
6	63	80	10215	10842	11498	7321.5	7384.2	9149.8
7	64	80	11294	11607	12288	7529.4	7560.7	9228.8
8	65	80	10637	10660	11316	7563.7	7566	9131.6
9	64	80	10818	10988	11456	7481.8	7498.8	9145.6
10	65	80	10518	10833	11332	7551.8	7583.3	9133.2
11	63	80	10019	10500	10922	7301.9	7350	9092.2
12	64	80	10580	11143	11890	7458	7514.3	9189
13	65	80	10563	11577	11376	7556.3	7657.7	9137.6
14	63	80	9654	10043	10576	7265.4	7304.3	9057.6
15	65	80	10278	10894	10906	7527.8	7589.4	9090.6
16	67	80	11047	11412	11442	7804.7	7841.2	9144.2
17	64	80	10557	11399	11326	7455.7	7539.9	9132.6
18	63	80	10137	10812	11306	7313.7	7381.2	9130.6
19	61	80	10423	11162	11622	7142.3	7216.2	9162.2
20	62	80	10174	10701	11218	7217.4	7270.1	9121.8
21	65	80	10113	10149	10742	7511.3	7514.9	9074.2
22	63	80	9945	10376	10770	7294.5	7337.6	9077
23	64	80	10488	10924	11274	7448.8	7492.4	9127.4
24	65	80	10517	11237	11360	7551.7	7623.7	9136
25	63	80	10338	10985	11472	7333.8	7398.5	9147.2
26	65	80	10360	11201	11350	7536	7620.1	9135
27	65	80	10558	11257	11268	7555.8	7625.7	9126.8
28	64	80	10926	11375	11960	7492.6	7537.5	9196
29	66	80	10762	10825	11356	7676.2	7682.5	9135.6
30	65	80	10666	11384	11396	7566.6	7638.4	9139.6

Table A7: Number of vehicles, total distance travelled and total route cost obtained by the SM, NH and CW heuristics for the Problem Set # 6

Instance #	No. of vehicles		Total distance			Total route cost		
	SM/NH	CW	SM	NH	CW	SM	NH	CW
1	66	80	12451	11503	11638	7845.1	7750.3	9163.8
2	64	80	11463	10499	10790	7546.3	7449.9	9079
3	64	80	11779	10834	11176	7577.9	7483.4	9117.6
4	66	80	11782	10855	10812	7778.2	7685.5	9081.2
5	66	80	12913	11551	11858	7891.3	7755.1	9185.8
6	66	80	12209	11079	11022	7820.9	7707.9	9102.2
7	64	80	11510	11210	11136	7551	7521	9113.6
8	66	80	11734	10826	11134	7773.4	7682.6	9113.4
9	63	80	10822	10192	10452	7382.2	7319.2	9045.2
10	65	80	12286	10884	11444	7728.6	7588.4	9144.4
11	65	80	12152	10922	11128	7715.2	7592.2	9112.8
12	62	80	12446	11180	11934	7444.6	7318	9193.4
13	65	80	11653	10906	10992	7665.3	7590.6	9099.2
14	65	80	11339	10570	10650	7633.9	7557	9065
15	63	80	11347	10655	11012	7434.7	7365.5	9101.2
16	67	80	12546	11287	11556	7954.6	7828.7	9155.6
17	66	80	11715	10416	10808	7771.5	7641.6	9080.8
18	63	80	12357	11523	11832	7535.7	7452.3	9183.2
19	64	80	12402	10643	11080	7640.2	7464.3	9108
20	65	80	12387	11664	12106	7738.7	7666.4	9210.6
21	64	80	11530	10904	11174	7553	7490.4	9117.4
22	66	80	11547	10668	10926	7754.7	7666.8	9092.6
23	64	80	11822	11006	11496	7582.2	7500.6	9149.6
24	64	80	11754	10770	11126	7575.4	7477	9112.6
25	64	80	12081	10392	10818	7608.1	7439.2	9081.8
26	65	80	11964	10820	11156	7696.4	7582	9115.6
27	64	80	11477	10118	10622	7547.7	7411.8	9062.2
28	65	80	12385	11690	11630	7738.5	7669	9163
29	64	80	11514	11341	11268	7551.4	7534.1	9126.8
30	64	80	11789	10939	11150	7578.9	7493.9	9115

Table 1: Delivery and pickup demands of the customers expressed as fractions of the vehicle capacity

Customer location	Delivery demand	Pickup demand
1	0.3	0.3
2	0.4	0.4
3	0.3	0.4
4	0.4	0.5
5	0.2	0.2
6	0.5	0.4
7	0.3	0.3
8	0.2	0.1

Table 2: Route costs between each pair of locations ('0' represents the depot)

Location	0	1	2	3	4	5	6	7	8
0	∞	2	7	6	8	6	9	2	7
1	2	∞	8	5	8	7	11	1	7
2	7	8	∞	6	3	12	3	7	14
3	6	5	6	∞	4	12	9	4	13
4	8	8	3	4	∞	14	6	7	15
5	6	7	12	12	14	∞	14	8	3
6	9	11	3	9	6	14	∞	10	16
7	2	1	7	4	7	8	10	∞	8
8	7	7	14	13	15	3	16	8	∞

Table 3: Routes, cumulative delivery loads of routes (expressed as fractions of the vehicle capacity and given alongside the routes) and total route costs obtained by SM, NH and CW

	SM	NH	CW
Route 1	0-1-7-3-4-0 (1)	0-2-6-4-0 (1)	0-8-4-2-0 (1)
Route 2	0-5-8-3-4-2-0 (1)	0-1-8-5-0 (0.7)	0-5-6-3-0 (1)
Route 3	0-2-4-6-0 (0.6)	0-7-3-4-0 (0.9)	0-1-7-0 (0.6)
Total route cost	80	60	72

Table 4: Experimental setup and corresponding tables of results

	Delivery demand = {0.6, 0.7, 0.8, 0.9, 1} Pickup demand = {0.6, 0.7, 0.8, 0.9, 1}	Delivery demand ={0.6, 0.7, 0.8, 0.9, 1} Pickup demand ={0.1, 0.2, 0.3, 0.4, 0.5}	Delivery demand ={0.1, 0.2, 0.3, 0.4, 0.5} Pickup demand ={0.6, 0.7, 0.8, 0.9, 1}
No. of Customers = 50	Problem set # 1 Table A2	Problem set # 2 Table A3	Problem set # 3 Table A4
No. of Customers = 80	Problem set # 4 Table A5	Problem set # 5 Table A6	Problem set # 6 Table A7

Table 5: Best-performed heuristics under different demand conditions and objective functions

	Objective function			
	Distance		Cost	
	Delivery demand		Delivery demand	
	High	Low	High	Low
Pickup demand				
High	CW	NH	NH	NH
Low	SM	CW	SM	CW

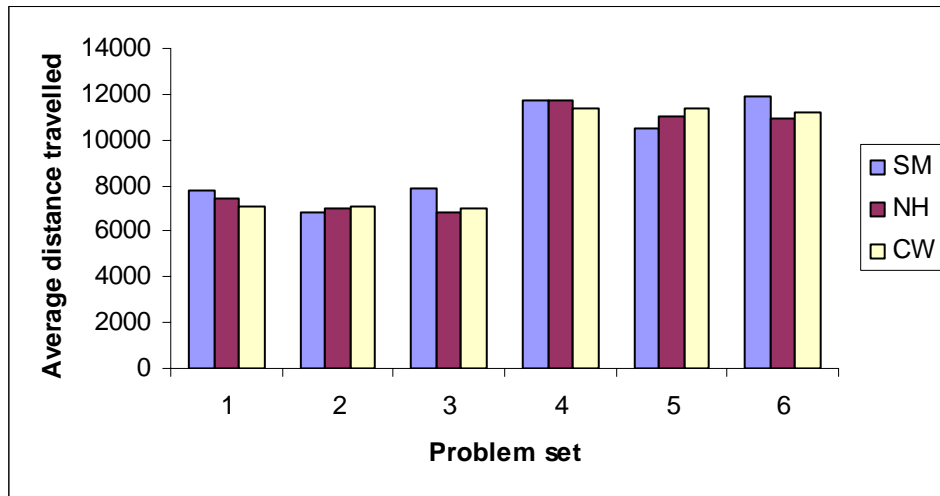


Fig. 1: Average distance travelled given by the heuristics for different problem sets

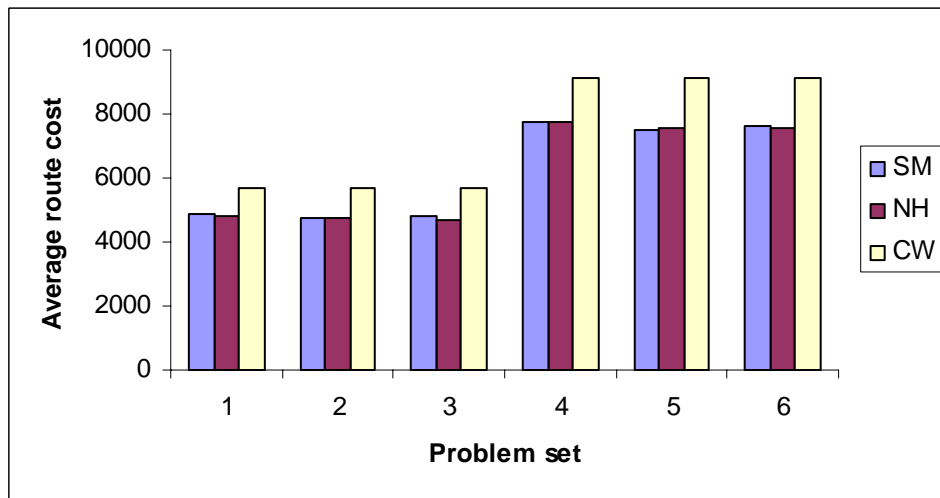


Fig. 2: Average route costs given by the heuristics for different problem sets