Capacity Choice under Demand Uncertainty: Effects of Production Postponement and Product Flexibility

Abstract: *This paper deals with the optimal capacity choice under demand uncertainty. A single period two product model with stochastic demand has been developed to determine the optimal capacity level that maximizes the expected profit. Dedicated plant with no production postponement strategy has been considered as base case. The model has been extended to examine the effect of production postponement and product flexibility on optimal capacity decision. While it is apparent that the cost of over-production has been eliminated under production postponement, the other major benefit depends on whether the products have been produced in a single product flexible plant rather than dedicated plants. It has been shown that investment in flexible plant makes sense only if the possibility of production postponement exists. The model has been extended to multi-product situation with correlation in demand. Simulated data based optimization procedure has been applied to solve the multi-product problem as the same is analytically intractable. The concept of PdPPF Index has been introduced to observe the effect of production postponement on product flexible plant. Finally the effects of imposing service level objective on firm's optimal profit and capacity have been studied for both dedicated and flexible plant strategies.*

Keywords: Demand Uncertainty; Capacity Planning; Production Postponement; Product Flexibility; Stochastic Programming; Service Level

1. Introduction:

Up to the middle of the last century, the paradigm of manufacturing had an emphasis on the mass production, mass markets and standard design. The existence of national market and absence of foreign competitors helped firms to act in the seller's market. Over the years the complexity in business environment has increased due to globalization and rapid technological advances. The changing nature of global business has led to highly competitive markets. Increased competition has changed the nature of demand in the market place both in terms of product variety as well as uncertainty associated with the product demands. This has increased challenges in all facets of manufacturing. Capacity planning in such scenario assumes complexity as one has to deal with the trade off between the cost of investment in excess capacity and the opportunity loss from not meeting the demand due to capacity constraint.

In the context of production decision under capacity planning objective, two distinct situations may arise: (a) the firm has to decide on capacity as well as the production quantity before the demand has been realized, (b) while capacity needs to be decided a priori, the firm can decide on production after the demand is realized. The above two have been normally referred to as "No Postponement" and "Production Postponement" respectively. While it has been apparent that the cost of over-production has been eliminated under production postponement, the other major benefit depends on whether the products have been produced in a single product flexible plant rather than dedicated plants. Yang et al. (2004) have argued that both flexibility and postponement are "reactive adaption behaviors" as they deal with the consequences of uncertainty rather than attacking the causes of uncertainty.

Product flexibility has been recognized as an important tool for coping with demand uncertainties. However, investment and management issues regarding product flexibility have been recently incorporated in operation management models (Bish and Wang, 2004). It is intuitive that in the presence of production postponement, the firm stands to gain from product flexibility by exploiting the differences in the realized demands of the individual products. The capacity decision being taken considering aggregate demand of all the products; at the production stage potential benefit exists in terms of utilizing the idle capacity due to the below average realized demand for one product by the higher than average realized demand for another product. On the other hand, as product flexibility allows production of different products in the same plant, it would typically involve higher marginal cost of investment compared to dedicated plant. This motivates to look at the economics of dedicated plants versus product flexible plant in the context of capacity planning decision. For this purpose, a single period multi-product model with stochastic demand has been developed to determine the optimal capacity level that maximizes the expected profit. Dedicated plant with no production postponement strategy has been considered as base case. This model is similar to the classical newsboy model with capacity as decision variable. The model has been extended to consider (a) Dedicated plant with production postponement and (b) Flexible plant with production postponement. The base model as well as (a) above, are essentially extension of Mieghem and Dada (1999) for multi product case. In literature, the extensions (a) and (b) have been modeled as two stage stochastic programming problem; where, in the first stage the firm decides the capacity that maximizes the expected profit. In the second stage, demands have been realized and the firm decides on production quantity.

In this paper the stochastic programming problem has been solved to determine the capacity level which maximizes the expected profit. However, in case of multiple products following correlated multivariate demand distribution, the problem becomes analytically intractable. Because of the analytical intractability, most of the literatures have come out with characterization of optimal solution with possibilities and dominant conditions. (Some of them have been discussed in literature review.) To make the problem analytically tractable, for three-product case, where demands follow correlated multivariate distribution, finite discretization of the random data allows writing the expectation in the form of summation and helps to solve the stochastic problem.

The rest of the paper is as follows. Relevant literature survey has been done in section 2. In section 3, the models for expected profit maximization and opportunity loss minimization under postponement and product flexibility have been introduced and shown that simulated data based optimization gives very good approximation of the analytical results in case of two-product example. In section 4, analysis has been done for multi product case. The results and insights of the analyses have also been shown in this section. In section 5, service level constraint has been added to observe its effect on various strategies. Section 6 concludes the paper.

2. Literature Survey:

The choice of dedicated and flexible plant combination for capacity planning under demand uncertainty has been considered by Fine and Freund (1990), followed by Meighem (1998) and Bish and Wang (2004). Eppen et al (1989) have considered capacity planning problem under risk and presented a mixed integer programming model based on a scenario planning approach. Peronne et al (2002) have tried to capture the economic advantage of flexible resource over the dedicated one.

Fine and Freund (1990) have worked with n different product families which can be produced in n dedicated plants or in a single flexible plant. K possible states of demand with known probability have been assumed. The market demand has been realized by the firm only after investment in capacity for the combination of dedicated and flexible plant. The capacity acquisition cost, revenue and production costs has been known. With the objective of profit maximization, followings have been included in their findings:

a) There is no guarantee of getting a unique optimal solution if the total no of products is greater than or equal to three. This has been shown by a counter example.
b) Shadow values at optimality for dedicated and flexible capacity constraint have been obtained. They have shown that, for a particular state of market demand, shadow value of flexible capacity is equal to the maximum of shadow values of dedicated capacities over all products. From that they have also derived the profitable condition for the investment in flexible capacity.
c) With the increase in dedicated capacity cost, dedicated capacity decreases, flexible capacity increases and vice-versa. Decrease in any type of capacity cost profit increases.
d) For downward sloping demand curves, in case of two products they have shown the relationship between the capacity, capacity cost and optimal profit for correlated demand scenario.

Mieghem (1998) has extended the works of Fine and Freund (1990) with same two product example with product one contribution is greater than that of product two. The benefits of product flexibility under uncertainty has been observed for the role of price and cost mix differentials in addition to demand correlation. He has expressed optimality condition in terms of dual variables. He has also highlighted the role of investment cost for choosing among possible investment strategies. For this he has defined two threshold values for flexible capacity cost. Similar to Fine and Freund (1990) he has observed substitution effect of marginal cost change on capacity. Similarly the investment in corresponding dedicated capacity has been increased with higher price. Increase in price differential increases flexible capacity and decreases dedicated capacity of less profitable product. He has proposed capacity investment strategy for perfectly positively correlated and perfectly negatively correlated demand under different conditions. Contradicting Fine and Freund (1990) he has shown that investment in flexible plant can give better benefit even in case of perfectly positively correlated demand if there is price difference between the products.

In line with the above literatures, Bish and Wang (2004) have also considered two-product case considering continuous distribution which makes it different from Fine and Freund (1990) and price dependent demand which makes it different from Mieghem (1998). The problem is two stage stochastic programming in nature. They have divided demand space into six regions and for each region they have derived optimal closed form expression for optimal profit of stage two as a function of capacity vector. For stage one problem they have derived necessary and sufficient condition for optimal profit. They have also derived necessary and sufficient conditions for investment in flexible capacity. They have proposed capacity investment strategies under perfectly correlated demand conditions for different parameter values.

Eppen et al (1989) have considered a multiproduct, multiplant, multiperiod capacity planning problem. Three scenarios (or states of nature) have been specified for each year. They have argued that variance is not a good measure of risk in this environment and suggested an alternative based on expected downside risk. Their works have been based on following assumptions: 1) a retooling decision determines which products can be produced at a site as well as other cost and capacity parameters; 2) there is a changeover cost for shutting down a plant as well as for retooling it; 3) the demand has been realized before the production decision has been made and no inventory has been carried from period to period; 4) production levels can be altered within the time period in order to satisfy as closely as possible the demand that has been actually experienced; 5) the probability of a scenario occurring in a year is independent of earlier outcomes and the capacity of a plant depends upon the configuration chosen and at any period any plant should be under one and only configuration. Interest rate has been taken as 0.1. They have added a constraint for expected downside risk to the original problem of the form EDR (0) < 7.0, where 0 is the target value of desired profit. Expected profit and EDR has been calculated from histogram generated using 15 cases (3 scenarios and 5 periods).

Peronne et al (2002) have assumed the following for their theoretical model: 1) demands follow uniform distribution, 2) price depends on mean demand only and 3) the variable cost is same for both dedicated and flexible plants but investment costs are different. System wise profit has been maximized by maximizing each products profit. Investment cost, which has been expressed in terms of unit time multiplied by the service time of the product, in flexible plant has been depended on scope economy factors α and β. According to them, flexibility has been most effective when products with longest service times have been performed in most expensive dedicated resource. The cumulative scope economy factor α for flexible machine is (investment cost of flexible machine)/(investment cost of dedicated machine capable of producing the products that have been produced in flexible plant). This flexible investment cost is less than sum of the total dedicated cost and greater than any of the dedicated cost. Similarly, service time scope economy factor β for any product is (service time in flexible machine)/(total service time in dedicated machine capable of producing the products that have produced in flexible plant). The difference between flexible resource and dedicated resource has been presented in the form of hyperbola i.e. $\alpha\beta$ = constant.

By simulating truncated normal distribution in 10-product-10-plant case Jordan and Graves (1995) have shown that limited flexibility with single chain captures more than 90% of the benefit of total flexibility in terms of expected sales and capacity utilization. According to them, benefit of flexibility has been affected by two factors; demand correlation and total capacity relative to expected total demand. Negatively correlated products have been required to be in the same chain, but might not be in the same plant. If total capacity deviates far from the total expected demand, flexibility has no value. They have argued that there can not be single optimal plan; rather many near optimal plans exist. The product-plant links have been added based on the following rules: 1) try to equalize the no of plants (measure in total units of capacity) to which each product in the chain has been directly connected, 2) try to equalize the no of products (measure in total units of expected demand) to which each plant in the chain has been directly connected and 3) create circuit that encompasses as many plants and products as possible.

Fine and Freund (1990), Meighem (1998) and Bish and Wang (2004) have examined two product situation and analytically studied characteristics of the optimal solution for two product case. In contrast to them, simulated data based models, developed in this paper, have been capable of finding optimal profit and capacity under given parameter values for multiproduct case with complete characterization of demand correlation into the model. However separate values for marginal cost of capacity for dedicated and flexible plants have not been considered. Also, partial product flexibility discussed by Jordan and Graves (1995), has been considered as out of scope for this paper.

3. Two Product Cases:

Consider a manufacturer producing two products wants to set capacity level(s) before realizing the demands. Also consider that, after demand realization, there is no inventory carry over or backorder which can affect next period’s planning. Remaining inventory has been sold at discount, called salvage value. Take D_i as demand and d_i as the realized demand for the product i. The price of product i is P_i per unit, cost is C_i per unit and salvage value is S_i per unit. The firm can decide the production quantity Q_i before demand for product i has been realized or, it can wait till demand realization so that no over-production happens. Similarly firm can go for two dedicated plants with capacity K_i or single flexible plant to produce the products with capacity K. In the following subsections the possible cases has been discussed. For simplicity, subscript i have been omitted in case of dedicated plant strategies.

Assume that the manufacturer starts with no initial resource(s) and incurs investment cost C(K). For simplicity, also assume that C(K) is linear in K, i.e., $C(K) = C_K K$, where C_K depends on whether the firm is using dedicated technology or flexible technology. It has been considered that same amount of capacity has been required to produce one unit of each product, so capacity has been expressed as the number of product units that can be produced. Moreover, there is no cost associated with producing away from installed capacity. These types of assumptions are common in literature.

3.1. Analytical Findings for Two Products:

3.1.1. Dedicated Plant, No Production Postponement:

As there is no production postponement the firm needs to decide both the capacity and production before the demand realization. So there is no point in invest in capacity higher than the production level. In other words, in case of no production postponement K = Q

Possible two situations have been described below with the help of under production and over production costs:

Situation	Profit	Opportunity loss
$D > K$	$(P - C - C_K)K$	$(P - C - C_K)(D - K)$
$D \leq K$	$[(P - C - C_K) - (S - C - C_K)]D + (S - C - C_K)K$	$(S - C - C_K)(D - K)$

So expected profit = $E(\Pi)$

$$= \int_K^{\infty}\left[(P - C - C_K)K\right] f(d)dd + \int_0^K \left[\left[(P - C - C_K) - (S - C - C_K)\right]d + (S - C - C_K)K\right] f(d)dd$$

$$=(P - C - C_K)K - (P - S)\int_0^K\left[(K - d)\right] f(d)dd \quad \textbf{......... (1)}$$

Similarly expected opportunity loss = E(O)

$$= \int_K^{\infty}\left[(P - C - C_K)(d - K)\right] f(d)dd + \int_0^K\left[(S - C - C_K)(D - K)\right] f(d)dd$$

Now, $\frac{d}{dK}\int_0^K df(d)dd = Kf(K)$

$$\frac{\partial E(\Pi)}{\partial K} = (P - C - C_K) - (P - S)[F(K) + Kf(K) - Kf(K)] = (P - C - C_K) - (P - S)F(K)$$

Hence, $F(K) = \frac{(P - C - C_K)}{(P - S)}$ **……… (2)**

3.1.2. Dedicated Plant, Production Postponement:

In case of production postponement, production has been done only after demand realization. Hence there is no over production cost and Q = Min (D, K). However, there has been a need to consider overcapacity cost in this case. There can be two situations as described below:

Situation	Profit	Opportunity loss
$D > K$	$(P - C - C_K)K$	$(P - C - C_K)(D - K)$
$D \leq K$	$(P - C - C_K)D - C_K(K - D)$	$C_K(K - D)$

So expected profit

$$= E(\Pi) = \int_K^\infty [(P - C - C_K)K]\, f(d)dd + \int_0^K [(P - C - C_K)d - C_K(K - D)]\, f(d)dd$$

$$= (P - C - C_K)K - (P - C)\int_0^K [(K - d)]\, f(d)dd \quad \textbf{……… (3)}$$

Similarly expected opportunity loss

$$= E(O) = \int_K^\infty [(P - C - C_K)(d - K)]\, f(d)dd - \int_0^K [C_K(K - D)]\, f(d)dd$$

Hence, $F(K) = \frac{(P - C - C_K)}{(P - C)}$ **……… (4)**

For the strategies discussed above, following propositions have been developed.

Proposition 1: *Production postponement always gives higher optimal capacity and profit for dedicated plant.*

Proof: As C > S, (P – C) < (P – S). Hence, F(K) in eq. (4) > F(K) in eq. (2), where, F(.) is c.d.f. of the distribution and F(.) increases monotonically in K.

Again (P – C) < (P – S) implies $E(\Pi)$in eq. (3) > $E(\Pi)$in eq. (1). ■

Proposition 2: *For normally distributed demand, in absence of production postponement, optimal capacity increases with the increase in variance as long as* $\frac{(P - C - C_K)}{(P - S)} \geq 0.5$*, else optimal capacity decreases with the increase in variance.*

Proof: Consider demand follows normal distribution with mean μ and standard deviation σ. Then, $F(K) = \Phi(\frac{K-\mu}{\sigma})$.

From eq. (2), $K = \mu + \sigma\, \Phi^{-1}\left[\frac{(P-C-C_K)}{(P-S)}\right]$. ■

For the rest of this sub-section proofs have been done taking $\frac{(P-C-C_K)}{(P-C)} \geq 0.5$ for both the products.

Proposition 3: *For normally distributed demand, optimal profit decreases with the increase in variance.*

Proof: $\int_0^K [(K-d)]\, f(d)dd = \int_0^K [(K-d)] \frac{1}{\sigma\sqrt{2\Pi}} e^{-\frac{1}{2}(\frac{d-\mu}{\sigma})^2} dd = (K-\mu)F(K) + \sigma^2\{f(K) - f(0)\}$.

With increase in σ this part increases, which in turn reduces $E(\Pi)$in eq. (1) and eq. (3). ■

3.1.3. Product flexible Plant, No Production Postponement:

When there is no production postponement, it has been shown that there is no added benefit from being product flexible. On the other hand, the investment required might be more for achieving product flexibility. Take total capacity = K, where $K = Q_1 + Q_2$. Below possible situations and the profit values corresponding to those situations have been presented.

Situation	Profit
$D_1 > Q_1$, $D_2 > K - Q_1$	$(P_1 - C_1 - C_K)Q_1 + (P_2 - C_2 - C_K)(K - Q_1)$
$D_1 > Q_1$, $D_2 \leq K - Q_1$	$(P_1 - C_1 - C_K)Q_1 + (P_2 - C_2 - C_K)D_2 + (S_2 - C_2 - C_K)(K - Q_1 - D_2)$
$D_1 \leq Q_1$, $D_2 > K - Q_1$	$(P_1 - C_1 - C_K)D_1 + (S_1 - C_1 - C_K)(Q_1 - D_1) + (P_2 - C_2 - C_K)(K - Q_1)$
$D_1 \leq Q_1$, $D_2 \leq K - Q_1$	$(P_1 - C_1 - C_K)D_1 + (S_1 - C_1 - C_K)(Q_1 - D_1) + (P_2 - C_2 - C_K)D_2 + (S_2 - C_2 - C_K)(K - Q_1 - D_2)$

Hence, $E(\Pi) = \int_{K-Q_1}^{\infty} \int_{Q_1}^{\infty} [(P_1-C_1)Q_1 + (P_2-C_2)(K-Q_1)] f(d_1,d_2)dd_1dd_2$

$+ \int_0^{K-Q_1} \int_{Q_1}^{\infty} [(P_1-C_1)Q_1 + (P_2-C_2)d_2 + (S_2-C_2)(K-Q_1-d_2)] f(d_1,d_2)dd_1dd_2$

$+ \int_{K-Q_1}^{\infty} \int_0^{Q_1} [(P_1-C_1)d_1 + (S_1-C_1)(Q_1-d_1) + (P_2-C_2)(K-Q_1)] f(d_1,d_2)dd_1dd_2$

$+ \int_0^{K-Q_1} \int_0^{Q_1} [(P_1-C_1)d_1 + (S_1-C_1)(Q_1-d_1) + (P_2-C_2)d_2 + (S_2-C_2)(K-Q_1-d_2)] f(d_1,d_2)dd_1dd_2 - C_K K$

……… (5)

Proposition 4: *For independent demands, flexibility does not generate any extra profit compared to corresponding dedicated plants, as long as productions have not been postponed.*

Proof: Considering demands are independent, i.e. $f(d_1,d_2) = f(d_1)f(d_2)$, the expression for expected profit works out as,

$$E(\Pi) = (P_1 - C_1 - C_K)Q_1 + (P_2 - C_2 - C_K)(K - Q_1)$$

$$-(P_1 - S_1)\int_0^{Q_1}[Q_1 - d_1]\, f(d_1)dd_1 - (P_2 - S_2)\int_0^{K-Q_1}[K - Q_1 - d_2]\, f(d_2)dd_2$$

Now, $\frac{\partial E(\Pi)}{\partial K} = (P_2 - C_2 - C_K) - (P_2 - S_2)F_2(K - Q_1) = 0.$

Or, $F_2(K - Q_1) = \frac{(P_2 - C_2 - C_K)}{(P_2 - S_2)} = F_2(Q_2).$

Similarly, $\frac{\partial E(\Pi)}{\partial Q_1} =$

$$(P_1 - C_1 - C_K) - (P_2 - C_2 - C_K) - (P_2 - S_2)F_2(Q_1) - (P_2 - S_2)F_2(K - Q_1)\frac{\partial (K - Q_1)}{\partial Q_1} = 0.$$

Or, $F_1(Q_1) = \frac{(P_1 - C_1 - C_K)}{(P_1 - S_1)}$. ■

3.1.4. Product Flexible Plant:

Take, total capacity = K, where $Q_1 + Q_2 \leq K$

Without the loss of generality, it has also been considered product 1 gives more contribution, i.e. $(P_1\text{-}C_1) \geq (P_2\text{-}C_2)$. Hence the firm will always try to meet the demand of product 1 first and after that it will go for product 2.

Possible situations and the profit and opportunity cost values corresponding to those situations are:

Situation	Profit	Opportunity loss
$D_1 + D_2 > K$	$(P_1 - C_1 - C_K)D_1 + (P_2 - C_2 - C_K)(K - D_1)$	$(P_2 - C_2 - C_K)(D_1 + D_2 - K)$
$D_1 + D_2 \leq K$	$(P_1 - C_1 - C_K)D_1 + (P_2 - C_2 - C_K)D_2 - C_K(K - D_1 - D_2)$	$C_K(K - D_1 - D_2)$

$$E(\Pi) = \int_0^{\infty}\int_{K\text{-}d_1}^{\infty}[(P_1\text{-}C_1)d_1 + (P_2\text{-}C_2)(K\text{-}d_1)]f(d_1,d_2)dd_1dd_2$$

$$+\int_0^{\infty}\int_0^{K\text{-}d_1}[(P_1\text{-}C_1)d_1 + (P_2\text{-}C_2)d_2]f(d_1,d_2)dd_1dd_2 - C_K K$$

Again considering demands are independent, i.e. $f(d_1,d_2) = f(d_1)f(d_2)$, the expression for expected profit works out as,

$$E(\Pi)= \left[(P_1 - C_1) - (P_2 - C_2)\right]\mu_1$$

$$+ (P_2 - C_2 - C_K)K - (P_2 - C_2)\int_0^{\infty}\left[\int_0^{K-d_1}[K - d_1 - d_2]\, f(d_2)dd_2\right] f(d_1)dd_1 \textbf{ .... (6)}$$

Where μ_1 represents mean demand for product 1.

$$\text{Now, } \frac{\partial}{\partial K}\left\{\int_0^{\infty}\left[\int_0^{K-d_1}[K - d_1 - d_2]\, f(d_2)dd_2\right] f(d_1)dd_1\right\} = \int_0^{\infty}\left[\frac{\partial}{\partial K}\int_0^{K-d_1}[K - d_1 - d_2]\, f(d_2)dd_2\right] f(d_1)dd_1$$

$$= \int_0^{\infty}[F_2(K - d_1)]f_1(d_1)dd_1 = \frac{(P_2 - C_2 - C_K)}{(P_2 - C_2)} \quad \textbf{......... (7)}$$

Proposition 5: *For independent and normally distributed demands having negligible probability of having demand less than zero*

a) Flexible plant optimal capacity is less than corresponding dedicated plant total capacities.

b) With the increase in demand variance, optimal capacity of the flexible plant increases, but the increase in optimal capacity is less than corresponding total increase in dedicated plant optimal capacity.

Proof: Consider demand for product i follows normal distribution with mean μ_i and standard deviation σ_i.

$$\text{Now, } \int_0^{\infty}[F_2(K - d_1)]f_1(d_1)dd_1 \cong \int_{-\infty}^{\infty}[F_2(K - d_1)]f_1(d_1)dd_1 = \text{Prob}(D_1 + D_2 \le K) = \Phi\left(\frac{K - \mu_1 - \mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right)$$

$$\text{From eq. (7), } K = \mu_1 + \mu_2 + \sqrt{\sigma_1^2+\sigma_2^2}\left\{\Phi^{-1}\left[\frac{(P_2 - C_2 - C_K)}{(P_2 - C_2)}\right]\right\}$$

$$\text{For dedicated plants, total capacity} = K_D = \mu_1 + \sigma_1\Phi^{-1}\left[\frac{(P_1 - C_1 - C_K)}{(P_1 - C_1)}\right] + \mu_2 + \sigma_2\Phi^{-1}\left[\frac{(P_2 - C_2 - C_K)}{(P_2 - C_2)}\right]$$

$$K_D - K = \sigma_1\Phi^{-1}\left[\frac{(P_1 - C_1 - C_K)}{(P_1 - C_1)}\right] + \sigma_2\Phi^{-1}\left[\frac{(P_2 - C_2 - C_K)}{(P_2 - C_2)}\right] - \sqrt{\sigma_1^2 + \sigma_2^2}\left\{\Phi^{-1}\left[\frac{(P_2 - C_2 - C_K)}{(P_2 - C_2)}\right]\right\}$$

$$\text{As } (P_1 - C_1) \ge (P_2 - C_2),\ \Phi^{-1}\left[\frac{(P_1 - C_1 - C_K)}{(P_1 - C_1)}\right] \ge \Phi^{-1}\left[\frac{(P_1 - C_1 - C_K)}{(P_1 - C_1)}\right]$$

$$\text{Hence, } K_D - K \ge \left[\sigma_1 + \sigma_2 - \sqrt{\sigma_1^2+\sigma_2^2}\right]\Phi^{-1}\left[\frac{(P_1 - C_1 - C_K)}{(P_1 - C_1)}\right]$$

As $\sigma_1 + \sigma_2 \ge \sqrt{\sigma_1^2+\sigma_2^2}$, $K_D \ge K$. This proves the first part.

With the increase in σ_i, K_D and K both increases, but the increase in $\sigma_1+\sigma_2$ is more compared to $\sqrt{\sigma_1^2+\sigma_2^2}$. This proves the second part. ■

Although intuitive, however if one wants to establish the following, analytical difficulty happens in case of flexible plant profit.

For independent and normally distributed demands having negligible probability of having demand less than zero

a) Flexible plant optimal profit is more than corresponding dedicated plant total profits.

b) With the increase in demand variance, optimal profit of the flexible plant decreases, but the decrease in optimal profit is less than corresponding total decrease in dedicated plant optimal profit.

For this purpose one needs to show: $E(\Pi) \geq E(\Pi_D)$.

Where, from eq. (6),

$$E(\Pi)=\left[\left(P_1-C_1\right)-\left(P_2-C_2\right)\right]\mu_1+\left(P_2-C_2-C_K\right)K$$

$$-\left(P_2-C_2\right)\int_0^{\infty}\left[\int_0^{K-d_1}\left[K-d_1-d_2\right]f(d_2)dd_2\right]f(d_1)dd_1$$

From eq. (3), total profit for dedicated plants

$$=E(\Pi_D)=\left(P_1-C_1-C_K\right)K_1-\left(P_1-C_1\right)\int_0^{K_1}\left[(K_1-d_1)\right]f(d_1)dd_1$$

$$+\left(P_2-C_2-C_K\right)K_2-\left(P_2-C_2\right)\int_0^{K_2}\left[(K_2-d_2)\right]f(d_2)dd_2$$

The derivation of flexible plant profit, $E(\Pi)$ has not been tried.

3.2. Simulated Data Based Optimization Procedure:

3.2.1. Methodology:

In the last section it has been observed that even for two product case with demands following independent distribution, finding a closed form solution for optimal profit and corresponding capacity is extremely difficult. The complexity increases if the demands are not independent. Only in some specific cases analytical calculation of stochastic programming is possible as the evaluation of expected value of demand involves calculation of multivariate integrals. A finite discretization of the random data allows writing the expectation in the form of summation and helps to solve the stochastic problem. In this sub-section this methodology has been developed. The model of flexible plant has been considered for this purpose.

The model for flexible plant:

$$\Pi_{\text{Flexible Plant}} = \sum_{i=1}^{m}[(P_i - C_i)\text{Min}(D_i, K_i)] - C_{\text{Flexible Plant}}K$$

Take, $Z_i = \text{Min}(D_i, K_i)$ = Production quantity of product i, where, $\sum_{i=1}^{m} K_i = K$.

The deterministic version of the flexible plant model (where d_i values are known with certainty) can be written as:

$$\begin{aligned} &\text{Maximize} && \sum_{i=1}^{m}[(P_i - C_i)Z_i] - C_{\text{Flexible Plant}}K \\ &\text{Subject to:} && Z_i \le D_i \quad \forall i \\ & && \sum_{i=1}^{m} Z_i - K \le 0 \\ & && Z_i, K \ge 0 \quad \forall i \end{aligned}$$

But in real life d_i values are not known. One has the idea of the distribution of the d_i values only and before the realization of these values one need to set the capacity K. To summarize this, time sequence is as follows (Wagner, 1993, Ch. 16, p. 667);

a) *First stage*: Manufacturer selects level of K.
b) *Random event*: Values of d_i are known and are independent of K.
c) *Second stage*: Manufacturer selects the level of Z_i, the production quantity.

Given the time sequence, manufacturer selects K for which expected profit has been maximized. The problem can be formulated as stochastic programming with recourse in the following way:

First stage problem:

$$\max_{K\ge 0} \Pi(K) = E\left[\Pi^*(K, \mathbf{D})\right] - C_{\text{Flexible Plant}}K$$

Second stage problem:

$$\begin{aligned} &\Pi^*(K, \mathbf{d}) = \max_{\mathbf{Z}\ge 0} \Pi(K, \mathbf{d}) = \max \sum_{i=1}^{m}[(P_i - C_i)Z_i] \\ &\text{Subject to:} \quad Z_i \le d_i \quad \forall i \\ &\qquad\qquad\qquad \sum_{i=1}^{m} Z_i - K \le 0 \end{aligned}$$

Here, $\mathbf{D} = (D_1, D_2, \ldots, D_m)$, $\mathbf{d} = (d_1, d_2, \ldots, d_m)$ and $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_m)$. Also E(.) is the expectation operator.

Now, consider only three values of **d** are possible with known probability p_js where $\sum_{j=1}^{3} p_j = 1$.

Hence the possible cases are (considering two product case): d_{11}, d_{12} with probability p_1; d_{21}, d_{22} with probability p_2 and d_{31}, d_{32} with probability p_3. Since K has been determined before the realization of the demand values, these variables will also come in stochastic programming formulation. The remaining decision variables Z_i have been determined after the realization of the demand. Hence, they have been noted as Z_{ij} for i = 1, 2 and j = 1, 2, 3.

As long as the decision variables in the first stage (here capacity K) do not depend on the realization of the random event, the two stage problem can be expressed as a single optimization model like below:

$$\text{Maximize} \quad \sum_{j=1}^{3}\left\{\sum_{i=1}^{2}\left[(P_i - C_i)Z_{ij}\right]\right\}p_j - C_{\text{Flexible Plant}}K$$

$$\text{Subject to:} \quad \begin{aligned} & Z_{ij} \le d_{ij} && \forall\, i, j \\ & \textstyle\sum_{i=1}^{2} Z_{ij} - K \le 0 && \forall\, j \\ & Z_{ij}, K \ge 0 && \forall\, i, j \end{aligned}$$

Here, the stochastic programming version of the problem has more number of constraints compared to its deterministic version.

As the first stage variable K do not depend on the outcome of the j^{th} scenario, objective function can be rewritten as:

$$\text{Maximize} \quad - C_{\text{Flexible Plant}}K + E\left[\left\{\sum_{i=1}^{2}\left[(P_i - C_i)Z_{ij}\right]\right\}p_j\right]$$

Finally, the distribution of **d** has been approximated by taking large number of values generated from the distribution. So all p_j values are equally likely and the model becomes:

$$\text{Maximize} \quad \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{2}\left[(P_i - C_i)Z_{ij}\right] - C_{\text{Flexible Plant}}K$$

$$\text{Subject to:} \quad \begin{aligned} & Z_{ij} \le d_{ij} && \forall\, i, j \\ & \textstyle\sum_{i=1}^{2} Z_{ij} - K \le 0 && \forall\, j \\ & Z_{ij}, K \ge 0 && \forall\, i, j \end{aligned}$$

However, with this procedure one trade off has been necessary. On one side, with the increase in number of products sample sets needs to be increased, otherwise the gap between sample statistic and parameter value increases. On the other side, with the increase in sample values the complexity of the problem increases and with the increase in number of products the complexity increases exponentially. Hence, to keep the accuracy of the results high, too many products have not been considered.

As discussed earlier in proposition 4, flexible plant without production postponement is not better option compared to multiple dedicated plants. So this strategy has been omitted in this section. The additional notations used in the models have been shown below:

d_{ij} = Demand for product i at iteration j

$C_{\text{Strategy}\ S}$ = marginal cost of capacity for strategy S

3.2.2. Models:

Two Product, Dedicated Plants, No Production Postponement:

$\Pi_{\text{Strategy }1} = \sum_{i=1}^{2}[P_i \text{Min}(D_i, K_i) + S_i \text{Max}(K_i - D_i, 0) - C_i K_i] - C_{\text{Strategy }1} \sum_{i=1}^{2} K_i$

Take, $Z_i = \text{Min}(D_i, K_i)$ = Sales quantity of product i in the primary market

Then, $\text{Max}(K_i - D_i, 0) = -\text{Min}(D_i - K_i, 0) = -\text{Min}(D_i, K_i) + K_i = -Z_i + K_i$

<u>Hence the simulated data based optimization model becomes,</u>

Maximize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{2}\left[(P_i - S_i)Z_{ij}\right] - \sum_{i=1}^{2}(C_{\text{Strategy }1} + C_i - S_i)K_i$

Subject to:

$$Z_{ij} \leq d_{ij} \quad \forall\, i, j$$
$$Z_{ij} - K_i \leq 0 \quad \forall\, i, j$$
$$Z_{ij}, K_i \geq 0 \quad \forall\, i, j$$

Two Product, Dedicated Plants, Production Postponement:

$\Pi_{\text{Strategy }2} = \sum_{i=1}^{2}[(P_i - C_i)\text{Min}(D_i, K_i)] - C_{\text{Strategy }2} \sum_{i=1}^{2} K_i$

Take, $M_i = \text{Min}(D_i, K_i)$ = Production quantity of product i

<u>Hence the simulated data based optimization model becomes,</u>

Maximize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{2}\left[(P_i - C_i)M_{ij}\right] - C_{\text{Strategy }2}\sum_{i=1}^{2} K_i$

Subject to:

$$M_{ij} \leq d_{ij} \quad \forall\, i, j$$
$$M_{ij} - K_i \leq 0 \quad \forall\, i, j$$
$$M_{ij}, K_i \geq 0 \quad \forall\, i, j$$

Two Product, Flexible Plant:

This strategy has been discussed already in the previous sub-section.

3.3. Comparison between Analytical and Simulated Data Based Procedure:

To find optimal capacity levels and maximum profit and corresponding optimal capacity values for the three strategies discussed above, working has been done on two different parameter sets. The values have been generated by both analytical (wherever possible) and simulated data based procedure. In both the examples demands have been considered to be followed independent normal distribution with given parameters. Using these parameters 10,000 demand scenarios has been generated. Percent deviation has been calculated using the following formula:

Percent deviation = (Simulated data based result– Analytical result)×100/(Analytical result)

Example 1: Consider marginal cost of Capacity for any case = 4.

Other Parameters are shown below:

Data	Price	Cost	Salvage Value	Mean Demand	Std. dev. of Demand
Product 1	15	9	5	100	25
Product 2	13	8	3	200	40

Analytical based results	Case 1		Case 2		Case 3	
	Capacity	Profit	Capacity	Profit	Capacity	Profit
Product 1	78.96	130.0	89.23	145.5	260.3	--
Product 2	148.74	129.8	166.34	144.0		

Simulated data based results	Case 1		Case 2		Case 3	
	Capacity	Profit	Capacity	Profit	Capacity	Profit
Product 1	79.24	130.7	89.59	146.2	260.74	334.2
Product 2	148.25	129.4	166.03	143.6		

Percent deviation	Case 1		Case 2		Case 3	
	Capacity	Profit	Capacity	Profit	Capacity	Profit
Product 1	0.35	0.54	0.40	0.48	0.17	--
Product 2	-0.33	-0.31	-0.19	-0.28		

Example 2: Consider marginal cost of Capacity for any case = 4.

Other Parameters are shown below:

Data	Price	Cost	Salvage Value	Mean Demand	Std. dev. of Demand
Product 1	15	9	5	200	40
Product 2	13	8	3	100	25

Analytical based results	Case 1		Case 2		Case 3	
	Capacity	Profit	Capacity	Profit	Capacity	Profit
Product 1	166.34	288	182.77	312.7	260.3	--
Product 2	67.96	56.2	78.96	65.0		

Simulated data based results	Case 1		Case 2		Case 3	
	Capacity	Profit	Capacity	Profit	Capacity	Profit
Product 1	166.05	288.2	182.79	312.6	260.32	434.6
Product 2	68.06	56.5	78.9	65.2		

Percent deviation	Case 1		Case 2		Case 3	
	Capacity	Profit	Capacity	Profit	Capacity	Profit
Product 1	-0.17	0.07	0.01	-0.03	0.01	--
Product 2	0.15	0.53	-0.08	0.31		

From the above examples one can conclude that the results found using simulated data based optimization procedures are very close to the results found using analytical procedures (deviations are less than 0.5% for most of the cases). Now, in the next sections, multivariate analysis and correlation will be introduced, and this becomes extremely difficult if not impossible to solve by analytical method and obtain closed form solution for optimum profit and capacity levels. Hence for the rest of the paper, whenever it has been required to maximize profit for the optimal capacity levels, simulated data based optimization has been used.

4. Multi Product Cases:

4.1. Methodology:

In this section multivariate normal demand distribution has been considered, so that it can capture the effects of correlation on profit level. Normal numbers have been generated by using variance-covariance matrix. The demands of the products $D_i \in R_+$ are random draws from a multivariate normal distribution function. For product i, realization of demand is d_i, the mean of the marginal distribution is μ_i, the variance is σ_i^2, and the covariance of the joint distribution is $\sigma_{ik} = \rho_{ik}\sigma_i\sigma_k$, where $1 \geq \rho_{ik} \geq -1$ for $i \neq k$.

For three products following correlated multivariate distribution, finite discretization of random parameter allows writing the expectation in the form of summation and makes the problem tractable. The random multivariate normal numbers have been produced by pre-multiplying a vector of random univariate normal numbers by the Cholesky decomposition of the Variance–Covariance matrix (**V**) according to the formula:

$$\mathbf{Z} = \boldsymbol{\mu} + \mathbf{LX} \quad \text{........ (8)}$$

Where,

Z = a vector of random multivariate normal numbers

μ = a vector of mean of the marginal distribution

X = a vector of random univariate normal numbers

L = the Cholesky decomposition of the covariance matrix.

Here the values derived from the Cholesky decomposition have been stored in the lower triangle and main diagonal of a square matrix; elements in the upper triangle of the matrix are 0.

If variance–covariance matrix is real, symmetric and positive definite, then Cholesky decomposition exists.

Positive-definiteness:

An arbitrary matrix is positive definite if and only if all the principal sub-matrices have a positive determinant.

4.1.1. Cholesky decomposition Algorithm:

V=LL^T

Start with **L**=*0*

for i=1… m do

Subtract from $v_{i,i}$, the dot product of the ith row of L with itself and set $l_{i,i}$ to be the square root of this.

for j=i+1,…,m

Subtract from $l_{i,j}$, the dot product of the ith and jth rows of L and set $l_{j,i}$ to be this result divided by $l_{i,i}$.

4.1.2. Example of Cholesky Decomposition:

Consider **V**, a [3×3] matrix, as given below:

$$\mathbf{V}=\begin{bmatrix}10 & 2 & -4\\ 2 & 15 & 1\\ -4 & 1 & 6\end{bmatrix}$$

Matrix is real, symmetric and positive definite. Hence Cholesky decomposition exists. The steps are shown below:

$$\mathbf{L^1}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix};$$

For i = 1, $v_{1,1}$ = 10 and 1st row of **L** = [0 0 0]; the dot product = 0×0 + 0×0 + 0×0 = 0; hence, $l_{1,1} = \sqrt{(v_{1,1} - \text{dot product})} = \sqrt{(10 - 0)} = 3.16$.

$$\mathbf{L^2}=\begin{bmatrix}3.16 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix};$$

Given i = 1, for j = 2, 1st row of **L** = [3.16 0 0] and 2nd row of **L** = [0 0 0]; the dot product = 3.16×0 + 0×0 + 0×0 = 0; hence $l_{2,1} = (v_{2,1} - \text{dot product})/\ l_{1,1} = 2/3.16 = 0.63$.

Given i = 1, for j = 3, 1st row of **L** = [3.16 0 0] and 3rd row of **L** = [0 0 0]; the dot product = 3.16×0 + 0×0 + 0×0 = 0; hence $l_{3,1}$ = ($v_{3,1}$ – dot product)/ $l_{1,1}$ = – 4/3.16 = – 1.26.

$$\mathbf{L^3} = \begin{bmatrix} 3.16 & 0 & 0 \\ 0.63 & 0 & 0 \\ -1.26 & 0 & 0 \end{bmatrix};$$

For i = 2, $v_{2,2}$ = 15 and 2nd row of **L** = [0.63 0 0]; the dot product = 0.63×0.63 + 0×0 + 0×0 = 0.3969; hence, $l_{2,2} = \sqrt{(v_{2,2} - \text{dot product})} = \sqrt{(15 - 0.3969)} = 3.82$.

$$\mathbf{L^4} = \begin{bmatrix} 3.16 & 0 & 0 \\ 0.63 & 3.82 & 0 \\ -1.26 & 0 & 0 \end{bmatrix}$$

Given i = 2, for j = 3, 2nd row of **L** = [0.63 3.82 0] and 3rd row of **L** = [-1.26 0 0]; the dot product = 0.63× (–1.26) + 3.82×0 + 0×0 = 0; hence $l_{3,2}$ = ($v_{3,2}$ – dot product)/ $l_{2,2}$ = (1 – (–0.79))/3.82 = 0.47.

$$\mathbf{L^5} = \begin{bmatrix} 3.16 & 0 & 0 \\ 0.63 & 3.82 & 0 \\ -1.26 & 0.47 & 0 \end{bmatrix};$$

For i = 3, $v_{3,3}$ = 6 and 3rd row of **L** = [– 1.26 0.47 0]; the dot product = (– 1.26)×(– 1.26) + 0.47×0.47 + 0×0 = 1.8; hence, $l_{3,3} = \sqrt{(v_{3,3} - \text{dot product})} = \sqrt{(6 - 1.8)} = 2.04$.

Finally $\mathbf{L} = \begin{bmatrix} 3.16 & 0 & 0 \\ 0.63 & 3.82 & 0 \\ -1.26 & 0.47 & 2.04 \end{bmatrix}$

Now, as per eq. (1) one only needs to generate **X**, column vector of standard normal random numbers. Then by the use of eq. (1), each set of **X** has been used generates one set of random numbers from multivariate normal distribution. In this way, 10,000 sets of samples have generated for the purpose. It has been seen that, with this large number the samples, statistics follow original distribution parameters.

Alike previous section, here also 10,000 demand data sets have been used for the optimization procedure. Models for simulated data based optimization are same as two product cases (section 3.3), except total number of products in these cases are 3. Hence, in this sub-section opportunity loss models for the above-mentioned strategies have been introduced.

4.2. Opportunity Loss Models:

Multi Product, Dedicated Plants, No Production Postponement:

$$\Pi_{Strategy\ 1} = \sum_{i=1}^{m}[(P_i - C_i)Max(D_i - K_i, 0) + (S_i - C_i)Max(K_i - D_i, 0)] + C_{Strategy\ 1}\sum_{i=1}^{m} K_i$$

<u>Hence the model becomes,</u>

Minimize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[(P_i - C_i)U_{ij} - (S_i - C_i)V_{ij}\right] + C_{Strategy\ 1}\sum_{i=1}^{m} K_i$

Subject to: $D_{ij} - K_i = U_{ij} - V_{ij} \quad \forall\, i, j$

$U_{ij}, V_{ij}, K_i \geq 0 \quad \forall\, i, j$

Multi Product, Dedicated Plants, Production Postponement:

$$\Pi_{Strategy\ 2} = \sum_{i=1}^{m}[(P_i - C_i)Max(D_i - K_i, 0)] + C_{Strategy\ 2}\sum_{i=1}^{m} K_i$$

<u>Hence the model becomes,</u>

Minimize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[(P_i - C_i)U_{ij}\right] + C_{Strategy\ 2}\sum_{i=1}^{m} K_i$

Subject to: $D_{ij} - K_i = U_{ij} - V_{ij} \quad \forall\, i, j$

$U_{ij}, V_{ij}, K_i \geq 0 \quad \forall\, i, j$

Multi Product, Flexible Plant:

$$\Pi_{Strategy\ 3} = \sum_{i=1}^{m}[(P_i - C_i)Max(D_i - K_i, 0)] + C_{Strategy\ 3}K$$

<u>Hence the model becomes,</u>

Minimize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[(P_i - C_i)U_{ij}\right] + C_{Strategy\ 3}K$

Subject to: $\sum_{i=1}^{m} D_{ij} - K = \sum_{i=1}^{m}\left(U_{ij} - V_{ij}\right) \quad \forall\, j$

$U_{ij}, V_{ij}, K_i \geq 0 \quad \forall\, i, j$

As intuitive, capacity values in case of profit models and opportunity loss models are same.

4.3. Findings:

Variance and coefficient of variation represent two common measures of individual level of demand uncertainty. On the other hand change in correlation changes aggregate level demand uncertainty keeping variance unchanged. In the numerical analysis, considering three-product environment, the effect of these uncertainties on optimal expected total profit and corresponding total capacity level has been tried to capture for all the strategies discussed above. μ_i = 500, P_i = 80, C_i = 20, S_i = 5 for all products and $C_{Strategy\ S}$ = 10 for all strategies. However, incorporating differences in cost of capacities of dedicated and flexible plants can be easily done. To check the effects of uncertainty following parameter sets have been considered: changes in coefficient of variation = {0.05, 0.1, 0.15, 0.2}, changes in variance = {2000, 4000, 6000, 8000, 10000}, changes in correlation = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5} for all products. The results are tabulated in Appendix A.1. The graphs have been shown in Appendix E. The observations have been discussed below:

1) Except when correlation is negative, the capacity of the flexible plant remains in between total capacity of the dedicated plants having no postponement and the same having production postponement, with last one giving the highest capacity. However, for highly positively correlated (0.99) demands, under postponement, the capacity of flexible plant becomes equal to aggregate capacity of dedicated plants. For negatively correlated demands, flexible plant optimal capacity is always the least. Intuitively, production postponement tends to increase the capacity as a result of elimination of overproduction, while the flexibility reduces the capacity due to pooling effect. For highly negatively correlated demands, aggregate demand variance almost reduces to zero and capacity approaches total mean demand value. For example in our case minimum possible correlation is – 0.5, as variance–covariance matrix does not remain positive definite below this value. At this level of correlation, irrespective of variance, total demand realization becomes 1500. Hence there is no aggregate level of uncertainty at this value and flexible plant capacity also remains at 1500.
2) In terms of profit, flexible plant always remains the best choice, followed by dedicated plant with production postponement; dedicated plants having no postponement give the least profit. For negatively correlated demand benefit from flexible plant is intuitive as below average realized demand for one product has been compensated by the higher than average realized demand for another product. For example alike capacity, with correlation of – 0.5, profit remains unaffected by the variance level. However, for highly positively correlated (0.99) demands, under postponement, the profit from flexible plant becomes equal to that of dedicated plant.
3) For dedicated plant strategies, profit and capacity remains unaffected with the change in correlation coefficient. However, with the reduction in correlation flexible plant optimal profit increases and capacity decreases.

Now the effects of uncertainties have been discussed on the strategies for a) change in demand differential, b) change in price differential, c) change in price and d) change in capacity cost.

a) When the effects of change in demand differential have been examined, mean demands for the products have been considered as follows: {500, 500, 500}; {400, 500, 600} and {250, 500, 750}. This helps to observe the effects on three levels of demand differential, {0, 200, 500}, average mean demand unchanged, where first one corresponds to the base case. Other values remain same: P_i = 80, C_i = 20, S_i = 5 and $C_{\text{Strategy S}}$ = 10. The results have been tabulated in Appendix A.1. The graphs have been shown in Appendix E. it has been observed that, changes in demand differential, has no effect on optimal profit and capacity for any of the three strategies, but individual profits and capacities change.

b) For examining the effects of change in price differential, prices for the products have been considered as follows: {80, 80, 80}; {60, 80, 100} and {40, 80, 120}. This helps to observe the effects on three levels of price differential, {0, 40, 80}, where first one corresponds to the base case and average product price remains unchanged. Other values remain same: μ_i = 500, C_i = 20, S_i = 5 and $C_{\text{Strategy S}}$ = 10. The results have been tabulated in Appendix A.2. The graphs have been shown in Appendix E. Increase in price differential decreases capacity for all three strategies. However, the effect of price differential on optimal total profit is not much.

c) For observing the effects of change in price, three price levels, {40, 55, 80} have been considered, where all the products have same price. Other values remain same: μ_i = 500, C_i = 20, S_i = 5 and $C_{\text{Strategy S}}$ = 10. The results have been tabulated in Appendix A.3. The graphs have been shown in Appendix E. The effects of no postponement and postponement on optimal total capacity and profit have been discussed below.

    1) In case of dedicated plant with no production postponement strategy when price of the product is low (40), capacity is less than the expected total demand and capacity decreases with the increase in variance. Similarly, when price of the product is high (80), capacity is greater than the expected total demand and capacity increases with the increase in variance. The reason is quite simple; when price is low, then cost of overstocking (20 + 10 – 5 = 25) exceeds the cost of understocking (40 – 20 – 10 = 10) and the capacity is maintained at a lower side. With the increase in variance, expected loss from overstocking increases more compared to expected loss from understocking.

Hence the capacity also reduces. When the price is high exactly opposite happens (cost of overstocking = 25 < cost of understocking = 50). When cost of overstocking and cost of understocking are almost same (at price 55 the value is 25), capacity has been maintained near to expected demand value and capacity remains indifferent with the change in variance. However optimal expected profit always reduces with the increase in variance or coefficient of variation as with the increase in individual uncertainty both the expected understocking and expected overstocking cost increases. These results are in line with the analytical findings in two product case.

2) In case of production postponement, capacity increases and profit reduces with the increase in variance for both dedicated and flexible plants. However, when the cost of overcapacity and the cost of undercapacity both remain same, optimal total capacity level remains almost equal to total mean demand in both dedicated and flexible plants having production postponement and remains unaffected by the changes in variance and correlation. For example, when price is 55, cost of undercapacity = 55 – 20 – 10 =25; when price is 40, cost of undercapacity = 40 – 20 – 10 = 10, where the cost of overcapacity = capacity cost = 10. In the first case total capacity is higher than mean demand, 1500. However, in the second case total capacity approaches the mean demand and remains unaffected by variance. The reason is quite simple. In case of production postponement there is no chance of overproduction, but there is always cost of underproduction due to capacity constraint and is same as undercapacity cost. As long as the cost of overcapacity does not exceed the cost of undercapacity, firm always gains from higher realized demand by maintaining higher capacity. At the same time, there is no loss from low realized demand except having idle capacity. But if the overcapacity cost is higher, the firm only tries to maintain capacity at mean demand level.

d) For looking into the effects of change in capacity cost, three levels, {5, 10, 15} have been considered, where all the strategies have same capacity costs. Other values remain same: μ_i = 500, P_i = 80, C_i = 20 and S_i = 5. The results have been tabulated in Appendix A.4. The graphs have been shown in Appendix E. Increase in cost of capacity reduces both capacity and profit in all cases.

In all the cases discussed above, change in correlation has no effect on unmet demand percentage for dedicated as well as flexible plant. However, with highly negatively correlated demand, when aggregate demand variance becomes negligible, flexible plant has no unmet demand (See Appendix C).

As flexible plant is effective only if there is production postponement, an index called 'PdPPF Index' has been introduced to check the effect of production postponement on flexible plant profit where 'PdPPF' stands for 'Production Postponement effect on Flexible plant'. The index is calculated as below:

$$\text{PdPPF Index} = \frac{\text{Profit}_{\textit{Dedicated Plant, Postponement}} - \text{Profit}_{\textit{Dedicated Plant, No Postponement}}}{\text{Profit}_{\textit{Flexible Plant}} - \text{Profit}_{\textit{Dedicated Plant, No Postponement}}} \times 100\%$$

A reduction in PdPPF index with the increase in a particular parameter indicates that the abovementioned effect reduces and suggests that manufacturer can invest more in product flexible technology. Hence, this PdPPF index can also be considered as the proxy of the value of product flexibility. Although, profit decreases with the increase in variance for all strategies, PdPPF index remains unaffected in variance or coefficient of variation; which means, the value of product flexibility has not been affected by the individual level demand uncertainty. However, PdPPF index decreases with the decrease in correlation. With the increase in negative correlation, manufacturer's incentive to invest in product flexibility increases. PdPPF index also decreases with the increase in price differential or with the decrease in marginal cost of capacity. So it can be concluded that the value of product flexibility increases in price differential. Product flexibility becomes more effective when higher price differential or lower marginal cost of capacity has been combined with lower correlation. With the increase in negative correlation, the price differential effect diminishes with increase in negative correlation, but the effect of marginal cost of capacity increases with the increase in negative correlation. The explanations have been given below (For PdPPF Index values see Appendix B.5):

a) With the change in price differential, the total profits of dedicated plant (both postponement and no postponement) strategies have not been affected much. They also remain unaffected with the change in correlation. However, the profit of flexible plant increases with decrease in correlation due to pooling effect. With the increase in price differential, this additional gain from flexibility becomes less effective. In other words, correlation effect acts better in flexible plant when price differential is low. The explanation is simple. With the increase in price differential, the flexible firm increases the option to allocate more of it's resource to the high price product, so that it can always meet the demand of high price product, even at the cost of low price product. As a result in case of high correlation also, flexible firm profit is more compared to dedicated plant with postponement. As a result, for different price differentials, PdPPF Index converges with decrease in correlation. The same can be observed in graph also (See Appendix E, Graph 6(a)).

b) With the reduction in correlation, PdPPF Index decreases irrespective of the price of the product. However, with very high correlation (0.99) they converge at value =1. The reason is quite intuitive. At 0.99 correlation value, there is no additional gain from flexible plant over dedicated plant with postponement. So the optimal profit in both cases remains same and the PdPPF Index value becomes unity. With the change in correlation the optimal profit in dedicated strategies do not change, but the profit of flexible plant increases due to pooling effect. Hence, PdPPF Index decreases. However, with the reduction in product price, this additional gain from flexibility becomes less effective (with low price product the flexible plant profit range reduces). In other words, correlation effect acts better in flexible plant when prices of the products are high. But, at the same time, with the decrease in price the extra benefit of postponement reduces as the cost of understocking reduces and capacity has been maintained at lower side. Hence, the difference between profits in dedicated plant strategies reduces. As a result with the decrease in correlation PdPPF Index diverges. The same can be observed in graph also (See Appendix E, Graph 6(b)).

c) In case of change in capacity cost, the structure of the graph and explanation on PdPPF Index is same as the effect of change in price. The same can be observed in graph also (See Appendix E, Graph 6(c)).

5. Service level constraint in multi product case:

In today's customer oriented business, maximizing profit is not the only target for firms. They also need to consider the service level as a satisfying objective. Here, service level means the expected number of cases where the demand has been met. Maximizing expected profit being the main objective of the firm, service level objective has been taken as constraint to the models. The aggregate service level has been calculated by averaging individual service levels, based on the assumption that same cost of stock-out occasions for products.

5.1. Mixed Integer Models for Satisfying Aggregate Service Level:

When constraints have been added for satisfying service level in models presented in section 3.2.2 one can go for the following argument. Given A = a large number, I = binary variable, D = demand of product, Z = Sales level, if $A*I \geq D - Z$, then

$D - Z > 0 \rightarrow I = 1$

$D - Z = 0 \rightarrow I = 0, 1$

$D - Z < 0 \rightarrow I = 0, 1$

Then, if "total $I \leq$ a given value" has been used as a constraint, it will try to assign zero to I values, whenever required. In other words, it will try to make $D - Z \leq 0$. So unmet demand instances will be reduced upto the required level.

Multi Product, Dedicated Plants, No Production Postponement (Service level ≥ 70%):

Maximize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[(P_i - S_i)Z_{ij}\right] - \sum_{i=1}^{m}(C_{Strategy\ 1} + C_i - S_i)K_i$

Subject to:

$$
\begin{aligned}
&Z_{ij} \leq d_{ij} && \forall\, i, j\\
&Z_{ij} - K_i \leq 0 && \forall\, i, j\\
&d_{ij} - Z_{ij} - A \times I_{ij} \leq 0 && \forall\, i, j\\
&\textstyle\sum_{j=1}^{n}\sum_{i=1}^{m} I_{ij} \leq 0.3mn &&\\
&I_{ij}\ \text{binary} && \forall\, i, j\\
&Z_{ij}, K_i \geq 0 && \forall\, i, j\\
&A = \text{Big positive number} &&
\end{aligned}
$$

Multi Product, Dedicated Plants, Production Postponement (Service level ≥ 90%):

Maximize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[(P_i - C_i)M_{ij}\right] - C_{Strategy\ 2}\sum_{i=1}^{m} K_i$

Subject to:

$$
\begin{aligned}
&M_{ij} \leq d_{ij} && \forall\, i, j\\
&M_{ij} - K_i \leq 0 && \forall\, i, j\\
&d_{ij} - M_{ij} - A \times I_{ij} \leq 0 && \forall\, i, j\\
&\textstyle\sum_{j=1}^{n}\sum_{i=1}^{m} I_{ij} \leq 0.1mn &&\\
&I_{ij}\ \text{binary} && \forall\, i, j\\
&M_{ij}, K_i \geq 0 && \forall\, i, j\\
&A = \text{Big positive number} &&
\end{aligned}
$$

Multi Product, Flexible Plant, Production Postponement (Service level ≥ 90%):

Variable $M_{ij} \geq 0 \ \ \forall\, i, j;$ $K \geq 0;$ I_{ij} binary $\forall\, i, j$

Maximize $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[(P_i - C_i)M_{ij}\right] - C_{Strategy\ 3}K$

Subject to:

$$M_{ij} \leq d_{ij} \qquad \forall\, i, j$$
$$\sum_{i=1}^{m} M_{ij} - K_i \leq 0 \qquad \forall\, i, j$$
$$d_{ij} - M_{ij} - A \times I_{ij} \leq 0 \qquad \forall\, i, j$$
$$\sum_{j=1}^{n} \sum_{i=1}^{m} I_{ij} \leq 0.1mn$$
$$I_{ij} \text{ binary} \qquad \forall\, i, j$$
$$M_{ij}, K_i \geq 0 \qquad \forall\, i, j$$

A = Big positive number

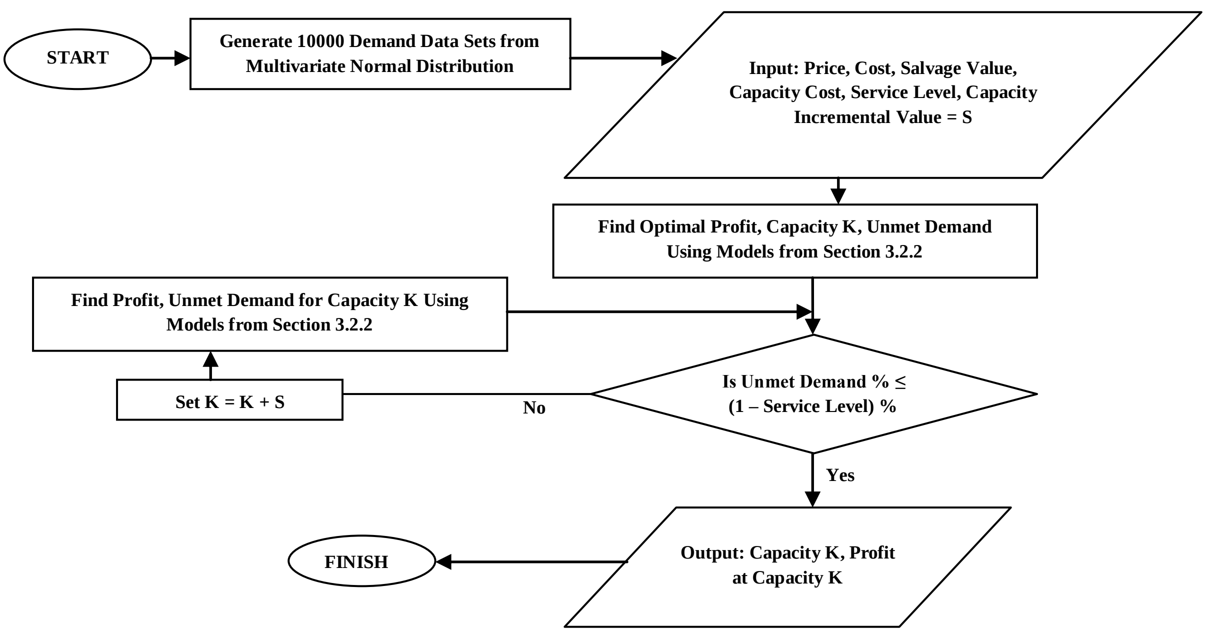
When one goes for satisfying individual service level only fourth constraint changes as below:

$$\sum_{j=1}^{n} I_{ij} \leq 0.3n \qquad \forall\, i \text{ (When required service level} \geq 70\%\text{) and,}$$

$$\sum_{j=1}^{n} I_{ij} \leq 0.1n \qquad \forall\, i \text{ (When required service level} \geq 90\%\text{)}$$

The problem with MIP models discussed above is that in many cases it can not perform with more than 100 demand data sets. So alternate approach has been adopted, which has been discussed next.

5.2. Simulation Diagram for Checking Required Service Level:



5.3. Results and Findings:

To compare the results with section 4.3 same parameter values have been kept. The observations have been discussed below (For PdPPF Index values with SLC see Appendix B.5):

1) Observations regarding PdPPF Index: PdPPF decreases with decrease in correlation in both SLC and without SLC. At same correlation level, with SLC, PdPPF increases with decrease in price differential, increase in price and decrease in capacity cost. However, without SLC PdPPF values converges at highly negative correlation when change in price differential happens, and it converges at highly positive correlation when change in price or change in capacity cost happens. On the other hand, when service level constraint has been imposed, PdPPF Index diverges with decrease in correlation for different price differential levels as well as for different price levels. However, with different capacity cost PdPPF Index remains parallel. The explanations have been given below:

a) As the service level constraints in any of the price differential levels are not violated much, the differences between profits in dedicated plant strategies are alike in this case compared to the cases without SLC. Hence, when operated under SLC, for different price differential levels, PdPPF Index converges with decrease in correlation. The same can be observed in graph also (See Appendix E, Graph 7(a)).
b) With the decrease in the product price, rate of decrease in Unmet Demand Percentage (hence, UD %) is more in case of dedicated plant with postponement compared to flexible plant. On the other hand, dedicated plant with no postponement has been affected most with service level constraint. As a result, the less the product price, the more profit reduction happens for dedicated plant strategies to fulfill the service level requirement. When price values are 80 and 55, flexible plant service levels are not violated and, in case of price = 40, a small reduction in profit happens to maintain the service level. Hence, even with correlation = 0.99, the profits between dedicated plant with postponement and flexible plant differs and this difference increases with decrease in product price. However, with the decrease in correlation, dedicated plant (both postponement and no postponement) profits with SLC remain same as optimal profit (without SLC) and UD % do not change in correlation for dedicated plants. As with the reduction in product price, additional gain from flexibility becomes less effective, the denominator of the PdPPF Index shows less increase with the reduction in correlation when product price is low. On the other hand, the reduction in differences between profits in dedicated plant strategies are less compared to the cases without SLC. As a result, as product price decreases, the PdPPF Index decreases less with the reduction in correlation. In other words, when

operated under SLC, for different product prices, PdPPF Index diverges with increase in correlation. The same can be observed in graph also (See Appendix E, Graph 7(b)).

c) In case of change in capacity cost, the reason for non convergence at correlation = 0.99 is same as the effect of price change. However, as the service level constraints in any of the capacity cost levels are not violated much, the reduction in differences between profits in dedicated plant strategies are alike in this case compared to the cases without SLC. Hence, when operated under SLC, for different capacity cost levels, PdPPF Index remains parallel with the change in correlation. The same can be observed in graph also (See Appendix E, Graph 7(c)).

2) Observations regarding reduction in profit from imposing Service Level Constraint (SLC): Reduction in Profit Percentage = RP % = [(Profit with SLC – Profit without SLC) ×100/ Profit without SLC]. More negative value of reduction in profit means more decrease in profit with SLC (See Appendix D).

a) In case of dedicated plant with postponement, correlation has no effect on RP %. The RP % decreases with correlation in case of flexible plant.
b) With increase in variance and decrease in price of products, RP % decreases for both dedicated and flexible plant. However the effect is less in case of flexible plant.
c) Change in price differential or change in capacity cost has little effect on the RP %.

6. Conclusion:

This paper deals with the optimal capacity planning under demand uncertainty. Simulated data based optimization procedure used in this paper helped to solve the multi-product two stage stochastic linear programming which is otherwise analytically intractable. The effect of production postponement increases profit, but flexible plant may generate higher profit compared to dedicated plants depending on the cost of flexible technology. For dedicated plant strategies, profit and capacity remains unaffected with the change in correlation coefficient. However, with the reduction in correlation flexible plant optimal profit increases and capacity decreases. On the other hand, change in demand differential or price differential has no effect on aggregate capacity or profit for any of the three strategies. But profit reduces with the reduction in price or increase in capacity cost.

The PdPPF index introduced in the paper is helpful in deciding on choice between dedicated and product flexible plant. The value of flexibility has not been affected by the change in individual demand uncertainty, but effectiveness of product flexibility increases with negatively correlated demands. The change in demand differential has no effect on aggregate capacity or aggregate profit level in any of the three strategies, however, increase in price differential or decrease in marginal cost of capacity increases the value of flexibility.

Change in correlation has no effect on unmet demand percentage for dedicated as well as flexible plant. However, with highly negatively correlated demand, when aggregate demand variance becomes negligible, flexible plant has no unmet demand. On the other hand, when service level constraint has been imposed, PdPPF Index diverges with decrease in correlation for different price differential levels as well as for different price levels. However, with different capacity cost PdPPF Index remains parallel.

The main contribution of this paper is threefold. First, the procedure of finding optimal profit and capacity has been developed for dedicated and flexible plant facing multivariate correlated demand distribution, which is otherwise analytically intractable. Second, PdPPF Index has been introduced as a proxy for value of product flexibility to find several meaningful insights based on the changes in various parameters. Third, the service level objective has been added to look into the problem from multi-objective angle.

Several extensions to the models are possible. We are currently working on price dependent demand scenario to accommodate price postponement into our model and observe the effect of product substitutability. Another extension on which we are also working is to extend the models for multi-period scenario.

References:

[1] E. K. Bish and Q. Wang, "Optimal investment strategies for flexible resources, considering pricing and correlated demands," Operations Research, vol. 52, pp. 954-964, 2004.

[2] G. D. Eppen, R. K. Martin, and L. Schrage "A Scenario Approach to Capacity Planning," Operations Research, vol. 37, pp. 517-527, 1989.

[3] C. H. Fine and R. M. Freund, "Optimal investment in product-flexible manufacturing capacity," Management Science, vol. 36, pp. 449-467, 1990.

[4] W. C. Jordan and S. C. Graves, "Principles on the benefits of manufacturing process

flexibility," Management Science, vol. 41, pp.577-94, 1999.

[5] G. Perrone, M. Amicob, G. L. Nigrob, and S. N. L. Diegab, "Long term capacity decisions in uncertain markets for advanced manufacturing systems incorporating scope economies," European Journal of Operational Research, vol. 143, pp. 125-137, 2002.

[6] J. A. Van Mieghem, "Investment strategies for flexible resources," Management Science, vol. 44, pp. 1071-1078, 1998.

[7] J. A. Van Mieghem and M. Dada, "Price versus production postponement: Capacity and competition," Management Science, vol. 45, pp. 1631 – 1649, 1999.

[8] H. M. Wagner, Principles of Operations Research, 2nd ed.: PHI, 1999.

[9] B. Yang, N. D. Burns, and C. J. Backhouse, "Management of uncertainty through postponement," International Journal of Production Research, vol. 42, pp. 1049–1064, 2004.

Appendix A: Results

Appendix A.1:

Parameter values: Price = 80, Cost = 20, Salvage value = 5 for each of the three products; Marginal cost of capacity = 10 for any type of plant

σ/μ = {0.05, 0.1, 0.15, 0.2}, σ^2 = {2000, 4000, 6000, 8000, 10000}

- ➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Mean Demand = {500, 500, 500}						**Mean Demand = {400, 500, 600}**						**Mean Demand = {250, 500, 750}**					
Change in σ/μ, ρ = 0.99						**Change in σ/μ, ρ = 0.99**						**Change in σ/μ, ρ = 0.99**					
1532	1574	1573	72965	73891	73895	1532	1572	1571	72891	73822	73825	1531	1573	1573	72888	73823	73826
1562	1642	1641	70828	72661	72668	1566	1648	1648	71000	72845	72853	1565	1643	1642	70911	72751	72758
1599	1719	1719	68973	71741	71752	1597	1715	1714	68872	71621	71633	1601	1721	1720	69028	71794	71804
1623	1779	1777	66731	70355	70370	1631	1791	1792	66986	70656	70671	1628	1789	1787	33779	70464	70476
Change in σ^2, ρ = 0.99						**Change in σ^2, ρ = 0.99**						**Change in σ^2, ρ = 0.99**					
1558	1631	1631	71387	73039	73045	1556	1628	1627	71335	72970	72977	1556	1630	1630	71349	72996	73304
1581	1683	1682	69781	72106	72116	1579	1682	1682	69680	72019	72028	1580	1681	1681	69795	72103	72112
1596	1726	1724	68551	71424	71437	1604	1731	1731	68678	71578	71590	1600	1726	1726	68722	71566	71577
1619	1762	1761	67742	71054	71069	1618	1765	1766	67705	71032	71045	1614	1759	1759	67832	71083	71096
1623	1783	1782	66686	70334	70349	1633	1793	1790	66828	70553	70569	1627	1789	1787	66778	70450	70465
Change in σ/μ, ρ = 0.5						**Change in σ/μ, ρ = 0.5**						**Change in σ/μ, ρ = 0.5**					
1532	1573	1558	72979	73899	74108	1532	1572	1560	72918	73847	74046	1532	1571	1559	72965	73882	74068
1565	1645	1621	70929	72771	73206	1566	1646	1619	71019	72856	73262	1567	1644	1622	70942	72788	73167
1597	1716	1677	68923	71662	72269	1598	1716	1676	69039	71764	72374	1597	1716	1680	68950	71696	72249
1629	1787	1734	66855	70522	71370	1633	1796	1743	66907	70625	71426	1631	1789	1743	66818	70519	71250
Change in σ^2, ρ = 0.5						**Change in σ^2, ρ = 0.5**						**Change in σ^2, ρ = 0.5**					
1557	1628	1606	71311	72950	73312	1558	1632	1608	71360	73026	73384	1558	1630	1606	71340	72991	73355
1581	1685	1650	69785	72123	72648	1582	1683	1648	69934	72238	72764	1582	1684	1650	69872	72196	72720
1598	1724	1681	68577	71428	72074	1596	1723	1681	68523	71377	72027	1600	1725	1684	68617	71485	72124
1612	1754	1706	67670	70935	71658	1616	1761	1710	67684	70984	71709	1612	1756	1707	67619	70904	71643
1629	1787	1734	66855	70522	71370	1631	1792	1735	66880	70571	71394	1629	1789	1736	66939	70598	71418
Change in σ/μ, ρ = 0.25						**Change in σ/μ, ρ = 0.25**						**Change in σ/μ, ρ = 0.25**					
1532	1573	1551	72940	73868	74204	1531	1571	1550	72909	73826	74154	1533	1572	1554	73015	73933	74228
1565	1646	1604	70925	72773	73433	1562	1642	1600	70826	72660	73314	1562	1642	1602	70831	72667	73269
1597	1717	1653	68920	71671	72650	1593	1714	1650	68659	71429	72391	1595	1714	1657	68785	71545	72438
1628	1793	1706	66824	70515	71858	1633	1793	1705	67033	70721	72038	1627	1792	1716	66763	70451	71643
Change in σ^2, ρ = 0.25						**Change in σ^2, ρ = 0.25**						**Change in σ^2, ρ = 0.25**					
1557	1629	1590	71290	72938	73530	1558	1631	1592	71379	73022	73621	1558	1629	1593	71366	73009	73584
1583	1685	1631	69933	72261	73099	1582	1686	1630	69785	72131	72967	1580	1680	1629	69799	72111	72929
1603	1726	1663	68722	71583	72599	1599	1722	1656	68658	71508	72534	1598	1722	1655	68712	71541	72600
1617	1763	1687	67793	71094	72252	1617	1760	1686	67725	71027	72184	1615	1760	1685	67691	70989	72158
1628	1793	1706	66824	70515	71858	1631	1792	1708	66842	70553	71858	1625	1786	1707	66660	70351	71649
Change in σ/μ, ρ = 0						**Change in σ/μ, ρ = 0**						**Change in σ/μ, ρ = 0**					
1533	1573	1542	72979	73906	74383	1532	1572	1543	72973	73893	74355	1531	1573	1545	72953	73872	74294
1566	1647	1587	70949	72804	73735	1563	1644	1583	70845	72691	73634	1562	1643	1586	70858	72700	73543
1595	1718	1624	68838	71598	73043	1597	1718	1627	68915	71675	73091	1598	1719	1639	68856	71628	72883
1626	1791	1666	66631	70340	72223	1631	1792	1670	66879	70575	72440	1631	1792	1685	66915	70595	72269
Change in σ^2, ρ = 0						**Change in σ^2, ρ = 0**						**Change in σ^2, ρ = 0**					
1557	1630	1575	71319	72964	73823	1559	1632	1577	71460	73108	73961	1557	1629	1575	71340	72982	73816
1583	1684	1607	69872	72207	73389	1582	1684	1605	69845	72180	73388	1581	1684	1606	69827	72153	73331
1596	1722	1627	68533	71380	72868	1599	1726	1629	68635	71499	72973	1600	1724	1630	68645	71509	72972
1615	1760	1648	67688	70993	72706	1615	1760	1649	67706	70993	72685	1615	1762	1652	67707	71018	72734
1626	1791	1666	66631	70340	72223	1626	1786	1663	66725	70395	72288	1627	1791	1665	66705	70407	72299

Change in σ/μ, ρ = – 0.25						**Change in σ/μ, ρ = – 0.25**						**Change in σ/μ, ρ = – 0.25**					
1531	1573	1529	72933	73858	74526	1532	1573	1531	72974	73897	74547	1532	1572	1535	72937	73864	74439
1565	1646	1560	70958	72799	74145	1563	1645	1560	70893	72744	74047	1565	1645	1569	70982	72813	73961
1595	1715	1587	68851	71608	73584	1597	1720	1591	68848	71634	73619	1596	1716	1602	68795	71577	73319
1629	1788	1617	66910	70563	73195	1628	1788	1620	66751	70347	73032	1629	1788	1640	66845	70511	72779
Change in σ^2, ρ = – 0.25						**Change in σ^2, ρ = – 0.25**						**Change in σ^2, ρ = – 0.25**					
1557	1631	1553	71330	72988	74198	1557	1630	1553	71268	72940	74147	1558	1629	1553	71349	72996	74205
1581	1683	1574	69810	72143	73832	1580	1683	1573	69801	72140	73819	1582	1684	1574	69855	72188	73868
1598	1723	1589	68578	71434	73487	1602	1724	1592	68757	71606	73663	1601	1727	1592	68697	71556	73628
1616	1759	1603	67666	70966	73390	1615	1758	1607	67734	71014	73376	1612	1757	1605	67680	70947	73305
1629	1788	1617	66910	70563	73195	1632	1791	1618	66781	70494	73179	1628	1793	1619	66826	70504	73161
Change in σ/μ, ρ = – 0.5						**Change in σ/μ, ρ = – 0.5**						**Change in σ/μ, ρ = – 0.5**					
1532	1573	1500	72941	73867	75000	1532	1572	1508	72949	73876	74871	1531	1572	1520	72931	73850	74649
1565	1644	1500	70917	72765	75000	1565	1643	1516	70913	72755	74735	1563	1644	1541	70881	72720	74312
1598	1717	1500	68840	71619	75000	1596	1718	1525	68868	71636	74614	1595	1717	1562	68919	71666	74046
1628	1786	1500	66868	70523	75000	1630	1788	1533	66811	70485	74497	1627	1791	1584	66870	70536	73728
Change in σ^2, ρ = – 0.5						**Change in σ^2, ρ = – 0.5**						**Change in σ^2, ρ = – 0.5**					
1557	1629	1500	71339	72988	75000	1558	1630	1500	71335	72986	75000	1558	1630	1500	71324	72978	75000
1581	1685	1500	69790	72135	75000	1582	1683	1500	69822	72163	75000	1580	1684	1500	69840	72158	75000
1603	1724	1500	68645	71521	75000	1601	1725	1500	68688	71533	75000	1599	1722	1500	68711	71537	75000
1615	1761	1500	67655	70961	75000	1614	1758	1500	67745	71015	75000	1616	1762	1500	67625	70957	75000
1628	1786	1500	66868	70523	75000	1630	1790	1500	66795	70487	75000	1628	1790	1500	66839	70509	75000

Appendix A.2:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; Marginal cost of capacity = 10 for any type of plant; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Price = {80, 80, 80}						**Price = {60, 80, 100}**						**Price = {40, 80, 120}**					
Change in ρ						**Change in ρ**						**Change in ρ**					
1623	1783	1782	66686	70334	70349	1614	1775	1699	66687	70314	70929	1563	1725	1500	67613	70869	72539
1629	1787	1734	66855	70522	71370	1611	1772	1659	66897	70443	71727	1566	1726	1501	67807	71060	73160
1628	1793	1706	66824	70515	71858	1617	1776	1640	66992	70580	72301	1566	1728	1502	67670	70938	73284
1626	1791	1666	66631	70340	72223	1619	1779	1618	67088	70674	72894	1566	1723	1501	67700	70945	73619
1629	1788	1617	66910	70563	73195	1616	1778	1580	66923	70511	73373	1565	1724	1500	67822	71077	74116
1628	1786	1500	66868	70523	75000	1619	1782	1500	66965	70589	75031	1566	1727	1500	67596	70870	74970

Appendix A.3:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5 for each of the three products; Marginal cost of capacity = 10 for any type of plant; σ^2 = {2000, 4000, 6000, 8000, 10000}

➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Price = {80, 80, 80}						**Price = {55, 55, 55}**						**Price = {40, 40, 40}**					
Change in σ^2, ρ = 0.99						**Change in σ^2, ρ = 0.99**						**Change in σ^2, ρ = 0.99**					
1558	1631	1631	71387	73039	73045	1501	1576	1576	34802	35898	35903	1425	1501	1501	13401	13932	13935
1581	1683	1682	69781	72106	72116	1500	1607	1607	33769	35288	35295	1391	1500	1500	12714	13465	13470
1596	1726	1724	68551	71424	71437	1498	1630	1630	32894	34737	34747	1368	1496	1495	12226	13130	13136
1619	1762	1761	67742	71054	71069	1500	1652	1652	32174	34325	34336	1349	1497	1498	11830	12867	12875
1623	1783	1782	66686	70334	70349	1500	1672	1671	31474	33922	33934	1328	1499	1499	11431	12596	12603

Change in σ^2, ρ = 0.5						Change in σ^2, ρ = 0.5						Change in σ^2, ρ = 0.5					
1557	1628	1606	71311	72950	73312	1502	1577	1563	34840	35930	36226	1424	1500	1500	13397	13925	14127
1581	1685	1650	69785	72123	72648	1500	1608	1587	33718	35253	35668	1391	1497	1500	12733	13474	13753
1598	1724	1681	68577	71428	72074	1502	1634	1610	32910	34783	35312	1366	1498	1498	12235	13139	13476
1612	1754	1706	67670	70935	71658	1502	1653	1626	32234	34381	34962	1351	1502	1502	11825	12877	13275
1629	1787	1734	66855	70522	71370	1502	1672	1641	31587	33996	34653	1334	1502	1499	11475	12647	13089
Change in σ^2, ρ = 0.25						**Change in σ^2, ρ = 0.25**						**Change in σ^2, ρ = 0.25**					
1557	1629	1590	71290	72938	73530	1500	1577	1554	34831	35915	36389	1422	1499	1499	13383	13915	14277
1583	1685	1631	69933	72261	73099	1498	1606	1572	33637	35176	35830	1392	1499	1499	12769	13501	13945
1603	1726	1663	68722	71583	72599	1501	1631	1593	32880	34754	35557	1372	1504	1502	12248	13170	13716
1617	1763	1687	67793	71094	72252	1497	1650	1606	32097	34265	35195	1351	1503	1503	11821	12880	13504
1628	1793	1706	66824	70515	71858	1500	1669	1622	31511	33929	34993	1335	1502	1502	11495	12654	13357
Change in σ^2, ρ = 0						**Change in σ^2, ρ = 0**						**Change in σ^2, ρ = 0**					
1557	1630	1575	71319	72964	73823	1500	1576	1544	34845	35927	36581	1424	1500	1499	13411	13934	14387
1583	1684	1607	69872	72207	73389	1499	1607	1562	33686	35223	36173	1392	1499	1500	12742	13484	14128
1596	1722	1627	68533	71380	72868	1498	1631	1575	32802	34681	35875	1365	1498	1497	12199	13116	13902
1615	1760	1648	67688	70993	72706	1498	1653	1586	32099	34276	35634	1347	1499	1498	11786	12841	13744
1626	1791	1666	66631	70340	72223	1503	1669	1601	31586	33993	35491	1334	1502	1501	11455	12626	13637
Change in σ^2, ρ = – 0.25						**Change in σ^2, ρ = – 0.25**						**Change in σ^2, ρ = – 0.25**					
1557	1631	1553	71330	72988	74198	1501	1576	1531	34841	35921	36863	1424	1500	1500	13407	13932	14563
1581	1683	1574	69810	72143	73832	1498	1605	1543	33711	35233	36566	1391	1498	1498	12744	13481	14375
1598	1723	1589	68578	71434	73487	1499	1631	1552	32852	34724	36375	1368	1499	1500	12238	13147	14241
1616	1759	1603	67666	70966	73390	1499	1649	1560	32132	34285	36175	1350	1500	1500	11827	12876	14128
1629	1788	1617	66910	70563	73195	1503	1671	1569	31589	34000	36104	1331	1500	1500	11412	12601	14016
Change in σ^2, ρ = – 0.5						**Change in σ^2, ρ = – 0.5**						**Change in σ^2, ρ = – 0.5**					
1557	1629	1500	71339	72988	75000	1500	1576	1500	34814	35898	37500	1424	1499	1500	13414	13935	15000
1581	1685	1500	69790	72135	75000	1500	1607	1500	33724	35248	37500	1394	1499	1500	12756	13492	15000
1603	1724	1500	68645	71521	75000	1500	1628	1500	32890	34745	37500	1370	1499	1500	12269	13169	15000
1615	1761	1500	67655	70961	75000	1499	1648	1500	32194	34328	37500	1348	1499	1500	11788	12841	15000
1628	1786	1500	66868	70523	75000	1502	1672	1500	31460	33910	37500	1330	1499	1500	11442	12608	15000

Appendix A.4:

Parameter values: Mean demand = 500, Price = 80, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

- ➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Capacity cost = 5						Capacity cost = 10						Capacity cost = 15					
Change in ρ						**Change in ρ**						**Change in ρ**					
1681	1912	1911	74632	79291	79300	1623	1783	1782	66686	70334	70349	1579	1706	1706	58809	61816	61835
1681	1912	1833	74808	79457	79951	1629	1787	1734	66855	70522	71370	1577	1704	1668	58916	61886	62920
1690	1922	1795	75345	79997	80795	1628	1793	1706	66824	70515	71858	1575	1698	1642	58873	61819	63461
1691	1917	1737	75171	79812	81000	1626	1791	1666	66631	70340	72223	1577	1701	1617	58839	61809	64244
1687	1917	1661	75144	79784	81439	1629	1788	1617	66910	70563	73195	1574	1699	1579	58789	61747	65146
1684	1907	1500	75203	79763	82500	1628	1786	1500	66868	70523	75000	1578	1701	1500	58783	61764	67500

Appendix B: Results with Service Level Constraint

Appendix B.1:

Parameter values: Price = 80, Cost = 20, Salvage value = 5 for each of the three products; Marginal cost of capacity = 10 for any type of plant

σ^2 = {2000, 4000, 6000, 8000, 10000}

➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Demand = {500, 500, 500}						**Demand = {400, 500, 600}**						**Demand = {250, 500, 750}**					
Change in σ^2, ρ = 0.99						**Change in σ^2, ρ = 0.99**						**Change in σ^2, ρ = 0.99**					
1589	1692	1661	71277	72848	72987	1587	1686	1656	71248	72794	72927	1588	1690	1630	71182	72759	72937
1608	1742	1742	69797	72020	72030	1610	1742	1742	69609	71893	71903	1612	1760	1680	69827	71980	72207
1625	1811	1779	68396	71059	71201	1633	1818	1787	68702	71387	71530	1628	1814	1723	68620	71292	71526
1646	1845	1813	67799	70889	71013	1648	1851	1822	67517	70696	70821	1645	1856	1766	67536	70723	70938
1658	1912	1851	66958	70342	70583	1660	1904	1843	66950	70293	70533	1659	1912	1792	66718	70146	70472
Change in σ^2, ρ = 0.5						**Change in σ^2, ρ = 0.5**						**Change in σ^2, ρ = 0.5**					
1588	1690	1637	71296	72855	73336	1589	1690	1638	71191	72781	73260	1587	1688	1604	71264	72806	73346
1611	1750	1678	69666	71887	72543	1612	1752	1680	69746	71967	72623	1614	1764	1651	69788	71973	72712
1629	1815	1746	68688	71344	72072	1629	1811	1711	68548	71222	72030	1633	1817	1686	68682	71367	72261
1641	1844	1766	67399	70534	71363	1650	1852	1744	67814	70949	71858	1644	1845	1706	67451	70590	71516
1652	1907	1791	66457	69868	70939	1661	1902	1770	66827	70273	71337	1663	1913	1738	67023	70408	71538
Change in σ^2, ρ = 0.25						**Change in σ^2, ρ = 0.25**						**Change in σ^2, ρ = 0.25**					
1587	1690	1621	71211	72777	73483	1590	1691	1622	71325	72887	73583	1586	1688	1590	71161	72720	73489
1612	1754	1660	69708	71940	72898	1610	1773	1661	69725	71843	72888	1611	1752	1627	69653	71881	72892
1632	1817	1688	68644	71325	72538	1629	1815	1689	68483	71176	72396	1632	1815	1662	68645	71329	72573
1648	1851	1714	67783	70913	72269	1649	1852	1715	67669	70815	72168	1646	1848	1684	67668	70792	72157
1654	1905	1761	66719	70096	71621	1667	1908	1734	66698	70136	71717	1656	1906	1704	66690	70095	71721
Change in σ^2, ρ = 0						**Change in σ^2, ρ = 0**						**Change in σ^2, ρ = 0**					
1587	1691	1606	71278	72839	73785	1589	1690	1606	71319	72877	73822	1588	1691	1576	71265	72823	73824
1610	1755	1637	69692	71917	73263	1612	1761	1634	69716	71885	73242	1614	1767	1608	69841	72021	73454
1631	1815	1661	68584	71269	72910	1629	1815	1659	68549	71235	72900	1630	1815	1628	68589	71269	72974
1645	1846	1678	67615	70738	72599	1647	1849	1682	67686	70820	72681	1645	1859	1653	67691	70778	72724
1661	1909	1699	66849	70256	72408	1658	1908	1697	66774	70191	72385	1669	1905	1669	66786	70271	72443
Change in σ^2, ρ = - 0.25						**Change in σ^2, ρ = - 0.25**						**Change in σ^2, ρ = - 0.25**					
1588	1690	1584	71271	72832	74097	1587	1690	1583	71196	72768	74025	1587	1690	1553	71224	72790	74174
1611	1755	1604	69760	71980	73782	1612	1765	1605	69700	71892	73746	1612	1753	1575	69818	72016	73864
1629	1814	1622	68626	71298	73528	1630	1816	1620	68599	71288	73528	1629	1814	1592	68566	71242	73535
1643	1851	1635	67595	70732	73281	1645	1851	1637	67556	70719	73255	1645	1847	1604	67648	70775	73365
1669	1899	1647	66791	70259	73122	1672	1912	1648	66942	70373	73294	1659	1914	1619	66754	70191	73200
Change in σ^2, ρ = - 0.5						**Change in σ^2, ρ = - 0.5**						**Change in σ^2, ρ = - 0.5**					
1588	1691	1500	71235	72809	75000	1588	1690	1500	71239	72809	75000	1588	1693	1500	71204	72790	75000
1612	1755	1500	69772	71988	75000	1611	1756	1500	69743	71967	75000	1613	1744	1500	69762	72032	75000
1630	1814	1500	68641	71298	75000	1630	1815	1500	68613	71283	75000	1631	1816	1500	68616	71291	75000
1646	1859	1500	67620	70724	75000	1646	1852	1500	67631	70772	75000	1646	1849	1500	67651	70785	75000
1648	1854	1500	67590	70753	75000	1660	1911	1500	66745	70168	75000	1661	1914	1500	66725	70174	75000

Appendix B.2:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; Marginal cost of capacity = 10 for any type of plant; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Price = {80, 80, 80}						**Price = {60, 80, 100}**						**Price = {40, 80, 120}**					
Change in ρ, σ^2 = 10000						**Change in ρ, σ^2 = 10000**						**Change in ρ, σ^2 = 10000**					
1658	1912	1851	66958	70342	70583	1685	1898	1703	66357	69971	70921	1684	1893	1676	66698	70098	72076
1652	1907	1791	66457	69868	70939	1682	1914	1670	66737	70271	71943	1682	1894	1650	66789	70126	72569
1654	1905	1761	66719	70096	71621	1679	1898	1642	66602	70141	72210	1689	1890	1625	66851	70297	73059
1661	1909	1699	66849	70256	72408	1668	1906	1614	66759	70153	72734	1688	1899	1596	67132	70484	73677
1669	1899	1647	66791	70259	73122	1687	1899	1583	66722	70288	73486	1702	1896	1589	66727	70244	73771
1648	1854	1500	67590	70753	75000	1676	1908	1500	66721	70181	74975	1685	1886	1500	66940	70330	75006

Appendix B.3:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5 for each of the three products; Marginal cost of capacity = 10 for any type of plant; σ^2 = {2000, 4000, 6000, 8000, 10000}

➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Price = {80, 80, 80}						**Price = {55, 55, 55}**						**Price = {40, 40, 40}**					
Change in σ^2, ρ = 0.99						**Change in σ^2, ρ = 0.99**						**Change in σ^2, ρ = 0.99**					
1589	1692	1661	71277	72848	72987	1590	1698	1579	34246	35385	35911	1575	1680	1590	12286	13091	13703
1608	1742	1742	69797	72020	72030	1618	1757	1607	32939	34615	35204	1623	1772	1621	10898	12173	13200
1625	1811	1779	68396	71059	71201	1642	1812	1630	32049	34067	34766	1638	1810	1651	10145	11700	12761
1646	1845	1813	67799	70889	71013	1650	1861	1651	31284	33485	34287	1653	1863	1653	9544	11190	12541
1658	1912	1851	66958	70342	70583	1680	1899	1670	30568	33111	34010	1665	1888	1678	8942	10801	12194
Change in σ^2, ρ = 0.5						**Change in σ^2, ρ = 0.5**						**Change in σ^2, ρ = 0.5**					
1588	1690	1637	71296	72855	73336	1592	1698	1564	34295	35428	36245	1573	1680	1561	12274	13078	13980
1611	1750	1678	69666	71887	72543	1619	1757	1588	32964	34630	35641	1604	1770	1590	11198	12169	13564
1629	1815	1746	68688	71344	72072	1640	1811	1608	31985	34015	35204	1641	1801	1620	10156	11799	13207
1641	1844	1766	67399	70534	71363	1650	1863	1629	31449	33581	34974	1646	1859	1617	9606	11171	12999
1652	1907	1791	66457	69868	70939	1685	1900	1644	30567	33148	34663	1659	1890	1650	8974	10771	12652
Change in σ^2, ρ = 0.25						**Change in σ^2, ρ = 0.25**						**Change in σ^2, ρ = 0.25**					
1587	1690	1621	71211	72777	73483	1591	1698	1555	34262	35411	36405	1575	1680	1559	12290	13091	14101
1612	1754	1660	69708	71940	72898	1622	1759	1579	33054	34714	35962	1602	1769	1589	11183	12161	13684
1632	1817	1688	68644	71325	72538	1648	1809	1591	31911	34019	35515	1641	1810	1591	10128	11705	13491
1648	1851	1714	67783	70913	72269	1652	1861	1610	31357	33531	35251	1646	1857	1616	9533	11141	13153
1654	1905	1761	66719	70096	71621	1683	1904	1626	30553	33143	35052	1671	1901	1619	8886	10759	13064
Change in σ^2, ρ = 0						**Change in σ^2, ρ = 0**						**Change in σ^2, ρ = 0**					
1587	1691	1606	71278	72839	73785	1590	1695	1544	34230	35376	36565	1575	1680	1560	12286	13090	14205
1610	1755	1637	69692	71917	73263	1622	1759	1565	33009	34696	36235	1601	1749	1558	11196	12320	13987
1631	1815	1661	68584	71269	72910	1622	1810	1576	32270	34058	35912	1637	1807	1588	10136	11696	13688
1645	1846	1678	67615	70738	72599	1651	1865	1588	31378	33526	35676	1648	1859	1586	9563	11156	13554
1661	1909	1699	66849	70256	72408	1683	1911	1600	30528	33060	35495	1671	1901	1618	8881	10750	13315
Change in σ^2, ρ = - 0.25						**Change in σ^2, ρ = - 0.25**						**Change in σ^2, ρ = - 0.25**					
1588	1690	1584	71271	72832	74097	1590	1697	1531	34254	35399	36868	1575	1678	1530	12273	13095	14501
1611	1755	1604	69760	71980	73782	1621	1757	1544	32984	34672	36598	1603	1770	1559	11199	12166	14201
1629	1814	1622	68626	71298	73528	1650	1811	1555	31910	34041	36353	1638	1809	1558	10170	11713	14097
1643	1851	1635	67595	70732	73281	1649	1863	1562	31393	33531	36238	1658	1860	1561	9467	11201	14011
1669	1899	1647	66791	70259	73122	1680	1911	1572	30457	32974	36034	1661	1890	1590	9022	10832	13774
Change in σ^2, ρ = - 0.5						**Change in σ^2, ρ = - 0.5**						**Change in σ^2, ρ = - 0.5**					
1588	1691	1500	71235	72809	75000	1588	1695	1500	34270	35395	37500	1575	1680	1500	12294	13096	15000
1612	1755	1500	69772	71988	75000	1620	1760	1500	32968	34646	37500	1603	1760	1530	11199	12252	14700
1630	1814	1500	68641	71298	75000	1650	1812	1500	31938	34052	37500	1640	1810	1500	10126	11698	15000
1646	1859	1500	67620	70724	75000	1649	1861	1500	31329	33500	37500	1646	1858	1500	9593	11187	15000
1648	1854	1500	67590	70753	75000	1680	1910	1500	30437	32974	37500	1680	1889	1500	8757	10844	15000

Appendix B.4:

Parameter values: Mean demand = 500, Price = 80, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

➔ Horizontally {**Capacity 1, Capacity 2, Capacity 3, Profit 1, Profit 2, Profit 3**}

Capacity Cost = 5						Capacity Cost = 10						Capacity Cost = 15					
Change in ρ, σ^2 = 10000						Change in ρ, σ^2 = 10000						Change in ρ, σ^2 = 10000					
1686	1916	1912	74899	79572	79580	1658	1912	1851	66958	70342	70583	1671	1911	1700	58724	60900	62070
1692	1919	1847	75387	80021	80537	1652	1907	1791	66457	69868	70939	1662	1890	1664	58198	60586	62618
1686	1915	1792	74987	79636	80449	1654	1905	1761	66719	70096	71621	1661	1898	1640	58276	60552	63292
1691	1919	1743	75288	79936	81091	1661	1909	1699	66849	70256	72408	1665	1914	1621	58372	60571	64140
1686	1918	1670	75119	79761	81408	1669	1899	1647	66791	70259	73122	1666	1903	1584	58439	60735	65197
1686	1916	1500	75099	79726	82500	1648	1854	1500	67590	70753	75000	1667	1910	1500	58453	60648	67500

Appendix B.5:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5 for each of the three products; Marginal cost of capacity = 10 for any type of plant; σ^2 = {2000, 4000, 6000, 8000, 10000}

➔ **Horizontally ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}**

PdPPF Index without Service Level Constraint																	
Price = {80, 80, 80}						Price = {55, 55, 55}						Price = {40, 40, 40}					
Change in σ^2						Change in σ^2						Change in σ^2					
99.64	81.91	73.57	65.69	57.81	45.04	99.55	78.64	69.58	62.33	53.41	40.36	99.44	72.33	59.51	53.59	45.42	32.85
99.57	81.66	73.53	66.39	58.01	45.01	99.54	78.72	70.18	61.80	53.31	40.36	99.34	72.65	62.24	53.54	45.19	32.80
99.55	81.53	73.79	65.67	58.18	45.26	99.46	77.98	70.00	61.15	53.14	40.24	99.34	72.84	62.81	53.85	45.38	32.95
99.55	81.87	74.03	65.86	57.65	45.01	99.49	78.70	69.98	61.58	53.25	40.22	99.23	72.55	62.92	53.88	45.59	32.78
99.59	81.22	73.32	66.33	58.12	44.95	99.51	78.57	69.44	61.64	53.40	40.56	99.40	72.61	62.24	53.67	45.66	32.77
PdPPF Index with Service Level Constraint																	
Price = {80, 80, 80}						Price = {55, 55, 55}						Price = {40, 40, 40}					
91.87	76.42	68.93	62.27	55.24	41.81	68.41	58.10	53.62	49.08	43.80	34.83	56.81	47.13	44.23	41.90	36.89	29.64
99.55	77.20	69.97	62.31	55.20	42.39	74.00	62.23	57.08	52.29	46.71	37.03	55.39	41.04	39.10	40.27	32.21	30.08
94.94	78.49	68.85	62.07	54.51	41.78	74.27	63.06	58.49	49.09	47.96	38.01	59.44	53.85	46.89	43.92	39.29	32.25
96.14	79.09	69.77	62.66	55.17	42.06	73.29	60.48	55.83	49.98	44.13	35.18	54.92	46.12	44.42	39.91	38.16	29.48
93.35	76.10	68.89	61.29	54.78	42.69	73.88	63.01	57.57	50.98	45.13	35.91	57.16	48.86	44.83	42.15	38.09	33.43

Appendix C: Unmet Demand Percentage

Appendix C.1:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; Marginal cost of capacity = 10 for any type of plant; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

➔ Horizontally {**Unmet demand % for cases 1, 2 and 3**}

Price = {80, 80, 80}			Price = {60, 80, 100}			Price = {40, 80, 120}		
Change in ρ			Change in ρ			Change in ρ		
33.01	16.4	5.53	34.69	17.76	8.29	41.82	25.25	16.64
32.92	16.35	5.52	34.71	17.82	8.3	41.84	25.29	16.61
33	16.39	5.52	34.65	17.78	8.27	41.82	25.31	16.58
32.97	16.44	5.51	34.59	17.78	8.28	41.84	25.32	16.57
33.03	16.38	5.49	34.64	17.79	8.26	41.77	25.25	16.48
32.94	16.39	0	34.67	17.78	0	41.9	25.28	0

Appendix C.2:

Parameter values: Mean demand = 500, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; Marginal cost of capacity = 10 for any type of plant; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

➔ Horizontally {**Unmet demand % for cases 1, 2 and 3**}

Price = {80, 80, 80}			**Price = {55, 55, 55}**			**Price = {40, 40, 40}**		
Change in ρ			**Change in ρ**			**Change in ρ**		
33.01	16.4	5.53	49.55	28.26	9.5	71.1	49.63	16.62
32.92	16.35	5.52	49.61	28.29	9.47	71.03	49.55	16.61
33	16.39	5.52	49.64	28.29	9.48	71.04	49.6	16.6
32.97	16.44	5.51	49.62	28.22	9.47	71.06	49.57	16.59
33.03	16.38	5.49	49.65	28.18	9.44	71.1	49.61	16.52
32.94	16.39	0	49.53	28.24	0	71.13	49.58	0

Appendix C.3:

Parameter values: Mean demand = 500, Price = 80, Cost = 20, Salvage value = 5, σ^2 = 10000 for each of the three products; ρ = {0.99, 0.5, 0.25, 0, – 0.25, – 0.5}

➔ Horizontally {**Unmet demand % for cases 1, 2 and 3**}

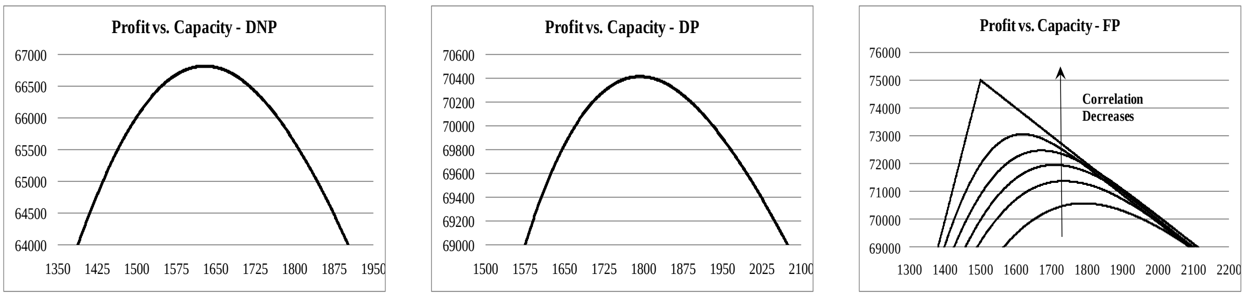
Capacity cost = 5			**Capacity cost = 10**			**Capacity cost = 15**		
Change in ρ			**Change in ρ**			**Change in ρ**		
26.28	8.17	2.76	33.01	16.4	5.53	39.61	24.68	8.29
26.37	8.16	2.76	32.92	16.35	5.52	39.59	24.66	8.31
26.31	8.18	2.76	33	16.39	5.52	39.6	24.64	8.28
26.31	8.19	2.73	32.97	16.44	5.51	39.61	24.68	8.25
26.3	8.22	2.73	33.03	16.38	5.49	39.59	24.68	8.25
26.33	8.18	0	32.94	16.39	0	39.64	24.67	0

Appendix D: Incremental Profit Value

Price = {80, 80, 80}											
Dedicated Plant, Postponement						**Flexible Plant, Postponement**					
-0.26	-0.13	-0.22	-0.17	-0.21	-0.25	-0.08	0.03	-0.06	-0.05	-0.14	0.00
-0.12	-0.33	-0.44	-0.40	-0.23	-0.20	-0.12	-0.14	-0.27	-0.17	-0.07	0.00
-0.51	-0.12	-0.36	-0.16	-0.19	-0.31	-0.33	0.00	-0.08	0.06	0.06	0.00
-0.23	-0.57	-0.25	-0.36	-0.33	-0.33	-0.08	-0.41	0.02	-0.15	-0.15	0.00
0.01	-0.93	-0.59	-0.12	-0.43	0.33	0.33	-0.60	-0.33	0.26	-0.10	0.00
Price = {55, 55, 55}											
Dedicated Plant, Postponement						**Flexible Plant, Postponement**					
-1.43	-1.40	-1.40	-1.53	-1.45	-1.40	0.02	0.05	0.04	-0.04	0.01	0.00
-1.91	-1.77	-1.31	-1.50	-1.59	-1.71	-0.26	-0.08	0.37	0.17	0.09	0.00
-1.93	-2.21	-2.11	-1.80	-1.97	-1.99	0.05	-0.31	-0.12	0.10	-0.06	0.00
-2.45	-2.33	-2.14	-2.19	-2.20	-2.41	-0.14	0.03	0.16	0.12	0.17	0.00
-2.39	-2.49	-2.32	-2.74	-3.02	-2.76	0.22	0.03	0.17	0.01	-0.19	0.00
Price = {40, 40, 40}											
Dedicated Plant, Postponement						**Flexible Plant, Postponement**					
-6.04	-6.08	-5.92	-6.06	-6.01	-6.02	-1.66	-1.04	-1.23	-1.27	-0.43	0.00
-9.60	-9.69	-9.93	-8.63	-9.75	-9.19	-2.00	-1.37	-1.87	-1.00	-1.21	-2.00
-10.89	-10.20	-11.12	-10.83	-10.91	-11.17	-2.85	-2.00	-1.64	-1.54	-1.01	0.00
-13.03	-13.25	-13.50	-13.12	-13.01	-12.88	-2.59	-2.08	-2.60	-1.38	-0.83	0.00
-14.25	-14.83	-14.98	-14.86	-14.04	-13.99	-3.25	-3.34	-2.19	-2.36	-1.73	0.00

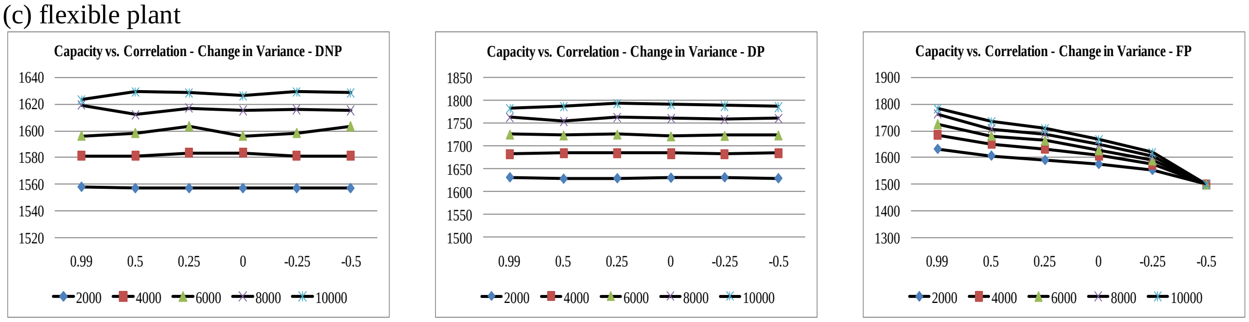
Change in Price Differential = {0, 40, 80}						Change in Capacity Cost = {5, 10, 15}					
Dedicated Plant, Postponement			Flexible Plant, Postponement			Dedicated Plant, Postponement			Flexible Plant, Postponement		
0.01	-0.49	-1.09	0.33	-0.01	-0.64	0.35	0.01	-1.48	0.35	0.33	0.38
-0.93	-0.24	-1.31	-0.60	0.30	-0.81	0.71	-0.93	-2.10	0.73	-0.60	-0.48
-0.59	-0.62	-0.90	-0.33	-0.13	-0.31	-0.45	-0.59	-2.05	-0.43	-0.33	-0.27
-0.12	-0.74	-0.65	0.26	-0.22	0.08	0.16	-0.12	-2.00	0.11	0.26	-0.16
-0.43	-0.32	-1.17	-0.10	0.15	-0.47	-0.03	-0.43	-1.64	-0.04	-0.10	0.08
0.33	-0.58	-0.76	0.00	-0.07	0.05	-0.05	0.33	-1.81	0.00	0.00	0.00

Appendix E: Graphs



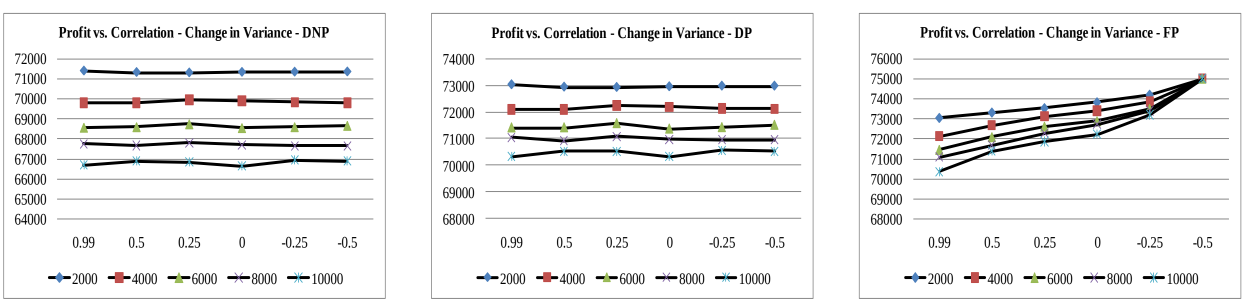
(a) (b) (c)

Fig 1: Optimal profit versus capacity for (a) dedicated plant no postponement, (b) dedicated plant postponement and (c) flexible plant



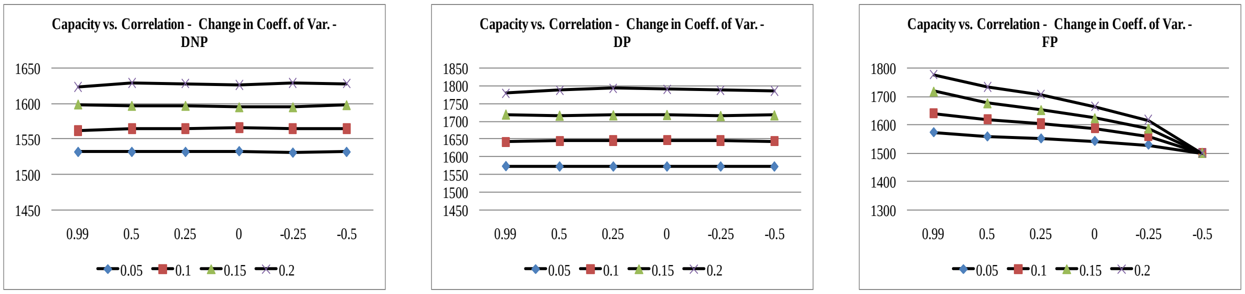
(a) (b) (c)

Fig 2: Optimal capacity versus demand correlation for (a) dedicated plant no postponement, (b) dedicated plant postponement and (c) flexible plant for different variance levels



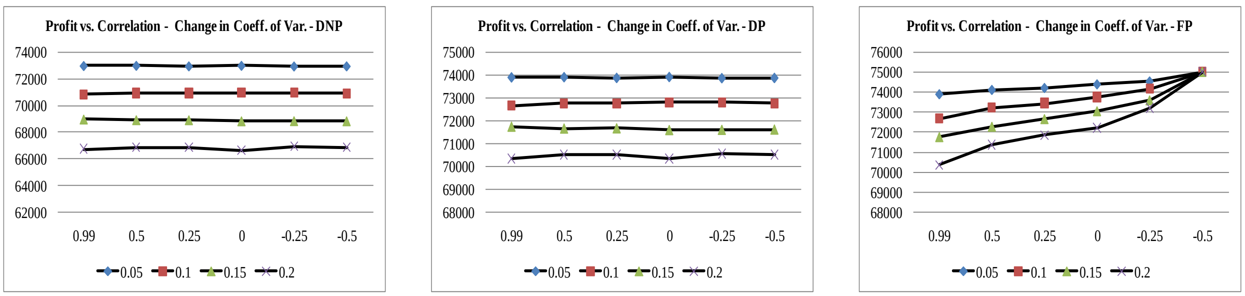
(a) (b) (c)

Fig 3: Optimal profit versus demand correlation for (a) dedicated plant no postponement, (b) dedicated plant postponement and (c) flexible plant for different variance levels



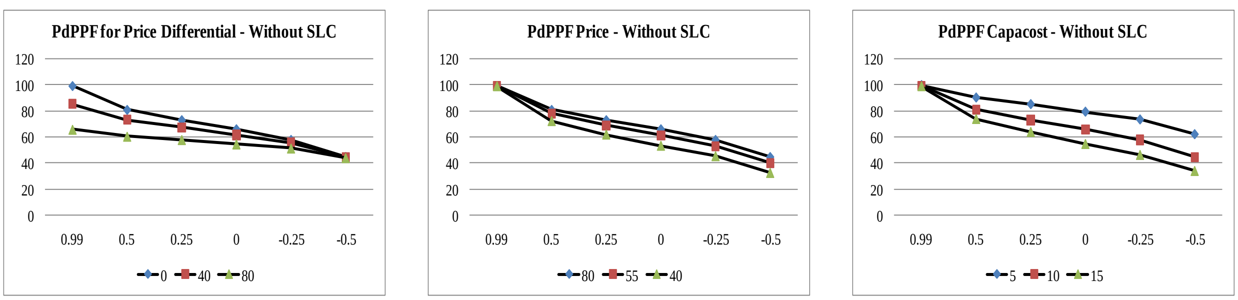
(a) (b) (c)

Fig 4: Optimal capacity versus demand correlation for (a) dedicated plant no postponement, (b) dedicated plant postponement and (c) flexible plant for different coefficient of variation levels



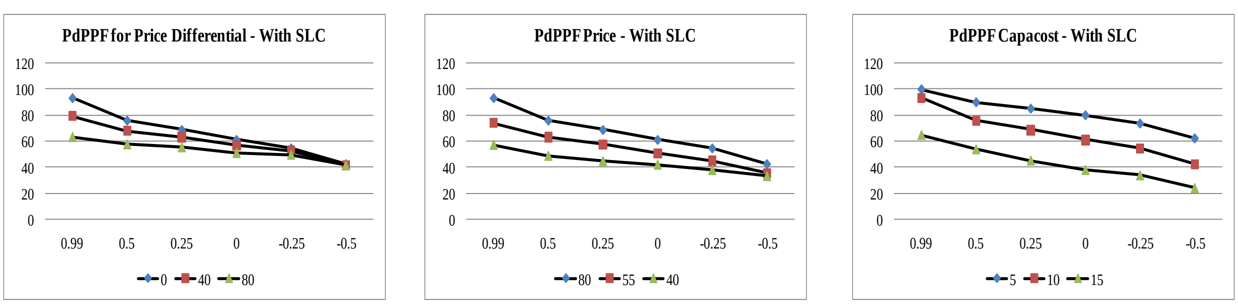
(a) (b) (c)

Fig 5: Optimal profit versus demand correlation for (a) dedicated plant no postponement, (b) dedicated plant postponement and (c) flexible plant for different coefficient of variation levels



(a) (b) (c)

Fig 6: PdPPF Index vs. correlation for change in (a) Price differential (b) Price and (c) Capacity Cost



(a) (b) (c)

Fig 7: Effect of Service Level Constraint (SLC) on PdPPF Index vs. correlation for change in (a) Price differential (b) Price and (c) Capacity Cost