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Financially Constrained Limited Clearance Sale Inventory Model

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Abstract

Recent rise in bank borrowing by retailers indicates that they are cash constrained and are unable to purchase their optimal order quantity on their own. The retailer's problem is further complicated by end-of-season markdown. If the cost of borrowing is high, then a retailer is better off by not borrowing at all. In case of markdown, a retailer with high leftover inventory is better off by selling a limited portion of her leftover inventory and disposing the remaining for free. Therefore, a retailer's optimal ordering decision has to strike a balance between these two aspects while facing an uncertain market demand. In this paper, we address these issues by modeling limited clearance sale inventory in the presence of financial constraint to determine optimal order quantity of a retailer. We show that financial constraint enables a retailer to earn higher profit when the market demand is less than her optimal order quantity. Subsequently, we design channel coordination mechanisms for a financially constrained supply chain using buyback and revenue-sharing contracts. The supplier can design either of these mechanisms only if the retailer shares the information about her own equity and borrowing interest rate with the supplier.

Keywords:

Financially constrained; coordination; contracts; limited clearance sale inventory

1 Introduction

In order to manage uncertainty due to stochastic demand, retailers of short life cycle products procure sufficient stock to protect against variation in market demand from its mean. While large corporations are able to adequately fund their inventory (Dada and Hu (2008)), small and medium sized enterprises (SMEs) and startup firms are particularly constrained by their working capital resulting in sub-optimal and hindered operational decisions (Bosma et al. (2004), Bastié et al. (2013)). In 2015 *Kiddyum*, a small firm from Manchester, won a contract to supply ready-meals to Sainsbury's, a large British supermarket chain[1]. Initially Kiddyum struggled with her cash flow due to Sainsbury's 60 day payment cycle; subsequently, Kiddyum entered into an agreement with Royal Bank of Scotland that facilitated early payment of bills in return for a small fee payment[2].

In absence of early payment mechanism, firms often opt for borrowing money through loan to finance for the gap between her production and payment (Dada and Hu (2008); Besbes et al. (2017)). Otherwise, cash or budget constraint might force a firm to stock less than her optimal stocking quantity leading to a possibility of lost sales (Buzacott and Zhang (2004)). In order to avoid situation of under-stocking, some firms opts for debt-financed inventory. Pharmacy sector startup firm *Pharmeasy* has recently raised INR 40 crore (5.6 million USD) of debt for working capital and strengthening inventory[3]. However, such debt financed inventory poses further problem for a firm if market demand is weak and leftover inventory is required to be cleared through a clearance sale (Avittathur and Biswas (2017)). Small-cap outsourcing firm of consumer electrical and appliances[4] - *Amber Enterprises* is attempting to trim her exposure to debt funds while the company has experienced inventory build-up due to weak market demand[5]; the company is expecting to gradually liquefy her

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[1]"Technology is revolutionising supply-chain finance" (Oct 12, 2017), The Economist, Retrieved from: https://www.economist.com/finance-and-economics/2017/10/12/technology-is-revolutionising-supply-chain-finance, Accessed on: Sept 13, 2018

[2]ibid.

[3]Variyar, M. (Sept 12, 2018) "Pharmeasy raises Rs 40 cr in debt", Economic Times, Retrieved from: https://economictimes.indiatimes.com/small-biz/startups/newsbuzz/pharmeasy-raises-rs-40-cr-in-debt/articleshow/65778944.cms, Accessed on: Sept 13, 2018

[4]Oberoi, R. (Sept 11, 2018) "Smallcap watch: 2 stocks that have been big draw among mutual funds", Economic Times, Retrieved from: https://economictimes.indiatimes.com/markets/stocks/news/smallcap-watch-2-stocks-that-have-been-big-draw-among-mutual-funds/articleshow/65766512.cms, Accessed on: Sept 13, 2018

[5]Karwa, K. (Aug 10, 2018), "", Moneycontrol, Retrieved from: https://www.moneycontrol.com/news/business/amber-

unsold inventory during second and third quarter of fiscal year 2018-19[6].

In these instances we observe that cash constrained retailers face challenge of designing an optimal ordering policy so that her loss due to over-ordering and subsequent clearance sale is minimized. Retailers often adopt limited clearance sale strategy to maximize her revenue from clearance sale and reduce loss from over-ordering (Avittathur and Biswas (2017)). Extant literature on ordering policy of budget constrained retailer does not incorporate the effect of complete or limited clearance sale (Dada and Hu (2008), Wuttke et al. (2016); Besbes et al. (2017)). Since retailers of short life cycle products have to place their order at the beginning of selling season without any knowledge of market demand, avoiding situation of over-ordering is almost impossible (Biswas and Avittathur (2018)). This observation raises a pertinent business question: how should a retailer, who follows limited clearance sale inventory (LCSI) to maximize her clearance revenue, design her ordering policy when she faces financial constraint? In this paper we design financially constrained (FC) LCSI model to determine optimal ordering policy of a retailer. We further design optimal wholesale price, buyback, and revenue-sharing contracts and investigate whether a FC retailer can attain channel coordination.

§2 reviews the literature relevant to this research. §3 describes financially constrained limited clearance sale inventory model. We also compare and contrast our model with those from extant literature to highlight our contribution. In §4 we discuss modeling of supply contracts for financially constrained limited clearance sale inventory model and conditions under which they achieve channel coordination. We discuss managerial implication of the model, highlight limitations of current research, and conclude in §5.

2   Literature Review

Ordering policy under financial constraint has recently started to gain attention in supply chain literature (Buzacott and Zhang (2004), Caldentey and Haugh (2009), Feng et al.

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enterprises-a-good-buy-despite-a-weak-q1-2827831.html, Accessed on: Sept 13, 2018
    [6]ibid.

(2015)). Though supply chain agents might face financial constraint, most of the key constructs are grounded in optimization of unconstrained problem (Dada and Hu (2008)). In the context of unconstrained ordering problem of a supply chain, Cachon (2003) provides a comprehensive review of channel coordination mechanisms. In this section, we first review the extant literature on ordering policy of financially constrained retailer using newsvendor model. Subsequently, we review the related literature on channel coordination strategy.

2.1 Ordering policy of a financially constrained retailer

Buzacott and Zhang (2004) have modeled per period available cash as a function of assets and liabilities of a firm. Based on dynamics of the production activities, valuation of assets and liabilities are also updated periodically. They demonstrate that the growth potential of firm is primarily constrained by her limited capital and dependence on bank financing. Dada and Hu (2008) have specifically looked into a firm's decision to finance her inventory by a bank. They show that a lender's interest rate decreases in the retailer's equity. Caldentey and Haugh (2009) have considered a dyadic supply chain with a FC retailer and a manufacturer. They prove that profitability of the supply chain increases as the retailer's constraint becomes binding. Raghavan and Mishra (2011) have analyzed short-term financing in a cash-constrained dyadic supply chain. They investigate a lender's decision to jointly finance the supplier as well as the retailer; they demonstrate that if one firm is severely cash constrained, then joint financing improves supply chain performance. Serel (2012) investigates quick response strategies of a supply chain with FC retailer. Financial constraint leads to higher initial order for a retailer. With decrease in available budget, a retailer places higher order for products with predictable demand. Moussawi-Haidar and Jaber (2013) integrate cash management and inventory lot sizing problems to analyze *optimal operational (how much to order and when to pay the supplier) and financial decisions (maximum cash level and loan amount)*. They demonstrate that with increase in retailer's return on cash, the optimal order quantity decreases. Jing and Seidmann (2014) compare between bank credit and trade credit for a dyadic supply chain with a supplier and a FC retailer. They

observe that in presence of limited liability a FC retailer's optimal order is equal to that of a retailer with no financial constraint. Feng et al. (2014) investigate a purchasing contract with options for a FC retailer. They identify retailer's optimal ordering strategy with limited capital and show a retailer's fixed order increases with bank financing. Jiang and Hao (2014) consider a dyadic supply chain model with a FC supplier and a manufacturer. They demonstrate that if the manufacturer offers an advance payment to pre-order from the supplier then it provides the supplier with insulation from capital restriction. Ni et al. (2017) analyze agency problem effect in context of a FC retailer and find that financial constraint restricts a retailer's optimal capacity decision, as borrowing rate increases with risk of default. Xiao and Zhang (2017) considered a cash constrained manufacturer who resorts to pre-selling to generate cash for production and then based on the on-hand cash, she decides her borrowing amount and production quantity. Jin et al. (2018) analyze, compare, and contrast between collaborative (bank financing with trade credit and bank financing with supplier's guarantee) and non-collaborative financing strategies (separate bank financing of budget constrained supplier and retailer). They show that collaborative financing strategies are dominating non-collaborative strategy for the supplier as well as the overall supply chain and the reverse logic holds for the retailer.

Most of these aforementioned studies either do not consider clearance sale (Buzacott and Zhang (2004), Dada and Hu (2008), Caldentey and Haugh (2009), Moussawi-Haidar and Jaber (2013), Jing and Seidmann (2014), Feng et al. (2014), Ni et al. (2017), Xiao and Zhang (2017), Jin et al. (2018)) or consider a fixed salvage value clearance sale (Raghavan and Mishra (2011), Serel (2012)). However, in the case of a FC supply chain clearance sale plays a crucial role in ordering policy and overall supply chain performance. Caldentey and Haugh (2009) have considered that FC retailer sells her product at a stochastic clearance price. Besbes et al. (2017) have shown that inventory financing through debt induces retailer to charge higher prices and offer slow discounting on products. They also indicate that these distortions lead to loss in revenue over time and inefficient supply chain performance. In this paper we investigate the influence of clearance on ordering policy of a FC retailer in further

detail by incorporating LCSI model as proposed by Avittathur and Biswas (2017).

In the next subsection, we review the relevant literature on channel coordination strategies for a FC supply chain.

2.2 Channel coordination strategy

Channel coordination strategies for a budget constrained supply chain can be further classified as: (i) coordination through flexible loan repayment (Dada and Hu (2008), Jing and Seidmann (2014), Feng et al. (2014)) and i(ii) coordination through supply contract design (Feng et al. (2015), Kouvelis and Zhao (2015), Xiao et al. (2017), Cao and Yu (2018)). First, we briefly review channel coordination through flexible loan repayment. Second, we review coordination through supply contract design as in this paper we focus on supply contract design for a FC retailer.

Dada and Hu (2008) have designed a channel coordinating non-linear loan schedule. Caldentey and Haugh (2009) analyze flexibility contract in the context of FC retailer. They conclude that a manufacturer always prefers flexible contract with hedging to that without hedging and a retailer's contract preference is dependent on model parameters. Lee and Rhee (2011) model a supplier's trade credit offer along with markdown allowance where the retailer decides the order quantity quantity and chooses the financing option between external debt or trade-credit. They have analyzed channel coordination effect of trade credit from a supplier's perspective. Jing and Seidmann (2014) demonstrate that trade credit is more effective mechanism compared to bank credit for eliminating double marginalization in a FC dyadic supply chain. Feng et al. (2014) analyze a purchasing contract with options for a FC retailer and prove that the retailer's profit increases with bank financing. Yang and Birge (2017) analyze *risk-sharing role of trade credit* to understand efficiency enhancing property of trade credit when a retailer partially shares her demand risk with the supplier. However, their work does not suggest any channel coordination mechanism.

We review related supply contract literature here. Feng et al. (2015) design a channel coordinating revenue sharing and buy back (RSBB) contract for a financially constrained

dyadic supply chain as channel coordination through revenue sharing and buyback contracts is not always feasible. Kouvelis and Zhao (2015) study contract design and coordination of a supply chain where both supply chain agents are cash constrained. In this context, they analyze revenue-sharing, buyback, and quantity discount contracts and demonstrate that quantity discount contract fails to coordinate the overall supply chain. In presence of default cost, revenue-sharing contract coordinates the overall supply chain. Chen (2015) analyzes wholesale price and revenue sharing contracts under manufacturer's trade credit financing and bank financing. In case of bank credit financing of the retailer, a channel coordinating revenue-sharing contract behaves identical to that of a retailer without capital constraint. However, Chen (2015) does not consider clearance sale of leftover inventory in the proposed model. In a dyadic supply chain, Xiao et al. (2017) show that a preselling-based incentive scheme motivates the manufacturer to increase his production quantity and to coordinate the supply chain. Xiao et al. (2017) further observe that one crucial element for channel coordination is a bidirectional compensation scheme in which all supply chain agents compensates each other for unsold items. Cao and Yu (2018) demonstrate that a dyadic supply chain with a FC retailer can be coordinated through quantity discount contract, revenue sharing contract and buyback contract. They also comment that a channel coordinating revenue sharing contract allows a FC retailer to earn more profit compared to an unconstrained retailer.

From the review of extant literature we observe that though clearance sale plays a crucial role in channel coordination of a FC supply chain (Besbes et al. (2017), Xiao et al. (2017)). To the best of our knowledge, analysis of channel coordinating supply contracts in presence of limited clearance sale for a FC retailer has not been done so far. In this paper we address this gap by analyzing three supply contracts for a FC retailer who adopts limited clearance sale strategy for her leftover inventory. In the next section, we develop the analytical model for a FC retailer with LCSI strategy and contrast it with existing newsvendor frameworks to clearly indicate our contribution.

3 Financially Constrained Limited Clearance Sale Inventory Model

In this section we first describe the financially constrained limited clearance sale inventory (FC-LCSI) model. We discuss different salient aspects of the proposed model and also indicate how the proposed model is different from classical newsvendor model. Subsequently we discuss the mathematical formulation of the same.

3.1 Model description and comparison with existing frameworks

We model FC-LCSI problem with two periods: normal selling period (T_1) and clearance-sale period (T_2). During the normal selling period, good is sold at an exogenous retail price, p, and either complete or limited amount leftover inventory is sold during clearance-sale period at an endogenous salvage price, $v(i)$, where i represents the leftover inventory after sale during T_1. At the beginning of T_1 the LCSI retailer[7] orders her quantity q and procures the same at a unit cost c. During T_1, the unit retail price p is set by the retailer such that $p > c$ (Dada and Hu (2008), Avittathur and Biswas (2017)). In LCSI framework, the clearance price v is decided at the end of period T_1 after observing leftover inventory i (Avittathur and Biswas (2017), Biswas and Avittathur (2018)). The objective of the LCSI model is to maximize the expected profit $E[\pi_{LC}(q)] = E[R_{T1}(\cdot)] + E[R_{T2}(\cdot)] - cq$ where $E[R_{T1}(\cdot)]$ and $E[R_{T2}(\cdot)]$ are the expected revenues in periods T_1 and T_2 respectively. Since in our case the LCSI retailer is also financially constrained, we further assume that she does not have sufficient capital to purchase her optimal fractile quantity q^* at cost cq^* where q^* represents the optimal order quantity for classical LCSI model without financial constraint. As a result, the retailer resorts to borrowing additional capital $B = cq - \eta$ from a bank at an interest rate r, where η designates the initial capital available with the retailer. At the end of period T_1 the retailer has to repay the bank with an amount $(1+r)B$. For the purpose of expositional simplicity, we do not consider the salvage revenue generated in period T_2 for the purpose of loan repayment since at the beginning of period T_1, the retailer cannot estimate either her

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[7]A LCSI retailer is one who sells her leftover quantity i during clearance-sale period T_2 following limited clearance sale principle as discussed in Avittathur and Biswas (2017) and Biswas and Avittathur (2018).

expected leftover quantity or the related salvage revenue. Chronological sequence of events is presented in figure 1.

Figure 1: Chronological sequence of events for a FC-LCSI retailer
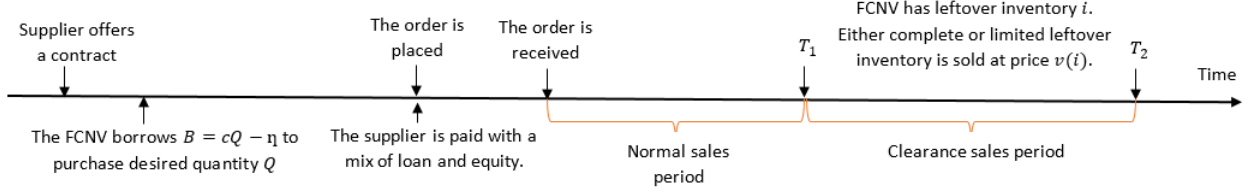
After incorporating the aspect of financial constraint in our model, we present the generalized optimization problem of a FC-LCSI retailer as follows:

$$q_C^* = \max_q E[\pi_{LC}(q)] = \max_q \{E[\pi_{T1}(q)] + E[R_{T2}(\cdot)]\} \tag{1}$$

$$subject\ to,\ \eta \le cq \tag{2}$$

where $E[\pi_{T1}(q)]$ and $E[R_{T2}(\cdot)]$ represent the expected profit of the retailer in period T_1 and expected limited clearance revenue in period T_2 respectively. In Table 1 we clearly present the differences and similarities between LCSI and newsvendor models in the presence of financial constraint. LCSI model has been developed by Avittathur and Biswas (2017). In this paper we extend their proposed LCSI model with incorporation of financial constraint.

Table 1: Comparison of limited clearance sale inventory and newsvendor frameworks

		Without Financial Constraint		With Financial Constraint	
		Newsvendor Model	LCSI Model	FC-NV Model	FC-LCSI Model
Indicative scholarly work		Arrow et al. (1951) Cachon (2003)	Avittathur and Biswas (2017)	This Paper	This Paper
Decision Parameters	Beginning of Period T_1	Order Quantity	Order Quantity	Order Quantity, Borrowing Amount	Order Quantity, Borrowing Amount
	End of Period T_1	None	Clearance Price, Clearance Sale Inventory	None	Clearance Price, Clearance Sale Inventory
Model Similarities	Exogenous input parameters	(a) Retail price of normal selling period T_1 (b) Salvage value (c) Marginal cost of production	(a) Retail price of normal selling period T_1 (b) Marginal cost of production	(a) Retail price of normal selling period T_1 (b) Salvage value (c) Marginal cost of production (d) Rate of Interest	(a) Retail price of normal selling period T_1 (b) Marginal cost of production (c) Rate of Interest
	Demand during T_1	Known stochastic distribution	Known stochastic distribution	Known stochastic distribution	Known stochastic distribution
Model Differences	At the beginning of period T_1	The retailer has no financial constraint for ordering her optimal quantity, Q^*	The retailer has no financial constraint for ordering her optimal quantity, Q^*	(a) The retailer has financial constraint, she cannot order her optimal quantity, Q^*, on her own. (b) She may take a bank loan (of amount B) to reach her optimal ordering level.	(a) The retailer has financial constraint, she cannot order her optimal quantity, Q^*, on her own. (b) She may take a bank loan (of amount B) to reach her optimal ordering level.
	Inventory disposal decision at the end of period T_1.	Entire leftover inventory is salvaged at an exogenous fixed salvage value.	Inventory up to some level is cleared at clearance price, rest is disposed off at zero salvage value.	Entire leftover inventory is salvaged at an exogenous fixed salvage value.	Inventory up to some level is cleared at clearance price, rest is disposed off at zero salvage value.
	Clearance price (in LCSI) and salvage value (in newsvendor)	Salvage value is known at beginning of T_1 based on retailer's past experience.	Clearance demand is a linear function of clearance price. This knowledge is based on retailer's past experience.	Salvage value is known at beginning of T_1 based on retailer's past experience.	Clearance demand is a linear function of clearance price. This knowledge is based on retailer's past experience.
Supply contracts analyzed		(a) Wholesale price (b) Buyback (c) Revenue sharing (d) Quantity discount (e) Sales rebate	(a) Wholesale price (b) Buyback (c) Revenue sharing (d) Sales rebate	(a) Wholesale price (b) Buyback (c) Revenue sharing	(a) Wholesale price (b) Buyback (c) Revenue sharing

Note: **FC:** Financially Constrained, **NV:** Newsvendor, **LCSI:** Limited clearance sale inventory

The demand during normal selling period (T_1) is represented by, x, and it is distributed over $[0, q_{max}]$. The demand is assumed to follow an increasing generalized failure rate (IGFR) distribution. The probability distribution and cumulative distribution functions of demand are represented by $f(\cdot)$ and $F(\cdot)$ respectively. We further assume the following: (i) $f(\cdot)$ and $F(\cdot)$ are differentiable over the entire range of demand $[0, q_{max}]$, (ii) $F(\cdot)$ is strictly increasing over $[0, q_{max}]$, and (iii) the boundary conditions of the distribution are: $F(0) = 0$ and $F(q_{max}) = 1$.

We assume that the clearance sale period demand[8] (d) is a function of the clearance price v. As demonstrated by Avittathur and Biswas (2017) and Biswas and Avittathur (2018), during period T_2 LCSI retailers set a higher clearance price, sell one portion of their leftover inventory, and dispose off remaining inventory at zero salvage value through product bundling. They particularly use this strategy for clearing large quantity of leftovers. Under such circumstances, *it is appropriate to model this scenario using linear demand function as in a linear demand function price elasticity is not constant and it is a function of clearance price itself* (Biswas and Avittathur (2018)). Clearance sale demand is represented by an inverse demand function: $v = a_v - b_v d$, where a_v is the maximum permissible price that a retailer can charge for a product in T_2, such that $0 \leq a_v \leq p$ and b_v is the sensitivity of price to the demand, such that $b_v \geq 0$. a_v and b_v are exogenous to our model and a LCSI retailer holds prior estimates of these parameters based on her past experiences of clearance sales. Therefore, we can express the clearance-sale revenue as follows: $R_{T2}(d) = a_v d - b_v d^2$. From the first-order condition of $R_{T2}(d)$ we observe that the clearance-sale revenue is maximized at a demand level, $s = a_v/2b_v$. As a result, in LCSI model any demand greater than s should not qualify for clearance sales and should be disposed off at a salvage value of zero. We can further note over here that, for $b_v = 0$ clearance price is constant at the value a_v and the retailer behaves like a newsvendor. In the next section we discuss the mathematical formulation of financially constrained LCSI model.

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[8]In LCSI model, relation clearance sale period demand (d) and leftover inventory (i) is as follows: $d \leq i$.

3.2 Mathematical formulation

We develop the framework for financially constrained LCSI retailer by combining LCSI framework (Avittathur and Biswas (2017)) with capital constrained newsvendor framework (Dada and Hu (2008)). During normal selling season T_1, the retailer's decision parameters are: (i) order quantity (q) and (ii) borrowing amount (B). During T_1, if the retailer's optimal stocking decision is presented by q^* then her optimal borrowing amount is: $B(q^*) = cq^* - \eta$. In order to ensure repayment of loan, the retailer has to at least sell quantity y during T_1; we can represent this minimum required quantity as follows: $y = (1+r)B/p = (1+r)(cq-\eta)/p$. After loan repayment, the retailer's expected profit in T_1 is expressed as follows:

$$E[\pi_{T1}(q)] = -\{\eta + (1+r)(cq - \eta)\overline{F}(y)\} + p\left(\int_y^q xf(x)dx + q \int_q^{q_{max}} f(x)dx \right) \quad (3)$$

where, $\overline{F}(y) = 1 - F(y)$. At the end of period T_1, the leftover inventory (i) of the retailer is given by: $i = (q - x)^+$. In clearance sale period T_2, the retailer's decision parameter is: z, where z represents the portion of leftover inventory i that the retailer decides to put up for clearance sales at a clearance price $v(z)$. From the discussion in §3.1, we can understand the following: (i) if $i = (q - x)^+ > s = a_v/2b_v$, the retailer will put up $z = s = a_v/2b_v$ amount of inventory for sale at a clearance price $v(z) = a_v/2$ and (ii) if $i = (q - x)^+ \leq s = a_v/2b_v$, the retailer will put up $z = i$ amount of inventory for sale at a clearance price $v(z) = a_v - b_v z = a_v - b_v i$. We additionally define the term $j = (q - s)^+$. Using these definitions we express three scenarios of leftover inventory, nature of clearance-sale, clearance-sale quantity and clearance price for different normal season demand in Table 2.

We also observe here: if $q_{max} \leq s = a_v/2b_v$, then the order quantity q at the beginning of T_1 and leftover inventory i at the end of T_1 would be always less than s. Under such circumstances, a limited clearance-sale situation would never arise. Therefore, we assume $q_{max} > s$ throughout our model. From Table 2 we can calculate the retailer's expected

Table 2: Nature of clearance sale and clearance quantity during period T_2

Demand (x) in period T_1	Leftover inventory (i) at the end of T_1	Nature of clearance sale	Quantity (z) to be sold during clearance sale period T_2	Clearance price $(v(z))$	Stock to be disposed off at zero clearance price
$j > x \geq 0$	$i = q - x > s$	Limited	$z = s$	$v(z) = a_v/2$	$i - s$
$q > x \geq j$	$s \geq i = q - x > 0$	Complete	$z = i$	$v(z) = a_v - b_v i$	–
$x \geq q$	–	Absent	$z = 0$	–	–

Note: This table is adapted from Avittathur and Biswas (2017).

revenue in period T_2 and this limited clearance sale revenue is expressed as follows:

$$E[R_{T2}(\cdot)] = (a_v - 2b_v q) \int_j^q F(x)dx + 2b_v \int_j^q xF(x)dx \qquad (4)$$

Derivation of equation 4 is presented in the appendix. Using the expressions of $E[\pi_{T1}(q)]$ and $E[R_{T2}(\cdot)]$ from equations (3) and (4) we calculate the expected profit function $E[\pi(q)]$ of a financially constrained LCSI retailer. Along with the financial constraint, as presented in equation (2), the optimization problem of a FC-LCSI retailer is rewritten as follows:

$$q_C^* = \max_q \left[-\{\eta + (1+r)(cq - \eta)\overline{F}(y)\} + p\left(\int_y^q xf(x)dx + q \int_q^{q_{max}} f(x)dx \right) + \right.$$

$$\left. (a_v - 2b_v q) \int_j^q F(x)dx + 2b_v \int_j^q xF(x)dx \right] \qquad (5)$$

$$subject\ to,\ q - \frac{\eta}{c} \geq 0 \qquad (6)$$

We observe from equations (5) - (6) that this optimization problem of a LCSI retailer is equivalent of the optimization of a centralized supply chain consisting of one supplier and one retailer where the retailer implement LCSI in period T_2. In the next section we present necessary and sufficient conditions for optimality for financially constrained LCSI model along with uniqueness of the solution.

3.3 Optimal solution and proof of uniqueness

Compared to the formulation of classical newsvendor problem where a retailer does not have any financial constraint, in our framework of financially constrained LCSI retailer is different in following aspects: (i) the term $(1+r)(cq-\eta)\overline{F}(y)$ represents the amount which is to be paid back to the bank along with its probability, (ii) the lower limit, y, of the integral captures the revenue which is in excess of the required payback amount, (iii) the term η represents the procurement that is financed by the retailer's own equity, and (iv) the last three terms of equation (5) represent the salvage revenue captured by the retailer in period T_2 through limited clearance sale. Our formulation of FC-LCSI problem extends the understanding of the retailer's problem considered by both Dada and Hu (2008) and Avittathur and Biswas (2017).

In presence of financial constraint and stochastic retail demand, Dada and Hu (2008) and Buzacott and Zhang (2004) have demonstrated that if the demand distribution has increasing failure rate (IFR) then the solution for optimal order quantity can be fully characterized by Karush–Kuhn–Tucker (KKT) conditions. We also use KKT conditions to characterize the optimal solution of aforementioned optimization problem of a FC-LCSI retailer. We present it in the following theorem.

Theorem 1. $\forall\ F^{-1}(\frac{p-c}{p-a_v}) > \eta/c$, *if the demand is IFR distributed then optimal order quantity (q_C^*) of a FC-LCSI retailer has following properties:*

i. $q_C^* \in [0, q_{max}]$ if the following condition holds: $\Delta(q) > \frac{c^2(1+r)^2}{p}f(y)$.

ii. The expression for optimal order quantity (q_C^*) is given below:

$$q_C^* = \begin{cases} \eta/c & \text{if, } \Delta(\eta/c) > p - (1+r)c \\ \hat{q}_C & \text{otherwise} \end{cases} \tag{7}$$

iii. $\hat{q}_C$ satisfies the following equation: $\Delta(q) = p - (1+r)c\overline{F}(y)$.

where, $y = (1+r)(\frac{cq-\eta}{p})$ and $\Delta(x) = (p - a_v)F(x) + 2b_v \int_{x-\frac{a_v}{2b_v}}^{x} F(u)du$.

In the first case, when $q_C^* = \eta/c$ the FC retailer uses her own equity η and decides not to borrow from bank. This scenario occurs if the interest rate, r, is high. Therefore, in the first case, the optimal order quantity is less than that of newsvendor order quantity: $q_C^* = \eta/c < F^{-1}(\frac{p-c}{p-a_v})$.

In Table 3 below, we illustrate the profit gains of a FC firm by adopting LCSI strategy compared to an equivalent firm without financial constraint. We consider following parametric values: (i) per unit retail price, $p = 10$, (ii) per unit cost, $c = 5$, (iii) firm's own equity, $\eta = 30$, (iv) borrowing rate, $r = 10\%$, (v) clearance sale parameters are $a_v = 4$, and $b_v = 0.5$, and (vi) market demand is uniformly distributed between the following limits: $U[5, 20]$. For these parameters, maximum clearance quantity is $s = a_v/2b_v = 4$. From Table 3 we can observe that FC-LCSI strategy offers gain to the retailer firm when market demand (x) is $x \in [q_{min}, q^*]$, i.e. less than her optimal order quantity. Subsequently, the retailer makes less profit than her non financially constrained counter part when market demand (x) is $x \in (q^*, q_{max}]$, i.e. more than her optimal order quantity. We further conduct additional numerical experiment for a FC retailer firm who adopts fixed clearance price strategy. We present those results in Table 4. In the case of fixed clearance price, we also observe similar behavior in retailer's profit.

In both cases we observe that the FC retailer's optimal order quantity is less than that of her unconstrained order quantity. As a result, her cost of procurement is comparatively less. During low market demand scenario, this works in the retailer's advantage as she is required to clear less leftover inventory through clearance sale. As a result, she manages to earn more profit compared to a retailer without FC. This result is counter intuitive in nature.

Table 3: Comparison of LCSI and FC-LCSI Models

Demand(x)	Optimal order quantity (q^*)		Cost of procurement		Leftover inventory		Clearance sale inventory	
	Without FC	With FC	Without FC	With FC	Without FC	With FC	Without FC	With FC
Low Demand Scenario								
7	14	12	70	63	7	5	4	4
8	14	12	70	63	6	4	4	4
9	14	12	70	63	5	3	4	3
10	14	12	70	63	4	2	4	2
11	14	12	70	63	3	1	3	1
High Demand Scenario								
14	14	12	70	63	0	0	0	0
15	14	12	70	63	0	0	0	0
16	14	12	70	63	0	0	0	0

Normal sale revenue		Clearance sale revenue		Total revenue		Retailer's profit		Profit Gain due to FC
Without FC	With FC	Without FC	With FC	Without FC	With FC	Without FC	With FC	
Low Demand Scenario								
70	70	8	8	78	78	8	15	7
80	80	8	8	88	88	18	25	7
90	90	8	7.5	98	97.5	28	34.5	6.5
100	100	8	6	108	106	38	43	5
110	110	7.5	3.5	117.5	113.5	47.5	50.5	3
High Demand Scenario								
140	120	0	0	140	120	70	57	-13
140	120	0	0	140	120	70	57	-13
140	120	0	0	140	120	70	57	-13

Note 1: Cost of Procurement is calculated by incorporating bank payment.
Note 2: Optimal ordering quantity and associated calculation for LCSI model (without FC) is adopted from Avittathur and Biswas (2017).

Table 4: Comparison of NV and FC-NV Models

Demand(x)	Optimal order quantity (q^*)		Cost of procurement		Leftover inventory		Normal sale revenue	
	Without FC	With FC	Without FC	With FC	Without FC	With FC	Without FC	With FC
Low Demand Scenario								
7	14	12	70	63	7	5	70	70
8	14	12	70	63	6	4	80	80
9	14	12	70	63	5	3	90	90
10	14	12	70	63	4	2	100	100
11	14	12	70	63	3	1	110	110
High Demand Scenario								
14	14	12	70	63	0	0	140	120
15	14	12	70	63	0	0	140	120
16	14	12	70	63	0	0	140	120

Clearance sale revenue		Total revenue		Retailer's profit		Profit Gain due to FC
Without FC	With FC	Without FC	With FC	Without FC	With FC	
Low Demand Scenario						
7	5	77	75	7	12	5
6	4	86	84	16	21	5
5	3	95	93	25	30	5
4	2	104	102	34	39	5
3	1	113	111	43	48	5
High Demand Scenario						
0	0	140	120	70	57	-13
0	0	140	120	70	57	-13
0	0	140	120	70	57	-13

Note 1: Fixed salvage value is 1.00.
Note 2: Optimal ordering quantity and associated calculation for LCSI model (without FC) is adopted from Cachon (2003).

In the second case, when $q_C^* = \hat{q}_C$ the retailer seeks additional loan from bank to order additional quantity after exhausting her own equity, η, if the interest rate, r, is low. We can further observe here that this optimal order quantity (q_C^*) that we have obtained for FC-LCSI model also represents the optimal order quantity of a centralized supply chain consisting of one supplier and one retailer.

The generalizability of our proposed LCSI model can be understood from the following: (i) for $b_v = 0$, LCSI model behaves like classical newsvendor model with a fixed clearance price of a_v and (ii) for $a_v = 0$ and $b_v = 0$, LCSI model behaves like classical newsvendor model with no clearance price. The optimal order quantities of a retailer for these cases can be easily derived from Theorem 1. We denote the optimal order quantity for a FC newsvendor with fixed salvage price[9] (a_v) as $[q_C^*]_{FS}$. In case of newsvendor with fixed salvage price, the function $\Delta(x)$ assumes a simple form: $\Delta(x) = (p - a_v)F(x)$. The value of $[q_C^*]_{FS}$ is presented below.

Proposition 1. $\forall F^{-1}(\frac{p-c}{p-a_v}) > \eta/c$, if the demand is IFR distributed, then a FC-NV retailer's optimal order quantity, $[q_C^*]_{FS}$, with fixed salvage price (a_v) is determined as follows:

$$[q_C^*]_{FS} = \begin{cases} \eta/c & \text{if, } \Delta(\eta/c) > p - c(1+r) \\ \hat{q} & \text{otherwise} \end{cases}$$

and, $\hat{q}$ satisfies the following condition:

$$\Delta(q) = p - c(1+r)\overline{F}(y)$$

where, $y = (1+r)(\frac{cq-\eta}{p})$ and $\Delta(x) = (p - a_v)F(x)$.

Proposition 1 represents optimal order quantity decision for a FC-NV retailer. From proposition 1 optimal order quanity of a FC-NV retailer with no clearance price can be readily calculated using $a_v = 0$ and we present the same below.

$\forall F^{-1}(\frac{p-c}{p}) > \eta/c$, if the demand is IFR distributed, then a FC newsvendor's optimal order quantity with no salvage price[10], is determined as follows:

---

[9] We denote fixed salvage price by the subscript FS

[10] We denote no salvage price by the subscript NS

$$[q_C^*]_{NS} = \begin{cases} \eta/c & \text{if, } pF(\eta/c) > p - c(1+r) \\ \hat{q} & \text{otherwise, } pF(\hat{q}) = p - c(1+r)\overline{F}\left((1+r)\left(\dfrac{c\hat{q} - \eta}{p}\right)\right) \end{cases}$$

This special case result signifies the generalizability of our proposed FC-LCSI model. This aforementioned optimal order quantity matches with that reported by Dada and Hu (2008).

In the next section, we analyze a dyadic decentralized supply chain consisting of one supplier and one FC-LCSI retailer. We specifically investigate optimal order quantity decision of the retailer and supply contract design(s) of the supplier.

4 Supply contracts for FC-LCSI model

In this section we analyze optimal supply contracts for a decentralized supply chain where a FC retailer employs LCSI strategy. In §3.3 we have derived the optimal order quantity for a centralized supply chain and have established the condition of concavity of central planner's profit function. For the purpose of expositional simplicity, we assume the following.

  i. Retailer's marginal cost of production is zero.

  ii. There is no penalty cost associated with under-stocking.

Using the results obtained in §3.3 , we investigate channel coordination strategies for a dyadic decentralized supply chain. In this context, we specifically study wholesale price, buy-back, and revenue sharing contracts. We further observe that in presence of any one of these supply contracts the expected profit function of the retailer is similar to equation 5. Therefore, using Theorem 1 we can conclude that there exists a unique optimal order quantity for each of these contracts. We present these optimal supply contracts in §4.1 - 4.3.

4.1 Wholesale price contract

The wholesale price contract is not only the simplest of all contract forms but also one of the most prevalent contract forms in practice (Cachon (2003), Biswas and Avittathur (2018)). In wholesale price contract, the supplier charges the retailer a fixed per unit wholesale price,

w_{WP} and thus the total payment made by the retailer to the supplier is: $T_{WP}(w_{WP}, q_{WP}) = w_{WP}q_{WP}$. The supplier's profit function is: $\pi_S^{WP}(w_{WP}) = (w_{WP} - c)q_{WP}^*(w_{WP})$, where $q_{WP}^*(w_{WP})$ represents the optimal order quantity chosen by the retailer for a given value of w_{WP}. The chronological sequence of events is presented below.

i. At the beginning of period T_1, the supplier announces her contract term, w_{WP}.

ii. Subsequently, the retailer decides her order quantity q_{WP} and pays the supplier $w_{WP}q_{WP}$ using her own equity, η, and bank borrowing, B.

iii. During period T_1, the retailer is expected to sell $E[min(q_{WP}, x)]$ in the market (where, x represents random market demand) and is expected to earn a profit of $E[\pi_{T1}^{WP}(q_{WP})]_R$.

iv. During period T_2, the retailer is expected to earn additional revenue $E[R_{T2}^{WP}(\cdot, q_{WP})]_R$ by using LCSI strategy.

The retailer chooses her optimal order quantity, q_{WP}^*, to maximize her expected profit, $E[\pi_R^{WP}(q_{WP})] = E[\pi_{T1}^{WP}(q_{WP})]_R + E[R_{T2}^{WP}(\cdot, q_{WP})]_R$, for a given value of w_{WP}. As the supplier is the Stackelberg leader, she solves for her optimal contract parameter, w_{WP}^*, by using backward induction method. We present the supplier's optimization problem for wholesale price contract as follows.

$$w_{WP}^* = \max_{w_{WP}} \pi_S^{WP}(w_{WP}) = \max_{w_{WP}} (w_{WP} - c)q_{WP}^* \tag{8}$$

$$\text{subject to, } q_{WP}^*(w_{WP}) = \max_{q_{WP}} E[\pi_R^{WP}(q_{WP})] \tag{9}$$

$$q_{WP} - \frac{\eta}{w_{WP}} \geq 0 \tag{10}$$

$$E[\pi_R^{WP}(q_{WP})] = -\{\eta + (1+r)(w_{WP}q_{WP} - \eta)\overline{F}(y)\} + p\int_{y_{WP}}^{q_{WP}} xf(x)dx +$$
$$pq_{WP}\int_{q_{WP}}^{q_{max}} f(x)dx + (a_v - 2b_v q_{WP})\int_{j_{WP}}^{q_{WP}} F(x)dx + 2b_v\int_{j_{WP}}^{q_{WP}} xF(x)dx \tag{11}$$

Using KKT conditions we characterize the optimal solution of aforementioned optimiza-

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tion problem of a supplier who sells her product through a FC-LCSI retailer using wholesale price contract. We present it in the following theorem.

Theorem 2. *When demand is IFR distributed and the supplier sells her products to a FC-LCSI retailer through a wholesale price contract then the optimal decisions of the supply chain are characterized as follows.*

    i. *For a given value of wholesale price, w_{WP}, the retailer's optimal order quantity, $q^*_{WP}(w_{WP})$, is given below.*

$$q^*_{WP}(w_{WP}) = \begin{cases} \eta/w_{WP} & \text{if, } \Delta(\eta/w_{WP}) > p - (1+r)w_{WP} \\ \hat{q}_{WP}(w_{WP}) & \text{otherwise} \end{cases} \tag{12}$$

    where, (a) $\hat{q}_{WP}(w_{WP})$ satisfies the following equation: $\Delta(q_{WP}) = p - (1+r)w_{WP}\overline{F}(y_{WP})$ and (b) $y_{WP} = \frac{1+r}{p}(w_{WP}q_{WP} - \eta)$.

    ii. *$\forall q^*_{WP}$, the supplier's optimal wholesale price contract, as expressed by w^*_{WP}, solves the following equation: $\partial\pi_S^{WP}(w_{WP})/\partial w_{WP} = 0$.*

where, $\Delta(x) = (p - a_v)F(x) + 2b_v \int_{x-\frac{a_v}{2b_v}}^{x} F(u)du$.

Similar to centralized supply chain, as presented by Theorem 1, the FC-LCSI retailer uses her own equity to place her order and does not borrow from the bank in the first case. In the second case, the retailer seeks additional loan from the bank to procure her optimal order quantity, $\hat{q}_{WP}$. By solving the supplier's optimization problem, we calculate the optimal wholesale price and it is presented below.

Lemma 1. *The supplier's optimal wholesale price, w^*_{WP}, is given by the following equation:*

$$w^*_{WP} = \begin{cases} c + \frac{q^*_{WP}}{1+r}\Delta'(q^*_{WP}), & \text{when } q^*_{WP} = \eta/w^*_{WP} \\ \left(\frac{p}{1+r}\right)\frac{c(1+r)\overline{F}(\hat{y}_{WP}) + \hat{q}_{WP}\Delta'(\hat{q}_{WP})}{p\overline{F}(\hat{y}_{WP}) + c\hat{q}_{WP}(1+r)f(\hat{y}_{WP})}, & \text{when } q^*_{WP} = \hat{q}_{WP}(w^*_{WP}) \end{cases} \tag{13}$$

where, $\Delta(x) = (p - a_v)F(x) + 2b_v \int_{x-\frac{a_v}{2b_v}}^{x} F(u)du$.

From Lemma 1 it is evident that WP contract does not coordinate the supply chain. Supply contract model with fixed salvage value ($b_v = 0$) can be readily derived from Theorem 2 and Lemma 1. The same is presented below in Proposition 2.

Proposition 2. *When demand is IFR distributed, the supplier sells her products to a FC-NV retailer with fixed salvage price (a_v) through a wholesale price contract then the optimal decisions of the supply chain are characterized as follows.*

i. *For a given value of wholesale price, w_{WP}, the retailer's optimal order quantity, $q_{WP}^*(w_{WP})$, is given below.*

$$
q_{WP}^*(w_{WP}) = \begin{cases} \eta/w_{WP} & \text{if, } \Delta(\eta/w_{WP}) > p - w_{WP}(1+r) \\ \hat{q}_{WP}(w_{WP}) & \text{otherwise} \end{cases}
$$

where, (i) $\hat{q}_{WP}(w_{WP})$ satisfies the following equation: $\Delta(q_{WP}) = p - w_{WP}(1+r)\overline{F}(y_{WP})$, (ii) $y_{WP} = \frac{1+r}{p}(w_{WP}q_{WP} - \eta)$, and (iii) $\Delta(x) = (p - a_v)F(x)$.

ii. *$\forall q_{WP}^*$, the supplier's optimal wholesale price, w_{WP}^*, solves the following equation:*

$$
w_{WP}^* = \begin{cases} c + \frac{q_{WP}^*}{1+r}(p - a_v)f(q_{WP}^*), & \text{if } q_{WP}^* = \eta/w_{WP} \\ \frac{p\{c(1+r)\overline{F}(\hat{y}_{WP}) + \hat{q}_{WP}(p - a_v)f(\hat{q}_{WP})\}}{(1+r)\{p\overline{F}(\hat{y}_{WP}) + c\hat{q}_{WP}(1+r)f(\hat{y}_{WP})\}}, & \text{otherwise} \end{cases} \tag{14}
$$

Proof. Proposition 2 follows from Theorem 2 and Lemma 1.

Proposition 2 presents optimal order quantity decision for a FC-NV retailer and corresponding optimal wholesale price contract for the supplier. In absence of clearance sale ($a_v = 0$), the retailer's optimal order quantity takes the following form:

$$
q_{WP}^*(w_{WP}) = \begin{cases} \eta/w_{WP} & \text{if, } pF(\eta/w_{WP}) > p - w_{WP}(1+r) \\ \hat{q}_{WP}(w_{WP}) & \text{otherwise} \end{cases}
$$

$\hat{q}_{WP}(w_{WP})$ satisfies the following equation: $pF(q_{WP}) = p - w_{WP}(1+r)\overline{F}(y_{WP})$. In this case,

the supplier's optimal wholesale price contract is calculated as follows:

$$
w_{WP}^* = \begin{cases} c + \frac{1}{1+r} p q_{WP}^* f(q_{WP}^*), & \text{if } q_{WP}^* = \eta/w_{WP} \\ \\ \frac{p\{c(1+r)\overline{F}(\hat{y}_{WP}) + p\hat{q}_{WP}f(\hat{q}_{WP})\}}{(1+r)\{p\overline{F}(\hat{y}_{WP}) + c\hat{q}_{WP}(1+r)f(\hat{y}_{WP})\}}, & \text{otherwise} \end{cases} \tag{15}
$$

From Theorem 2 and Proposition 2 we can observe that wholesale price contract coordinates the supply chain if $w_{WP}^* = c$. Similar to the case of retailer without financial constraint, as presented in Cachon (2003), wholesale price contract coordinates a supply chain if the supplier earns a non-positive profit when the retailer is financially constrained. $\forall w_{WP}^* > c$, the FC-LCSI retailer orders less than q_C^*. As a result, the overall profit level of the supply chain reduces compared to the centralized case and the supply chain does not coordinate.

4.2 Buyback contract

In buyback contract, the supplier charges the retailer a fixed per unit price (w_{BB}) at the beginning of period T_1; subsequently, at the end of period T_1, she buys back leftover quantity ($max(q_{BB} - x, 0)$) from the retailer at a fixed exogenous buyback rate (b), where x represents random market demand. Therefore, the effective transfer payment to be made by the retailer to the supplier is expressed as: $T_{BB}(w_{BB}, q_{BB}, b) = w_{BB}q_{BB} - bE[max(q_{BB} - x, 0)]$, for a given value of retailer's order quantity, q_{BB}. In period T_2, the supplier adopts LCSI strategy and earns additional revenue from clearance sale. The chronological sequence of events is presented below.

   i. At the beginning of period T_1, the supplier announces her contract terms, w_{BB} and b.

   ii. Subsequently, the retailer decides her order quantity q_{BB} and pays the supplier $w_{BB}q_{BB}$ using her own equity, η, and bank borrowing, B.

   iii. During period T_1, the retailer is expected to sell $E[min(q_{BB}, x)]$ in the market (where, x represents random market demand) and is expected to earn a profit of $E[\pi_R^{BB}(q_{BB})]$.

   iv. At the end of period T_1, the retailer sells back her leftover quantities, $max(q_{BB} - x, 0)$, to the supplier at per unit rate, b, and earns additional revenue, $bE[max(q_{BB} - x, 0)]$.

v. During period T_2, the supplier is expected to earn additional revenue, $E[R_{T2}^{BB}(\cdot, q_{BB})]_S$, by adopting LCSI strategy.

The retailer chooses her optimal order quantity, q_{BB}^*, to maximize her expected profit function, $E[\pi_R^{BB}(q_{BB})]$, for given values of supplier's contract parameters (w_{BB}, b). As the supplier is the Stackelberg leader, she solves for her optimal contract parameters, (w_{BB}^*, b), by using backward induction method. We present the supplier's optimization problem for buyback contract below.

$$w_{BB}^*(b) = \max_{w_{BB}} E[\pi_S^{BB}(w_{BB})] \tag{16}$$

$$subject\ to,\ q_{BB}^*(w_{BB}) = \max_{q_{BB}} E[\pi_R^{BB}(q_{BB})] \tag{17}$$

$$q_{BB} - \frac{\eta}{w_{BB}} \geq 0 \tag{18}$$

$$where,\ E[\pi_S^{BB}(q_{BB})] = (w_{BB} - c)q_{BB}^* - b\int_0^{q_{BB}^*} F(x)dx$$
$$+ (a_v - 2b_v q_{BB}^*)\int_{j_{BB}^*}^{q_{BB}^*} F(x)dx + 2b_v\int_{j_{BB}^*}^{q_{BB}^*} xF(x)dx \tag{19}$$

$$E[\pi_R^{BB}(q_{BB})] = -\{\eta + (w_{BB}q_{BB} - \eta)(1 + r)\overline{F}(y)\} + p\int_{y_{BB}}^{q_{BB}} xf(x)dx +$$
$$pq_{BB}\int_{q_{BB}}^{q_{max}} f(x)dx + b\int_0^{q_{BB}} F(x)dx \tag{20}$$

Using KKT conditions we characterize the optimal solution of aforementioned optimization problem of a LCSI supplier who sells her product through a FC retailer using buyback contract. We present it in the following theorem.

Theorem 3. *When demand is IFR distributed and the supplier sells her products to a FC-LCSI retailer through a buyback contract then the optimal decisions of the supply chain are characterized as follows.*

i. For a given buyback contract, (w_{BB}, b), the retailer's optimal order quantity, $q_{BB}^*(w_{BB}, b)$,

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is given below.

$$
q_{BB}^*(w_{BB}, b) = \begin{cases} \eta/w_{BB} & \text{if, } (p-b)F(\eta/w_{BB}) > p - w_{BB}(1+r) \\ \hat{q}_{BB}(w_{BB}, b) & otherwise \end{cases} \tag{21}
$$

where, (i) $\hat{q}_{BB}(w_{BB}, b)$ satisfies $(p-b)F(q_{BB}) = p - w_{BB}(1+r)\overline{F}(y_{BB})$ and (ii) $y_{BB} = \frac{1+r}{p}(w_{BB}q_{BB} - \eta)$.

ii. *$\forall q_{BB}^*$, the supplier's optimal buyback contract, as expressed by $w_{BB}^*(b)$, solves the following equation: $\partial E[\pi_S^{BB}(w_{BB})]/\partial w_{BB} = 0$.*

From Theorem 3 we observe that the supplier cannot coordinate the overall supply chain if the FC retailer decides to use only her own equity to place order. The supplier can coordinate the overall supply chain if and only if the FC retailer borrows additional loan from a bank to procure her optimal order quantity. Under such circumstances, it is possible for the supplier to design a channel coordinating buyback contract mechanism. This optimal channel coordinating buyback contract is presented below in Lemma 2.

Lemma 2. *In buyback contract, the supplier's channel coordinating per unit price, w_{BB}^*, satisfies the following equation:*

$$
(1+r)\{w_{BB}^*\overline{F}(y_{BB}^*) - c\overline{F}(y_C^*)\} = \Delta(q_C^*) - (p-b)F(q_C^*) \tag{22}
$$

where, (a) b is an exogenously decided buyback price, (b) $q_C^* = \hat{q}_C$, as presented in Theorem 1; (c) $\Delta(x) = (p-a_v)F(x) + 2b_v\int_{x-\frac{a_v}{2b_v}}^{x} F(u)du$, (d) $y_C^* = (1+r)(cq_C^* - \eta)/p$, and (e) $y_{BB}^* = (1+r)(w_{BB}^*q_C^* - \eta)/p$.

Lemma 2 is of particular significance. From equation (22) we can observe that the supplier can design a channel coordinating buyback contract if and only if she knows (i) the value of interest rate, r, and (ii) the value of retailer's own equity, η. Otherwise, it would not be possible for her to design a channel coordinating contract parameter, w_{BB}^*, for a exogenous value of buyback rate, b. Therefore, in the presence of financial constraint the supplier can

design channel coordination mechanism only under full information setting.

Channel coordinating buyback contract with fixed salvage value ($b_v = 0$) can be readily derived from Theorem 3 and Lemma 2. We present it below through Proposition 3.

Proposition 3. *When demand is IFR distributed, the supplier sells her products to a FC-NV retailer with fixed salvage price (a_v) through a buyback contract then the optimal decisions of the supply chain are characterized as follows.*

   i. *For a given buyback contract, (w_{BB}, b), the retailer's optimal order quantity, $q_{BB}^*(w_{BB}, b)$, is given below.*

$$
q_{BB}^*(w_{BB}, b) = \begin{cases} \eta/w_{BB} & \text{if, } (p - b)F(\eta/w_{BB}) > p - w_{BB}(1 + r) \\ \hat{q}_{BB}(w_{BB}, b) & \text{otherwise} \end{cases}
$$

   where, (i) $\hat{q}_{BB}(w_{BB}, b)$ satisfies the following equation: $(p - b)F(q_{BB}) = p - w_{BB}(1 + r)\overline{F}(y_{BB})$ and (ii) $y_{BB} = \frac{1+r}{p}(w_{BB}q_{BB} - \eta)$.

   ii. *When $q_{BB}^* = \hat{q}_{BB}$, the supplier's channel coordinating buyback contract, (w_{BB}^*, b), solves the following equation:*

$$
(1 + r)\{w_{BB}^*\overline{F}(y_{BB}^*) - c\overline{F}(y_C^*)\} = (b - a_v)F(q_C^*) \tag{23}
$$

   where, (a) b is an exogenously decided buyback price, (b) $y_C^* = (1 + r)(cq_C^* - \eta)/p$, and (c) $y_{BB}^* = (1 + r)(w_{BB}^* q_C^* - \eta)/p$.

Proof. Proposition 3 follows from Theorem 3 and Lemma 2.

Proposition 3 presents optimal order quantity decision for a FC-NV retailer and corresponding optimal buyback contract for the supplier. Comparing Proposition 3 with Theorem 3, we observe that the retailer's order quantity remains identical for both LCSI and fixed salvage value models as she is not responsible for clearance sale. However, the supplier's channel coordinating buyback contract changes for fixed salvage price and it is represented by equation (23).

From Proposition 3 we further observe that, in absence of clearance sale ($a_v = 0$), the retailer's optimal order quantity remains unchanged and the supplier's optimal buyback contract expression gets simplified as follows:

$$w_{BB}^* \overline{F}(y_{BB}^*) - c\overline{F}(y_C^*) = \frac{b}{1+r}F(q_C^*) \tag{24}$$

From Proposition 3, Lemma 2, and equation (24) we can understand the generalizability of Theorem 3. For FC-LCSI and FC-NV models, buy-back contract coordinates the overall supply chain and the supply chain profit is equal to that of the centralized case.

4.3 Revenue-sharing contract

In revenue-sharing contract, the supplier charges a fixed per unit price, w_{RS} at the beginning of period T_1; subsequently, at the end of period T_2, the supplier and the retailer distribute the total expected revenue (earned over periods T_1 and T_2) as per revenue-sharing agreement. The total expected revenue from sale over periods T_1 and T_2 is expressed as: $E[R_{RS}(\cdot)] = E[R_{T1}(\cdot, y, q_{RS})] + E[R_{T2}(\cdot, q_{RS})]$, where $E[R_{T1}(\cdot, y, q_{RS})]$ represents normal selling season revenue and $E[R_{T2}(\cdot, q_{RS})]$ is retailer's clearance sale revenue by adopting LCSI strategy. From this overall generated revenue, the supplier keeps a share $(1 - \phi)$ and the retailer keeps a share $\phi(1 \geq \phi \geq 0)$ of it.

The minimum required quantity that the retailer is required to sell to repay her borrowed amount with interest is computed based on the fraction of revenue that she keeps. Thus, retailer's minimum required quantity is defined as: $y = (1+r)B/\phi p = (1+r)(w_{RS}q_{RS} - \eta)/\phi p$, as she earns ϕp from the sale of one unit of finished goods. In case of revenue-sharing contract, the chronological sequence of events is presented below.

i. At the beginning of period T_1, the supplier announces her contract terms, w_{RS} and ϕ.

ii. Subsequently, the retailer decides her order quantity q_{RS} and pays the supplier $w_{RS}q_{RS}$ using her own equity, η, and bank borrowing, B.

iii. During period T_1, the retailer is expected to sell $E[min(q_{RS}, x)]$ in the market (where,

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x represents random market demand) and her expected revenue is $E[R_{T1}(\cdot, y, q_{RS})]$.

iv. During period T_2, the retailer is expected to earn an additional revenue, $E[R_{T2}(\cdot, q_{RS})]$, by adopting LCSI strategy.

v. At the end of period T_2, the retailer retains fraction, ϕ, of the total revenue earned, $\{E[R_{T1}(\cdot, y, q_{RS})] + E[R_{T2}(\cdot, q_{RS})]\}$ and pays the supplier the remaining fraction, $(1-\phi)$ of the total revenue.

In the case of revenue-sharing contract, the retailer chooses her optimal order quantity, q_{RS}^*, by maximizing her expected profit function, $E[\pi_R^{RS}(q_{RS})]$, for given values of supplier's contract parameters (w_{RS}, ϕ). As the supplier is the Stackelberg leader, she solves for her optimal contract parameters, (w_{RS}^*, ϕ), by using backward induction method. We present the supplier's optimization problem for revenue-sharing contract below.

$$w_{RS}^*(\phi) = \max_{w_{RS}} E[\pi_S^{RS}(w_{RS})] = \max_{w_{RS}} \left[(w_{RS} - c)q_{RS}^*(w_{RS}) + \right.$$
$$\left. (1 - \phi)\{E[R_{T1}(\cdot, y, q_{RS})] + E[R_{T2}(\cdot, q_{RS})]\} \right] \quad (25)$$

$$subject\ to,\ q_{RS}^*(w_{RS}) = \max_{q_{RS}} E[\pi_R^{RS}(q_{RS})] = \max_{q_{RS}} \left[- \{\eta + (w_{RS}q_{RS} - \eta)(1 + r)\overline{F}(y)\} \right.$$
$$\left. + \phi\{E[R_{T1}(\cdot, y, q_{RS})] + E[R_{T2}(\cdot, q_{RS})]\} \right] \quad (26)$$

$$q_{RS} - \frac{\eta}{w_{RS}} \geq 0 \quad (27)$$

$$where,\ E[R_{T1}(\cdot, y, q_{RS})] = p \int_{y_{RS}}^{q_{RS}} x f(x)dx + p q_{RS} \int_{q_{RS}}^{q_{max}} f(x)dx \quad (28)$$

$$E[R_{T2}(\cdot, q_{RS})] = (a_v - 2b_v q_{RS}) \int_{j_{RS}}^{q_{RS}} F(x)dx + 2b_v \int_{j_{RS}}^{q_{RS}} x F(x)dx \quad (29)$$

Using KKT conditions we characterize the optimal solution of aforementioned optimization problem of a supplier who sells her product through a FC-LCSI retailer using revenue-sharing contract. We present it in the following theorem.

Theorem 4. *When demand is IFR distributed and the supplier sells her products to a FC-LCSI retailer through a revenue-sharing contract then the optimal decisions of the supply chain are characterized as follows.*

    i. For a given revenue-sharing contract, (w_{RS}, ϕ), the retailer's optimal order quantity, $q_{RS}^*(w_{RS}, \phi)$, is given below.

$$q_{RS}^*(w_{RS}, \phi) = \begin{cases} \eta/w_{RS} & \text{if, } \Delta(\eta/w_{RS}) > p - \dfrac{1+r}{\phi}w_{RS} \\ \hat{q}_{RS}(w_{RS}, \phi) & \text{otherwise} \end{cases} \quad (30)$$

    where, (i) $\hat{q}_{RS}(w_{RS}, \phi)$ satisfies the following equation: $\Delta(q_{RS}) = p - \frac{1+r}{\phi}w_{RS}\overline{F}(y_{RS})$, (ii) $y_{RS} = \frac{1+r}{\phi p}(w_{RS}q_{RS} - \eta)$, and (iii) $\Delta(x) = (p - a_v)F(x) + 2b_v \int_{x-\frac{a_v}{2b_v}}^{x} F(u)du$.

    ii. $\forall q_{RS}^*$, the supplier's optimal revenue-sharing contract, as expressed by $w_{RS}^*(\phi)$, solves the following equation: $\partial E[\pi_S^{RS}(w_{RS})]/\partial w_{RS} = 0$.

From Theorem 4 we observe that the supplier cannot coordinate the overall supply chain if the FC retailer decides to use only her own equity to place order. This result is similar to that obtained in the case of buyback contract. The supplier can coordinate the overall supply chain if and only if the FC retailer borrows additional loan from a bank to procure her optimal order quantity. Under such circumstances, it is possible for the supplier to design a channel coordinating revenue-sharing contract mechanism. This optimal channel coordinating revenue-sharing contract is presented below in Lemma 3.

Lemma 3. *In revenue-sharing contract, the supplier's channel coordinating per unit price, w_{RS}^*, satisfies the following equation:*

$$w_{RS}^* = \phi c \frac{\overline{F}(y_C^*)}{\overline{F}(y_{RS}^*)} \quad (31)$$

where, (a) ϕ is an exogenously decided revenue-sharing fraction, (b) $q_C^* = \hat{q}_C$, as presented in Theorem 1; (c) $y_C^* = (1+r)(cq_C^* - \eta)/p$, and (e) $y_{RS}^* = (1+r)(w_{RS}^*q_C^* - \eta)/\phi p$.

From Lemma 3 we can observe that the supplier can design a channel coordinating

revenue-sharing contract if and only if she knows (i) the value of interest rate, r, and (ii) the value of retailer's own equity, η. This result is similar to that obtained in the case of buyback contract. Therefore, in the presence of financial constraint the supplier can design channel coordination mechanism only under full information setting.

From Lemma 3, we can further observe that the expression of supplier's channel coordinating revenue-sharing contract parameter remains identical for the following cases: (a) clearance sale with fixed salvage value ($a_v \neq 0$ and $b_v = 0$) and (b) no clearance sale ($a_v = 0$ and $b_v = 0$). Though the expression of optimal per unit price (w_{RS}^*) is not influenced by the choice of model in revenue-sharing contract, the optimal order quantity (q_{RS}^*) changes with the choice of model.

In other contract types, namely wholesale price and buyback, the optimal per unit prices calculated for FC-LCSI model degenerates into those calculated for FC-NV model (either with fixed salvage price or zero salvage price) and thus we can understand that FC-LCSI model represents a generalized case of single period inventory ordering problem when a FC retailer faces stochastic demand.

5 Conclusion

In many real life context the retailer is both financially constrained and her clearance price is a function of leftover inventory. However, classical NV model assumes a constant clearance price for salvage sale and does not consider financial constraint. In this paper, we have addressed this specific literature gap by designing a framework where the retailer is financially constrained and employs limited clearance sale strategy to improve her overall revenue. In this context, we present a generalized framework for a dyadic supply chain consisting one supplier and one FC-LCSI retailer. We have presented the concavity condition for retailer's profit function and have calculated the optimal order quantity for the FC-LCSI retailer. We observe that the optimal order quantity of FC-LCSI retailer is less than that of a retailer without financial constraint. Subsequently, we calculate optimal order quantity for two cases

of FC-NV model (with fixed clearance price and no clearance price) from FC-LCSI model and thus we establish generalizability of our proposed framework.

Subsequently, we investigate channel coordination aspect of such a dyadic supply chain. We find that wholesale price contract does not coordinate the supply chain. Both buyback and revenue-sharing contracts coordinate the supply chain if and only if the retailer decides to borrow in order to attain her optimal order quantity. If the retailer decides to procure using her only equity only, in that case the supplier is not able to coordinate the channel. In order to design channel coordinating buyback contract or revenue-sharing contract, the supplier needs to know the bank interest rate, r, and the retailer's equity, η. Otherwise, the supplier cannot design channel coordinating mechanism. Therefore, in case of a financially constrained supply chain channel coordination is dependent on the retailer's choice for revelation of these information to the supplier.

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