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# A Markov-based Diurnal Mobility Model for 3G Cellular Networks

**by**

**Samir K Sadhukhan**
SSA, IIM Calcutta, Diamond Harbour Road, Joka P.O., Kolkata 700 104 India

**Swarup Mandal**
Wipro Technologies Kolkata, India

**Partha Bhaumik**
Computer Sc. & Engg. Dept. Jadavpur University Kolkata, India

**&**

**Debashis Saha**
Professor, IIM Calcutta, Diamond Harbour Road, Joka P.O., Kolkata 700 104 India

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Samir K. Sadhukhan
IIM Calcutta, Joka, D. H. Road, Kolkata – 700104, India
samir@iimcal.ac.in

Swarup Mandal
Wipro Technologies, Kolkata, India
Swarup.mandal@wipro.com

Partha Bhaumik
Dept of Computer Science and Engineering, Jadavpur University, Kolkata, India
partha.bhaumik@gmail.com

Debashis Saha
MIS & Computer Sc group, IIM Calcutta, Joka, D. H. Road, Kolkata – 700104, India
ds@iimcal.ac.in

**Abstract.** User mobility is an important aspect for handoff planning in cellular networks because handoff depends on network topology (configuration of cells) and user mobility pattern. Mobility pattern largely depends on the user's behavior, geographical area, location, dwell time, direction and speed. While case studies are rare in public domain, most analytical and simulation studies adopt circular and hexagonal cell configuration. A pure random walk mobility model using hexagonal cells assumes that a user can move in all six directions from its current cell to its neighboring cells with equal probability (1/6) while that using circular cells assumes uniform distribution over some predefined fixed directions. However, such assumptions are not applicable in practical cases where users know their destination locations before the move. Such move is not purely random; rather it is somewhat direction-based and depends on present location and the destination of the user. For instance, a user often returns to the source when mobility is diurnal. In this paper we have studied direction-based diurnal mobility model of a user. We have formulated a Markov model for such movement and have given a theoretical upper bound of cell boundary crossings (expected number of handoffs) by the user. We also simulate the mobility environment and observe that a majority of complex and simple handoff comes from a small number of cells of the network.

**Keywords:** Network planning, Cellular network, Mobility model, Markov model, Direction-based, Diurnal.

## I Introduction

User mobility model is an important handle for planning and re-engineering of cellular networks, particularly in respect of handoff minimization. A reduction of number of handoff and better handoff management reduces the TCO of the network while improving the QOS. Since for cellular networks the main aspect of mobility is the handoff between cells, one is not particularly interested in every detail of the movement of a mobile terminal (MT). In the Random Walk Mobility Model [2], an MT moves from the current position to a new position by randomly choosing a direction and speed. The speed and direction are both uniformly distributed over a predefined range [MinSpeed, MaxSpeed] and $[0,2\pi]$ respectively. Each movement in a random Walk Mobility model occurs in either constant time interval t or a constant distance travelled d, at the end of which a new direction and speed are calculated [1]. The probabilities are often adjusted to practical observations of client behavior in cells. The Random Walk Mobility Model is one of the most widely used mobility models because it describes individual movements relative to cells [3]. Since this model is memory-less, there is no such concept as a path or consecutive movement. Therefore, MTs may stay in a vicinity of the starting cell for a rather long time. The Random Waypoint Mobility Model includes pause times between changes in direction and/or speed of the MT. An MT chooses a random destination and the distribution of speed is uniform over a predefined range of speed. The MT pauses for a specified time on reaching the destination. In Trace based models cellular operators keep records of mobility patterns of their users. These traces are a valuable source for the evaluation and improvement of handoff protocols. The only drawback is that usually such data is not publicly available and therefore cannot serve as benchmarks for the scientific community. In Fluid Flow Mobility Model, the individual MTs are modeled on a macroscopic level [3]. The Fluid-Flow Mobility Model represents movement of MTs in group on highways very well for cellular networks. The Fluid-Flow Mobility Model is not useful for individual movements including stopping and starting.

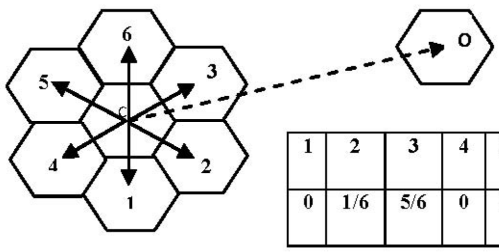
No one has attempted to estimate handoff for mobility driven network design that is needed by the operator. The required handoff estimation is based on some specific pattern such as diurnal mobility [4[[5]. No such model exists in the literature. If we use the generic mobility model the handoff estimation becomes far from the exact value leading to suboptimal result. In this case, our model is a perfect fit and it will provide more exact estimates of various handoffs. The existing generic model fails to capture the exact movement pattern of the users. Moreover, on-call handoff is less than non on-call handoff in the network. We have proposed a novel mobility model in this paper to estimate both on-call and non on-call handoffs while the user movements mimic our mobility model. Our direction-based mobility model is based on a range of mobility based parameters such as dwell time, location, destination, speed of an MT. In this model we have assumed that an MT is restricted to one of the two fixed directions based on its destination during the movement and can change the direction at the centre of the cell following a Markov model. Users rarely move in random speed and/or direction; rather they move towards some destinations in time synchronizing manner. The direction-based mobility model is useful to predict handoffs in the cellular network. In some cases, the mobility of MT is diurnal. In diurnal mobility, an MT starts from a source and move towards a destination and back to the source after a long dwell time in the destination cell (a diurnal tour). The diurnal mobility model is governed by the following assumptions: (1) a convex geographical region, (2) the distribution of sources and destinations over the convex region, (3) the distribution of initial speed of the MT, (4) The distribution of speed increment of the MT, (5) MTs are allowed to move from one cell to a neighbouring cell following Markov model based on source and destination, and (6) the distribution of long dwell time before MT starts from destination to source. We have theoretically calculated the number of handoffs on our proposed mobility model and verified the result in a simulated environment.

The paper is divided in five sections. Following introduction in section I, we have formulated a Markov model of direction-based mobility in section II. Some theoretical results are presented in section III. A simulation study is done in section IV. Section V concludes the paper.

## II Mathematical Model

Before we go into the mathematical model, let us consider a small example.

*Example:* In pure random mobility, a MT can move to any one of the six directions from its current cell with equal probability 1/6 to each direction. In direction-based movement, the probability distribution of direction for a MT is not uniform. Out of six directions, only two directions will contain the direction of movement from the current cell to the destination cell. Let $\theta_1$ and $\theta_2$ be the angles ($\theta_1 + \theta_2 = 60^0$) that is made by the two directions closest to the direction of the destination from the current cell. The probabilities of these two directions are set to $1-\theta_1/60$ and $1-\theta_2/60$ where as the probabilities of the other four directions are set to zero. In direction-based movement the probability distribution of direction depends on current cell and the destination of the MT.




| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| 0 | 1/6 | 5/6 | 0 | 0 | 0 |

Fig. 1. Probability distribution of direction of a MT in direction-based mobility at its current cell

Fig. 1 shows six directions 1 to 6 and the probability distribution of direction of a MT at its current cell. The direction of the destination is shown as O. The angles made by direction 3 and direction 2 with the direction O are assumed to be $10^0$ and $50^0$ respectively.

Fig. 2 shows that direction-based mobility without any obstacle, the two closest covering directions remains invariant across cells during the user movement.

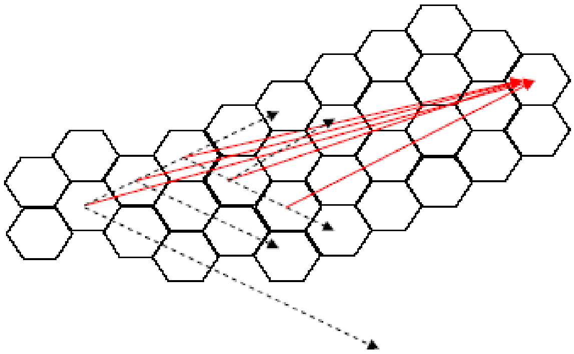


Fig. 2. In direction-based mobility a user will move either one of the two directions from any intermediate cell to reach the destination

Restricting a user movement in direction-based mobility to at most two invariant directions from current cell to its adjacent cells, a Markov model of cell transitions can be formulated. Fig. 3 shows a typical Markov model on direction-based mobility of an user from current cell (0,0) to the destination cell (2,1).

To formulate the transitional probabilities from one cell to its adjacent cells, let us consider C is the centre of the current cell and O is the centre of the destination cell of the MT. Let 1, 2, … 6 be the six directions of possible movements of an MT from any cell.

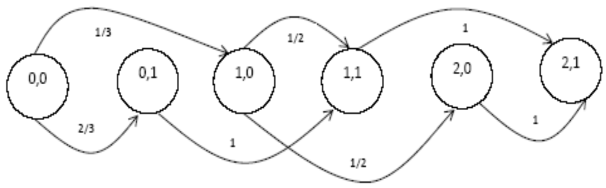



Fig. 3. Markov Model of direction-based mobility from (0,0) to (2,1)

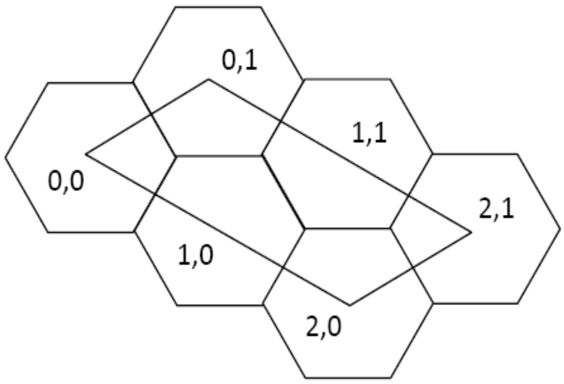



Fig. 4. MT moves within a parallelogram formed by the diagonal joining source to destination of MT during direction-based mobility model

Without loss of generality, we can assume that direction 1 and direction 2 from C are the closest covering of $\overrightarrow{CO}$ and assume C is the origin and the direction 1 and direction 2 are the positive direction of x and positive direction of y respectively inclined at an angle $60^0$. With this frame of reference, let (α,β) be the co-ordinates of O.

Using the Manhattan distance, the transition probability from cell $(i_1,j_1)$ to the neighboring cell $(i_2,j_2)$ in forward mobility (towards O) is

$$p_{(i_1,j_1),(i_2,j_2)} = \frac{(\alpha-i_1)(i_2-i_1)+(\beta-j_1)(j_2-j_1)}{\alpha-i_1+\beta-j_1} \text{ ----------- (1)}$$

Here, $i_1 \leq i_2 \leq \alpha$, $j_1 \leq j_2 \leq \beta$. The number of states will be (α+1)(β+1) in the Markov Model. As per our markov model and transition probability defined in direction-based mobility, the destination O is reachable from C with probability 1 in $\propto +\beta$ transition steps.

*Example:* In Figure 3, Markov model of an MT on direction-based mobility from source cell (0,0) to destination cell (2,1) is shown. There are (2+1)*(1+1) = 6 states in the model. Table 1 shows the 6 x 6 transition matrix P of the Markov model. The transition probabilities in P are calculated using the formula (1) above. Table 2 shows that the MT will reach the destination in 2+1=3 transition steps from (0,0) to (2,1) at its steady state.

Table 1: The transition matrix P of the Markov Model

| | (0,0) | (0,1) | (1,0) | (1,1) | (2,0) | (2,1) |
|---|---|---|---|---|---|---|
| (0,0) | 0 | 1/3 | 2/3 | 0 | 0 | 0 |
| (0,1) | 0 | 0 | 0 | 1 | 0 | 0 |
| (1,0) | 0 | 0 | 0 | ½ | ½ | 0 |
| (1,1) | 0 | 0 | 0 | 0 | 0 | 1 |
| (2,0) | 0 | 0 | 0 | 0 | 0 | 1 |
| (2,1) | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2: The 3-step transition matrix $P^3$

| | (0,0) | (0,1) | (1,0) | (1,1) | (2,0) | (2,1) |
|---|---|---|---|---|---|---|
| (0,0) | 0 | 0 | 0 | 0 | 0 | 1 |
| (0,1) | 0 | 0 | 0 | 0 | 0 | 0 |
| (1,0) | 0 | 0 | 0 | 0 | 0 | 0 |
| (1,1) | 0 | 0 | 0 | 0 | 0 | 0 |
| (2,0) | 0 | 0 | 0 | 0 | 0 | 0 |
| (2,1) | 0 | 0 | 0 | 0 | 0 | 0 |

# III Analysis

Theorem 1: If $(i_1,j_1)$ and $(i_2,j_2)$ be the coordinate of the source cell and destination cell respectively, then the number of handoffs of an MT in a direction-based mobility is be the Manhattan distance between the cells.

Proof: In the direction-based mobility every transition of an MT to its neighbouring cell reduces the Manhattan distance of destination by 1 from its current cell. Thus, a handoff count of the MT is increased by 1. When the MT reaches to its destination cell the handoff count reaches the maximum which is equal to Manhattan distance from its source to the destination.

Theorem 2: The number of cells visited by the MT is one more than the Manhattan distance from its source cell to the destination cell.

Proof: Since the MT is not visiting a cell twice during the direction-based mobility and every cell transition takes the MT closer to its destination cell, the number of cells visited by the MT is one more than the Manhattan distance between source cell and destination cell.

Theorem 3: In direction-based mobility of MT, the probability of transition to destination cell from its immediate previous cell is 1.

Proof: Just before the final transition, the MT is one cell away from the destination. The closest covering two directions from the previous cell will coincide with the line joining the centres of the cells. As per the expression of transition probability, both denominator and numerator will be one. Hence the transition probability to the destination is 1.

Theorem 4: If $(x_1,y_1)$ and $(x_2,y_2)$ be the cell coordinates of the source cell and destination cell respectively, then the number of cells with transition probability 1 happens in $|x_2 - x_1| + |y_2 - y_1|$ cells in possible region formed by the parallelogram formed from the diagonal joining $(x_1,y_1)$ and $(x_2,y_2)$.

Proof: Without loss of generality we assume $x_2 \geq x_1$ and $y_2 \geq y_1$. The Markov model will contain cells $(x_1,y_2)$, $(x_1+1,y_2)$ … $(x_2-1,y_2)$ with transition probability 1. Similarly the cells $(x_2,y_1)$, $(x_2,y_1+1)$,… $(x_2,y_2-1)$ will have transition probability 1. So the total number of cells with transition probability 1 in the direction-based mobility Markov model will be $|x_2 - x_1| + |y_2 - y_1|$.

Theorem 5: For the direction-based diurnal mobility from source cell to destination cell, the forward mobility and backward mobility contains exactly same set of cells in the Markov model but the forward and backward transition probability distributions differ for a cell.

Proof: The line joining from source cell to destination cell and the line joining destination cell to source cell coincides but opposite in direction. The two closest covering directions of this line from source cell is parallel and opposite to the closest covering directions of the line from destination cell. So the probability distribution of the direction in forward mobility is not same as probability distribution of direction in backward mobility for a cell. The Manhattan distance from source to destination is equal to the Manhattan distance from destination to source.
If ($x_1$,$y_1$) and ($x_2$,$y_2$) be the coordinate of the source cell and destination cells, then the number of cells in the Markov model is equal to $(|x_2 - x_1| + 1) * (|y_2 - y_1| + 1)$. Thus the proof is complete.

Theorem 6: The path traced by the MT during forward mobility may not be same as the path traced during backward mobility.

Proof: Clear from Theorem 5.

Theorem 7: The transition probability from source to destination under direction-based mobility is 1.

Proof: If ($x_1$,$y_1$) and ($x_2$,$y_2$) be the coordinate of the source cell and destination cell, then the number of cells in the Markov model is equal to $(|x_2 - x_1| + 1) * (|y_2 - y_1| + 1) = n\ (say)$. The path length between the source and destination is equal to the Manhattan distance between the cells is $|x_2 - x_1| + |y_2 - y_1| = m\ (say)$. Let P be the n x n initial transition matrix of the Markov model having n cells. Then in Pm the transition probability of source cell to destination cell will be 1 as per the Markov model and transition probability as defined.

Theorem 8: The Manhattan distance is the upper bound on the Euclidian distance between two cells.

Proof: Let ($x_1$,$y_1$) and ($x_2$,$y_2$) be the coordinate of the two cells. Then the Euclidian distance between the cells is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (x_2 - x_1)(y_2 - y_1)} \leq \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + 2(x_2 - x_1)(y_2 - y_1)} = |x_2 - x_1| + |y_2 - y_1|$, the Manhattan distance between ($x_1$,$y_1$) and ($x_2$,$y_2$).

## IV Simulation

We divide a rectangular area into multiple hexagonal cells created using a non orthogonal cartesian system inclined at 600. Each cell has exactly six neighbour cells except the boundary cells that have less than six neighbour cells. For each configuration of the network is represented by the triplet (MSC-RNC-NodeBs) specifying number of MSCs, number of RNCs and Number of NodeBs in the network. After the creation of the above synthetic cell structure, a specified set of residential locations and office locations (80% residential location and 20% office location) are assigned randomly in some cells. Users are assigned direction-based diurnal mobility in the network. For each NodeB, the number of handoff and the handoff type is recorded. Figure 5 shows the percentage of NodeBs having 80% of total handoff for each type. It is seen that with direction-based diurnal mobility, 80% of complex handoff occurs in 15% to 20% of the cell in the network while 80% of the simplest handoff occurs 40% to 50% of the cells in the network. Figure 6 shows the actual number of on-call handoff occurs during simulation with 100 to 200 MTs under direction-based mobility under different network configuration. The theoretical upper bound of the number of handoff is calculated following Theorem 1. It is observed that actual number on-call handoff is always less than its theoretical bound.

## V Conclusions

The paper has considered direction-based diurnal user mobility with hexagonal cell configuration and formulated a Markov mobility model. The cell transition probabilities are defined using Manhattan distance. We have simulated a diurnal mobility pattern and estimated the number of handoffs in the network. It is revealed that for uniform distribution of office & home location of the users, not all cells encountered similar handoff both in number and its type. Majority of the handoff occurs in small number of cells in the network. A more general mobility pattern is possible for a diurnal tour by setting a chain of destination locations before it comes back to the source. For further details of movement, one can consider the direction-based movement of MT within a parallelogram formed by the diagonal joining source to destination of MT in Figure 4. Since, we are interested in measuring handoffs in a cellular network we need not consider obstacles across cell boundaries. However we have not considered any freeway or way point in our geographical area. These can be overlaid on the direction-based diurnal mobility model for further investigation.

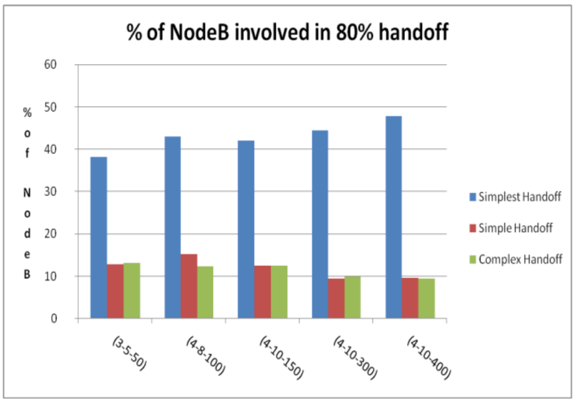



Fig. 5. % of cells showing different handoff types under direction-based diuranal mobility model.

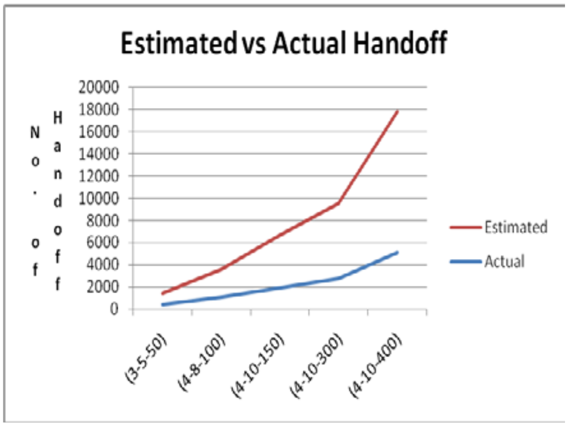



Fig. 6. Actual number of on-call handoff vs. Theoritical Estimate.

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