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Analytical Aspects of Short-run Growth

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Analytical Aspects of Short-run Growth

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Abstract:

This paper studies the short-run and medium-run aspects of economic growth under two alternative frameworks. These are: (i) a demand-determined growth model with excess capacity having Keynesian features; and (ii) a full-capacity Harrod-Domar model modified to include autonomous consumption. While much of growth theory is about the long-run understanding contemporary growth experience requires application of short-run analysis.

The analysis is applied to address the following questions, among others: (a) To what extent does a change in the rate of exponential growth signal a 'structural break'? (b) What are the appropriate analytical definitions of consumption-driven growth and investment-driven growth? (c) What observable characteristics of a growth path help identify if growth (or a change in growth) is consumption-driven in contrast to its being investment-driven?

The analysis brings out the relation between growth rates of autonomous consumption, investment and GDP, emphasizing the difference in conclusions in the two models.

Keywords: Short-run growth, consumption-driven growth, investment-driven growth, demand-determined growth, full-capacity growth.

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1. Introduction

This paper studies short-run and medium-run aspects of economic growth. The set of comprehensive equilibrium conditions of the long-run is not invoked. Instead of full equilibrium, we deal with contexts that have several Keynesian features such as demand determined output, excess capacity, unemployment and price rigidities. Moreover we shall analyze the 'transition dynamics' associated with out-of-steady-state behaviour.

Much of growth theory is about the long-run (for e.g. Solow [13]). Understanding contemporary growth experience however requires analyzing the short-run. Some full-equilibrium conditions would have to be given up. To take an example, if our interest is in studying recent Indian growth then it seems pointless to assume that there is full capacity utilization and full employment. We need a short-run framework.

Moreover, when analyzing short-run growth we can ill-afford to assume that the rate of growth is constant – or piecewise constant. The literature on Indian growth rates, however, mostly deals with output trends that are log-linear or piece-wise log-linear. That is like assuming that India is always on a steady-state, allowing only for the steady state to shift periodically ("structural breaks") but assuming that the new steady state is attained instantly. A moment's reflection should convince us that this is not the right way to proceed if the rate of growth is varying over time endogenously and the economy is far from being on a steady state. Thinking of an average rate that is constant or piecewise constant is going to be treacherous. We will forever be hunting for 'structural breaks' even though no such thing may have occurred.

The analysis attempted here is theoretical and preliminary. It is best viewed as "experiments" using a set of alternative models. The models are simple and conclusions can be stated in particularly transparent terms. It would be easy to extend these models to more realistic scenarios. The first model is a pure demand-driven model that dynamizes the simple multiplier model of the Keynesian cross.

2. Pure Demand Growth with Excess Capacity and Unemployment

We consider the simple Keynesian multiplier model in a *closed* economy in which there exists excess capacity and unemployment, i.e. the economy is demand-constrained. We assume that investment is wholly autonomous, consumption has an autonomous component and the marginal propensity to consume is constant.

If the consumption function is $C = \overline{C} + cY$, then the equilibrium condition that demand = income gives

$$D = Y = A/s \tag{1}$$

where $A \equiv I + \overline{C}$ represents autonomous demand. Taking this static model, we simply project it over time. Excess capacity and unemployment are assumed to persist over time. The proportional rates of growth autonomous consumption ($g_{\overline{C}}$) and autonomous investment (g_I) are both given exogenously: $g_{\overline{C}} = \alpha$ and $g_I = \beta$.

Let $\mu \equiv \dfrac{I}{I+\overline{C}}$ be the share of investment expenditure in total autonomous demand. For positive I and $\overline{C}$, we have $0 < \mu < 1$. Writing g_x for the rate of growth of x, the relation $A = I + \overline{C}$ implies

$$g_A = \mu\beta + (1-\mu)\alpha$$

From the multiplier relation (1) we have $g_Y = g_A - g_s$. Assume $g_s = 0$ and let $g = g_Y = g_A$. Then

$$g = \mu\beta + (1-\mu)\alpha, \qquad (2)$$

Note that μ is a function of time such that

$$\frac{d\mu}{dt} \equiv \dot{\mu} = \mu\,(1-\mu)\,(\beta-\alpha) \equiv \mu(1-\mu)\,\theta, \text{ where } \theta \equiv \beta - \alpha \qquad (3)$$

Since $0 < \mu < 1 \Rightarrow \mu\,(1-\mu) > 0$, it follows from (3) that for $0 < \mu < 1$,

(i) $\theta > 0 \Rightarrow \dot{\mu} > 0$ and $\mu \to 1$ as $t \to \infty$.

(ii) $\theta < 0 \Rightarrow \dot{\mu} < 0$ and $\mu \to 0$ as $t \to \infty$.

If $\mu = 0$ or 1, then $\dot{\mu} = 0$.

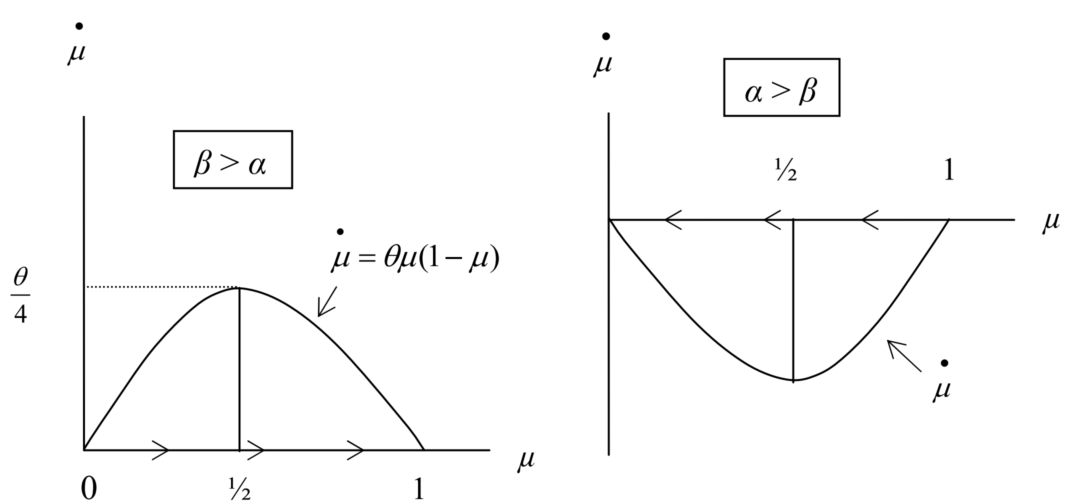

Fig 1: Share of investment in autonomous demand

From (2) we have $\dot{g} = (\beta-\alpha)\,\dot{\mu} = \theta\,\dot{\mu} = \mu(1-\mu)\,\theta^2 \qquad (4)$

It follows from (2) and (4) that for $0 < \mu < 1$, *(i)* $\alpha \neq \beta$, i.e. $\theta \neq 0 \Rightarrow \dot{g}(t) > 0$;

(ii) $\dot{g}(t) \geq 0$ for all $t \geq 0$ since θ and $\dot{\mu}$ always have the same sign.

Therefore the growth rate is non-decreasing, regardless of the values of α and β.

2.1. Properties of Demand Determined Growth

Let $g^* = max\ [\alpha, \beta]$ be called the 'leading rate'.

Let $g_{\min} = min\ [\alpha, \beta]$ be called the 'laggard rate'.

We take note of the following properties of the growth path.

Property I: The GDP growth rate increases over time.

If $0 < \mu(0) < 1$, then $g_{\min} < g(t) < g^*$ and $\dot{g}(t) > 0$ for all $t \geq 0$.

The result is intuitive. By virtue of the simple multiplier relation, the GDP growth rate (g) is equal to the rate of growth of autonomous demand (A). The latter is a weighted average of its two components $\overline{C}$ and I, the weights being the respective component shares. If $\overline{C}$ and I are positive, then $1 > \mu(0) > 0$ and that implies $g(t) > g_{\min}$. The weight on the faster growing component keeps increasing over time, pulling up the average rate of growth towards it.

Note: It is possible that the growth rate is negative; that requires $g_{\min}$ to be negative.

Property II(a): The GDP growth rate converges to the leading rate.

If $g(0) > g_{\min}$, then $g(t) \rightarrow g^*$ as $t \rightarrow \infty$

If $g(0) = g_{\min}$, then $g(t) = g_{\min}$ for all $t \geq 0$.

These properties are displayed in the phase diagrams Figs 1 and 2. The growth rate in a demand driven framework is pulled up ('driven') by whichever is the faster growing component of autonomous demand, i.e. by the leading rate. Provided that the economy starts strictly above $g_{\min}$ the growth rate $g(t)$ converges to g^*. Else it is stuck at $g_{\min}$.

This property of convergence will be used to throw light on what 'drives' growth – see below.

We shall also examine how *changes* in α and β affect $g(0)$ and $g(t)$. That too will throw light on what drives (changes in) growth.

It also follows from (1)-(4) that the growth curve $g(t)$ is S-shaped. We have $g = \theta\mu+\alpha$. Thus the absolute rate of change in g is proportional to the absolute rate of change in μ, varying directly with it when $\beta > \alpha$, and varying inversely with it when $\alpha > \beta$. Observing now Fig.1 that indicates the behaviour of μ, we can easily obtain Fig.2 below.

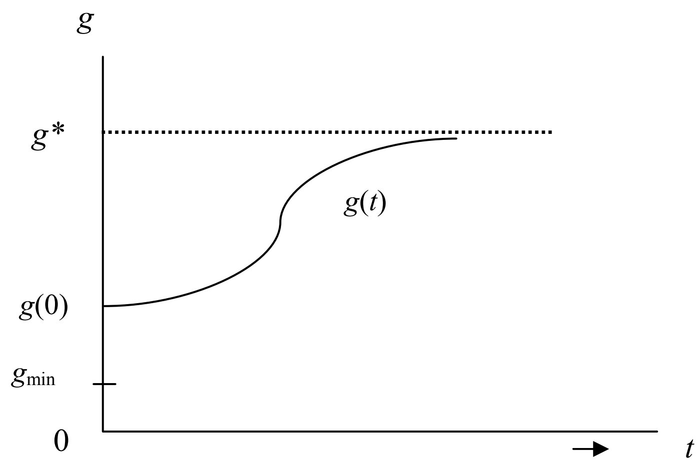

Fig 2: The growth curve in the demand model

If we were to draw a curve to depict the function $\mu(t)$ when $\beta > \alpha$, or the function $(1- \mu(t))$ when $\alpha > \beta$, that curve will likewise be S-shaped and have an asymptote 1. Such a curve will then display how the shares of investment and consumption in output will behave over time, since $(I/Y) = s\mu$ and $(C/Y) = 1- s\mu$. It is clear that the share of the faster growing demand component will converge to s or $(1 – s)$ as the case may be.

<u>Property II(b)</u>: The growth curves of g, μ, (I/Y) and (C/Y) are S-shaped. (When $\alpha > \beta$ then μ and (I/Y) are actually reverse S-shaped; $(1- \mu)$ and (C/Y) are S-shaped).

Initially if the growth rate is low, it will increase at an increasing absolute rate. In the later stages, the rate of increase will slow down as the growth rate nears the leading rate to which it will converge.

The behaviour of the demand shares may also be elaborated upon in the following manner:

<u>Property II(c)</u>:

(i) If $\alpha > \beta$, i.e. the leading growth rate is that of autonomous consumption, then

$\alpha > g_C(t) > g(t) > \beta$, where g_C is the growth rate of overall consumption. In this case (C/Y) and (C/I) rise over time.

<u>Reason</u>: Since $g(t)$ is a weighted average of α and β, it follows that $\alpha > g(t) > \beta$ for any $t < \infty$. Hence (I/Y) falls over time. For our closed economy, this means that (C/Y) rises over time. Therefore $[(C/Y)/ (I/Y)] = (C/I)$ rises over time.

(ii) If $\beta > \alpha$, i.e. the leading growth rate is that of investment, then $\alpha < g_C(t) < g(t) < \beta$. In this case (I/Y) and (I/C) rise over time.

<u>Reason</u>: Since $g(t)$ is weighted average of α and β, $\beta > g(t) > \alpha$ for any $t < \infty$. Then it must be the case that (I/Y) rises over time. For our closed economy, this means that (C/Y) falls over time. So it must be the case that $[(I/Y) /(C/Y)] = (I/C)$ rises over time.

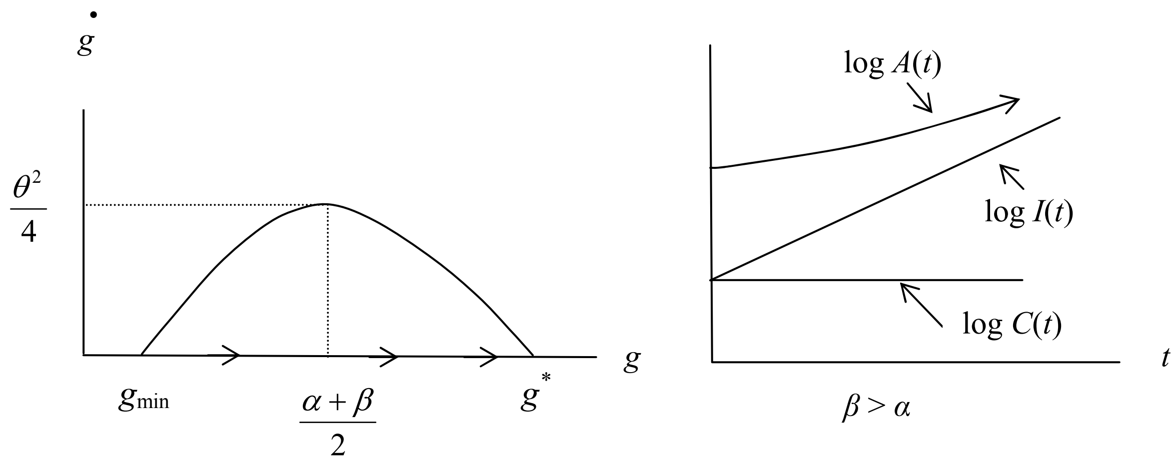

Fig 3: Dynamics of the growth rate

We now turn briefly to changes in α and β.

Recall that $g(t) = \mu(t)\beta + [1 - \mu(t)]\alpha$ and. Therefore we have (see Figs.4(a) and (b) below:

<u>Property III</u>

(a) A one-shot jump in $g_{\min}$ at date $t = 0$ shifts $g(0)$ and $g(t)$ up in the short-run (i.e. for all $t < \infty$).

(a1) The shifted path has a gentler slope: $\dot{g}(t)$ is reduced in the short-run.

(a2) The long-run growth rate is unaffected: g^* is unaffected and the shifted path still converges to g^*.

(b) A one-shot jump in g^* at $t = 0$ increases the growth rate both in the short-run as well as in the long-run.

(b1) The shifted path is uniformly steeper.

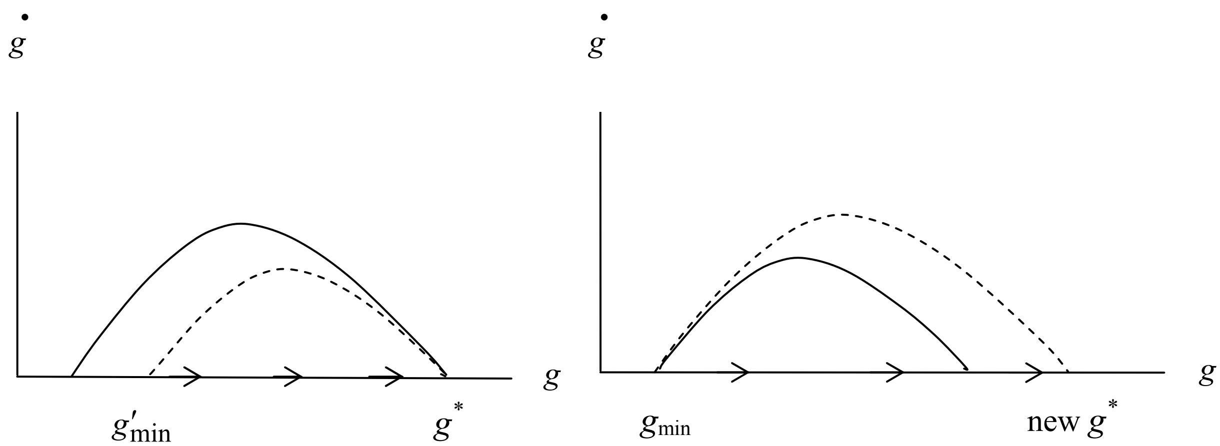

Fig 4: The effect of a rise in (a) $g_{\min}$ and (b) g^*

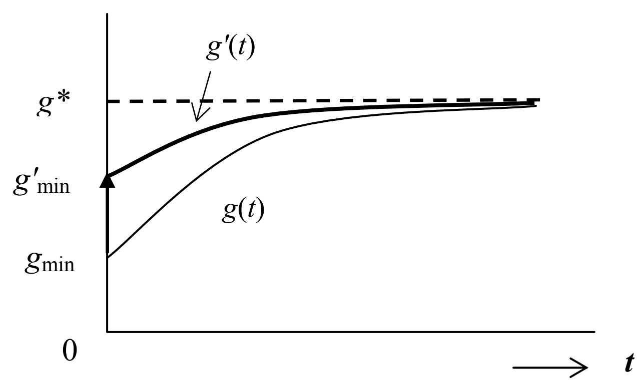

Fig 5: Effect (later stages) of an increase in the laggard growth rate ($g_{\min}$)

2.2. Discussion

2.2.1. Structural Breaks?

Save in the long-run, the rate of growth is never (almost never) constant. Variations in the growth rate reflect transition dynamics. If the initial growth rate is not maximal or minimal, then the growth rate will rise over time to the maximal level. The increase in the growth rate is clearly not the result of *changes* in any exogenous parameter or structural equation of the model.

There are two issues here. Firstly, observed changes in the growth rate could reflect approximation errors. If we approximate the continuous time growth curve of the model by a sequence of constant rate curves, we will end up with a sequence of ‘jumps’ in the average growth rate between successive intervals of time – see Fig.6 below. These jumps in growth rate do not reflect ‘structural breaks’. The increase in the growth rate is entirely endogenous to the model. A change in the average rate of exponential growth does not necessary signal a ‘structural break’.

The second point is about the economic meaning of the term 'structural break'. A 'jump' in the solution growth rate of the model is not necessarily a break. Suppose, as above, that there is a finite change in α or β. This will shift the growth path by a finite amount in the short-run – a 'jump'. Why should that signal a 'break'? Note that the solution path $g(t)$ is a continuously nth-differentiable function of the parameters α and β for any given t – refer to (6) below. A discrete change in these parameters would lead to a discrete change in $g(t)$ causing the path to jump or shift. It could be misleading to regard a discrete change in a parameter as a change in the structure of the model. A structural change is a regime change in a model that allows for the existence of multiple regimes. It is hasty to conclude that breaks in structure must have occurred whenever there is an observed discontinuity in the growth path or in its derivative function of time. A discrete change is roughly speaking a very large change in a very short interval of time.

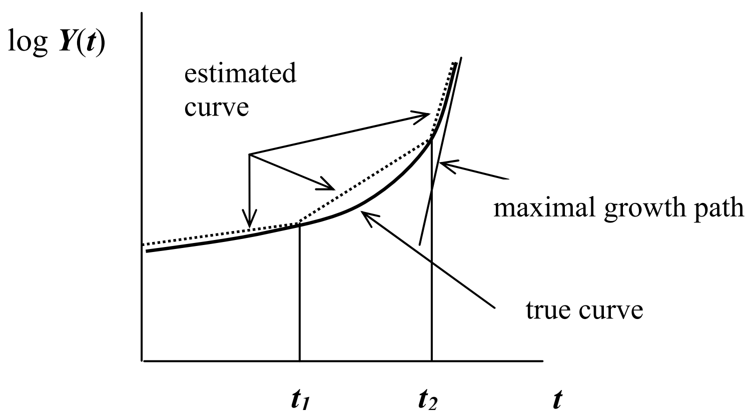

Fig 6: The Illusion of Structural Breaks at t_1 and t_2

To repeat, given the equations of the model and the levels at which the different exogenous parameters are fixed, the growth path is a continuously differentiable increasing function of time. One-shot changes in parameters may of course occur – time paths of these parameters could 'shift' – and that would lead to a jumps in the growth path and in rates of change. However the structure of the model may not have changed and nor may have there been a regime change.

2.2.2. Defining Consumption-driven and Investment-driven growth

What drives growth? What drives changes in growth? It is necessary to distinguish between those factors that are themselves the outcome of the model (the unknowns or endogenous variables) and those factors that are determined from outside the model and that have a role in determining the outcomes (exogenous parameters). To identify what drives growth we have to look at the exogenous parameters of the model and see how these affect GDP growth. The obvious candidates to be 'drivers' are (*i*) the rate of growth of autonomous consumption α and (*ii*) the rate of growth of autonomous investment β. In our model, there is convergence of the actual growth rate to the given leading rate. The laggard rate of growth represents an unstable steady state: if the initial growth rate is the minimal one it remains at that level, but a small perturbation sends the growth rate on an upward trajectory.

We may ask the following questions to identify what is driving growth.

(Q1) What determines the character of growth – e.g., is it positive, is it increasing, does it converge – given the parameters of the model?

(Q2) How sensitive is $g(t)$ to a (discrete) change in a chosen parameter in (a) the short-run, and (b) the long-run?

Properties I-III enable answers to these questions. We take up (Q1) first. The 'stability' property (see Property I and II above) suggests that it is the long-run rate of growth that drives the growth rate over time. The long-run rate is almost always the leading rate.

Definition: If autonomous investment has the leading rate then growth is investment-driven; for brevity we say that in this case 'growth is β-type'. If autonomous consumption has the leading rate, then growth is consumption-driven; i.e. 'growth is α-type'.

In this pure demand determined growth model the rate of growth $g(t)$ of GDP is the rate of growth of aggregate demand and that in turn is a weighted average of α and β. Almost always will $g(t)$ be pulled towards α or β depending on which is the larger of the two.

There is often a tendency in the less formal literature to regard constancy of growth rates to be the norm. It is assumed that the growth rate is constant for a while and then jumps to a new plateau. Whenever that happens and it is observed that the average growth rate in a recent period is different from that in the past, analysts rush to uncover some underlying change factor that could be responsible for causing the jump. A recent Indian question has been this: Is the increase in GDP growth in recent years the result of an increase in investment growth or is it the result of a 'consumption boom'? Based on our analysis, the following comments are in order:

(a) Even if neither autonomous rate – α or β – is positive, the GDP growth rate could still be increasing over time.

(b) Even if neither autonomous rate has changed, the calculated average GDP rate over a period of time could change from period to period. This need not be the result of any change in the parameters of the model such as the growth rate of autonomous consumption or investment.

(c) In this model, it is possible to infer from short-run observations of (I/C) whether it is α that is driving growth or it is β that is doing so.

2.2.3. Identifying growth drivers with the help of 'observables'

The last comment (c) above is significant. The above characterisation of consumption-driven and investment-driven growth is not in terms of readily 'observable' features of the growth path. Using the characterisation to identify the growth driver requires econometric estimation of the aggregate consumption function. Is there an equivalent characterisation in terms of observable variables? In particular, the question we ask is this: what does growth being consumption-driven – or alternatively investment-driven – imply about the short-run behaviour of observable ratios such as (C/Y) or (I/C)? In the present model, it is actually easy to tell from the behaviour of these ratios whether growth is α-type (i.e., consumption-driven) or β-type (i.e., investment-driven).

Consider $\alpha > \beta$. Then $g(t) > \beta$ and hence *Y/I* must be rising over time. For a closed economy this means *Y/C* must be falling over time, hence *C/Y* must be rising over time. Similarly if growth is β-type, then *C/Y* and *C/I* must be falling over time. (See Property II above and the remarks preceding it. See also section 2.3 [appendix A] below on the solution).

For this rather simplistic model then the identification question is a particularly easy one to solve. It would however be misleading to believe that we could begin by *defining* growth to be consumption-driven or α-type if *C/I* is increasing over time. This kind of characterisation-by-effect rather than characterisation-by-cause may not work. Firstly, the identification of cause by observing the outcomes *C/I* and *C/Y* works for *given* α and β but might get mixed up when either α or β changes. Moreover the identification is not robust to even simple extensions of the model.

Next we move on to (Q2) above.

2.2.4. Changes in α and β

The manner in which observables such as the growth rate $g(t)$ or the investment share $(I(t)/C(t))$ respond to a one-shot change – or 'jump' – in autonomous consumption growth (call it a 'consumption boom') is evident from the above (see also section 2.3 below). Suppose that growth is β-type – so that (I/C) is increasing over time – but there is a one-shot rise in α. If $\beta > \alpha$ is maintained then (I/C) will continue to rise over time. But simply observing that the share of investment in GDP is going up does not tell us whether there has been any change in α or β.

This case is illustrated in Fig. 5: growth is β-type but there is a one-shot rise in α. Though the long-run rate of GDP growth will be unaffected, in the short-run – i.e., for $t < \infty$ – $g(t)$ will *rise*. Assuming that growth continues to be β-type a consumption boom (one-shot increase in α) will have a positive growth effect in the short-to- medium run. There is however no effect of a change in the laggard rate on the long-run rate of growth [Figs. 5 and 10(a)].

What about the (I/C) path? That is still increasing over time, but has been *shifted* down and the *rate of increase* in (I/C) is now lower. Hence in the β-type regime if (I/C) is observed to *fall* then that is a signal that autonomous consumption growth has picked up.

A change in the leading rate does however affect the growth rate both in the short-run and the long-run.

These effects can be readily derived from the solutions to the differential equation given in Section 2.3 (appendix A) below. Note that:

$$\mu(t) = \frac{I(t)}{I(t) + \bar{C}(t)} = \frac{1}{1 + \phi\, e^{-(\beta-\alpha)t}} \tag{5}$$

where $\phi = \frac{1}{\mu(0)} - 1$ and $\mu(0) = \frac{I(0)}{I(0) + \bar{C}(0)}$

An upward shift in α reduces $\mu(t)$ for $t > 0$ and hence also $(I(t)/Y(t)) = s\mu(t)$ for each t. This happens because of a discrete fall in $\dot{\mu}(t)$ for $t \geq 0$.

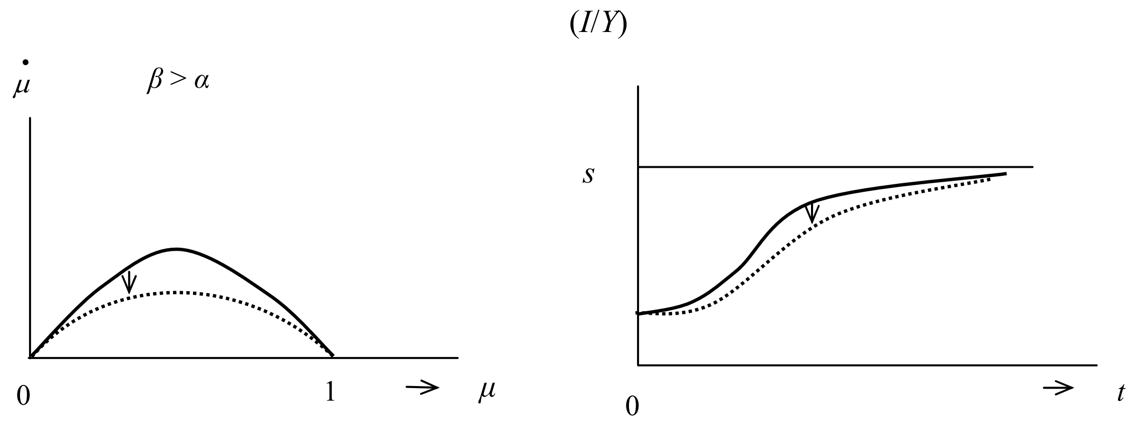

Fig 7: Effects of a one-shot increase in α

2.2.5. One-shot changes in the time constant level of autonomous consumption

It is important to differentiate between 'level effects' and 'growth effects'. Suppose that $\alpha = 0 < \beta$. What is the effect of a one-shot increase in the (time-constant) level of $\overline{C}$? This too is a consumption-driven change. Clearly there is a fall in $\mu(0)$, the share of investment in autonomous demand, and therefore $g(0)$, which is a weighted average of α and β, falls too. The rise in autonomous consumption level will raise the current *level* of GDP (as $Y = \frac{I + \overline{C}}{s}$) but since there will be no effect on $\dot{Y}$ (as $\dot{Y} = \frac{\dot{I}}{s}$ with $\overline{C}$ constant in time), $g(t) = \dot{Y}/Y$ *will actually fall*. Though there is an increase in the *level* of autonomous consumption, the growth rate *falls* in the short-run.

However the long-run rate of growth $g(t) = \beta$ remains unchanged.

2.2.6. Role of the marginal propensity to save

The above is a demand-constrained model. Unlike in neo-classical models that take-off from Solow, and unlike in models of capacity constrained output, a higher marginal propensity to save does not stimulate the rate of growth. In the Solow model, a higher marginal propensity to save increases the level of output per head in the long-run while leaving the long-run rate of growth unaffected. In the demand-constrained model considered above, the mps was taken to be constant over time. A one-shot increase in the given mps does not have any growth effect but it does have a level effect on output that is negative. The entire $Y(t)$ path gets shifted downward in a parallel fashion. This follows from the multiplier relation $Y(t) = A(t)/s$. A change in the time-constant parameter s at the initial date does not affect $\mu(t)$ or $g(t)$. However aggregate demand $Y(t)$ gets shifted at each date t.

Here too there is possibility of an approximation error. The shift that occurs in the model is an instantaneous adjustment of demand and output. In practice this adjustment will take time. It will clearly spill over into the next period if the adjustment is slow or the unit period is short. One may therefore tend to conclude that there is a fall in the short-run rate of growth, though strictly speaking there has been no growth effect; only a level effect.

Note that the saving ratio (or the average propensity to save) will clearly rise over time as long as growth is positive since we have $S/Y = s - (\overline{C}/Y)$. This is so even when growth is consumption driven.

2.3. Solutions and Proofs of the Properties

See Appendix A

3. Full-capacity Growth: Harrod-Domar Models

Plenty of questions are left unaddressed in the above demand-determined growth model. One question is this: For how long is it possible to have positive net investment in the face of persistent excess capacity? This is especially relevant when growth itself is investment driven growth. In the case of consumption driven growth, one is led to ask how it is possible to maintain high rates of consumption growth without expanding productive capacities at sufficiently high rates over time. There must be some production-consumption consistency that puts an upper bound on the consumption growth rate.

Moreover in the pure demand model there is nothing to distinguish between autonomous consumption and autonomous investment. These are equally useful in raising current production and there is no difference as far as the future goes – since excess capacity persists by assumption.

Let us drop the assumption of everlasting excess capacities. A simple exercise is to move to the class of models that is best described as 'Harrod-Domar models' (see Domar [2] & Harrod [3]). Any positive net investment adds to capacity. In order for these new capacities to be utilized fully, there must be an appropriate increase in aggregate demand. That requires an appropriate *increase* in investment. A constant level of net investment will cause capacity to grow at an arithmetic rate, but leave demand unchanged. Hence a constant path of investment tends to generate excess capacity. This would induce a cut back in the level of investment, making the problem worse. A constant level of investment is simply not sustainable over time; investment has to grow at a sufficiently rapid rate for investment to be sustainable. The Harrod-Domar model of short run growth was designed to address this issue and it was shown that investment must grow at one particular rate ('warranted rate') to keep the economy in a self-fulfilling equilibrium (perfect foresight) path. This rate is determined by the saving propensity and the productivity of capital.

3.1. Generalised Harrod-Domar Model with fixed Autonomous Consumption

We consider a generalised Harrod-Domar model by incorporating autonomous consumption expenditure. Assume that in a closed economy the productivity of capital B, the marginal propensity to save s and the level of autonomous consumption $\overline{C}$ have exogenously given constant paths over time.

Let $Y = BK$ be the production relation. $(1/B)$ is the capital-output ratio. With full-capacity utilization, market clearing entails $sBK = \dot{K} + \overline{C}$. Divide by K and let $g_K = I/K$. Then

$$g_K = sB - (\overline{C}/K) \tag{6}$$

Thus if $K(0) > \overline{C}/sB$, then $g_K(t) > 0$ for all $t \geq 0$. If $K(0) = \overline{C}/sB$ then $K(0) = K(t)$, $g_K(t) = 0$ for all $t \geq 0$, and if $K(0) < \overline{C}/sB$ then $g_K(t) < 0$ for all $t \geq 0$.

The following phase diagram shows the dynamics (Fig. 8).

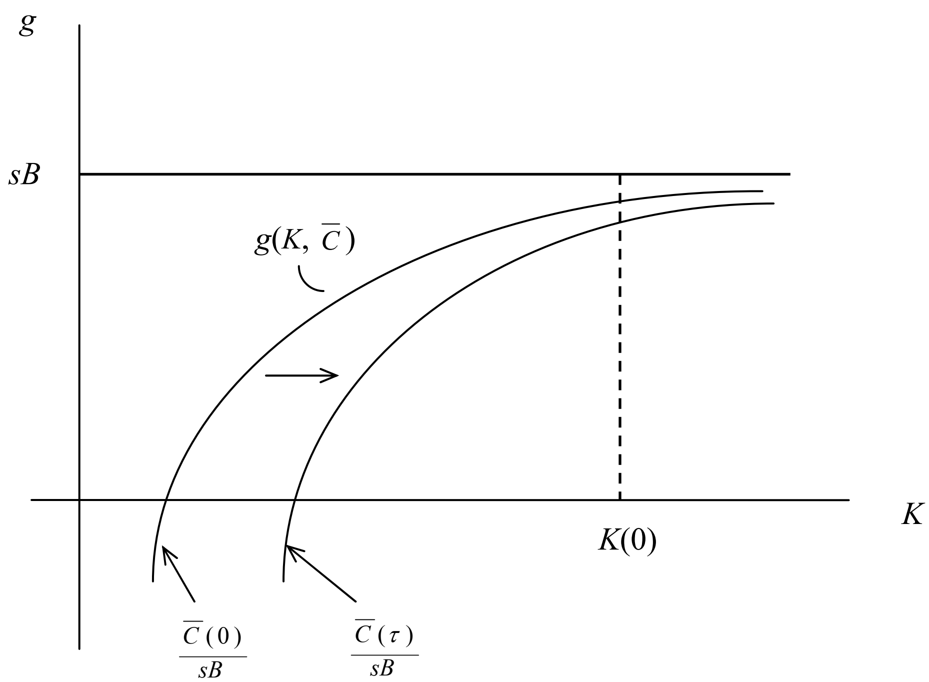

Fig 8: Dynamics of the growth paths for different levels of autonomous consumption

Solving for the path of $K(t)$ we get the following,

$$K(t) = (\frac{\overline{C}}{sB}) + [K(0) - \frac{\overline{C}}{sB}]e^{sBt} \tag{7}$$

Assume $K(0) > \frac{\overline{C}}{sB}$. Then as $t \to \infty$, $K(t) \to \infty$ and $\frac{\overline{C}}{K(t)} \to 0$. Therefore from (6) it follows that

$g_K \to sB$ from below. From (10), $I = sBK - \overline{C}$, hence $\dot{I} = \text{sB}\dot{K}$, i.e.

$$g_I = sB \tag{8}$$

Hence full capacity equilibrium requires investment to grow at a constant rate determined by the saving ratio and productivity of capital. This is exactly the Harrod-Domar rate.

Let $sB \equiv \beta$. In fact $g_I = \beta$ is the original Harrod-Domar rate of growth (in the absence of any autonomous consumption). Let the growth rate of autonomous consumption $\overline{C}$ be denoted by α.

It is important to note that the rates of growth of capital, investment and output *do* depend on the marginal savings propensity, in sharp contrast to the demand determined model.

3.1.1. Properties

Property V

A rise in the level of $\overline{C}$ reduces the short-run growth rate but does not affect the long-run growth rate of capital stock. The investment-growth rate is not affected by the level of $\overline{C}$ at any date.

From (6) it is evident that a rise in $\overline{C}$ reduces the short-run growth rate. However the long-run growth rate β remains as it is. From (8) since the investment growth rate does not depend on $\overline{C}$; its growth rate is not affected by any changes in $\overline{C}$.

Property VI

An upward shift in β raises both the short-run and the long-run capital stock and investment growth rates.

This property is immediate from (6) and (8).

3.1.2. Discussion

First consider the following discrete change: an upward shift in β caused by an upward shift in s. This shift creates jumps in the growth rates of capital and investment, both in the short and the long run. In this capacity constrained generalised Harrod-Domar model, 'investment driven' growth is simply an upward displacement of the marginal propensity to save, i.e. investment driven growth is actually 'saving-driven' growth (property VI). As β rises aggregate demand tries to fall (through a fall in income induced consumption) but it actually can't on the equilibrium path because the new capacity created must be fully utilized. This reflects fulfillment of expectations. It is necessary to raise investment to offset slackening demand so that capacity is fully utilised. That is why the investment growth rate has to go up on the expectations equilibrium path. This insight is the main conclusion of Harrod's pioneering work on growth.

This result is at sharp variance with the effect of a rise in β in the demand-based model with the Keynesian multiplier operative under condition of excess capacity. If we introduce induced investment via an accelerator, the rise in s and the resulting emergence of excess capacity would lead to a fall in investment – but that would only worsen excess capacity over time. While the rise in β raises the *equilibrium* rate of growth, the *actual* rate of growth may go down.

Next consider a one shot change in $\overline{C}$ with fixed s. This leads to what can be described as a consumption driven change. The immediate effect on the equilibrium rate of growth of capital g_K is to push it down. However, in the long run the equilibrium capital stock continues to grow at the Harrod-Domar rate β. Also note that there is no effect on the growth rate of investment g_I, either in the short or in the long run – see (8). Thus a one shot change in the autonomous consumption demand has no long run effect on the growth rates (property V).

It is interesting to see if a continuous upward drift in autonomous consumption modifies the results.

3.2 Generalized Harrod-Domar Model with growing Autonomous Consumption

Differentiating (6), with fixed β, we get

$$\begin{aligned}\dot{g}_K &= -\frac{\dot{\overline{C}}}{K} + \frac{\overline{C}}{K^2}\dot{K} = \frac{\dot{\overline{C}}/\overline{C}}{K/C} + \left(\frac{\overline{C}}{K}\right)\frac{\dot{K}}{K} = -\alpha\frac{\overline{C}}{K} + \frac{\overline{C}}{K}g_K \\ &= \frac{\overline{C}}{K}(g_K - \alpha) = (\beta - g_K)(g_K - \alpha) \qquad (9)\end{aligned}$$

From (9), $\dot{g}_K$ is positive either when $\alpha < g_K < \beta$ or $\beta < g_K < \alpha$.

However note from (6) that it is always the case that $g_K < \beta$ for positive $\overline{C}$. Hence for growth to be positive we must have $\alpha < g_K < \beta$. Note that $\dot{g}_K$ reaches a maximum as a function of g_K at $g_K = (\alpha + \beta)/2$.

Note that for full-capacity growth $g_Y = g_K$. We drop the subscripts henceforth for brevity.

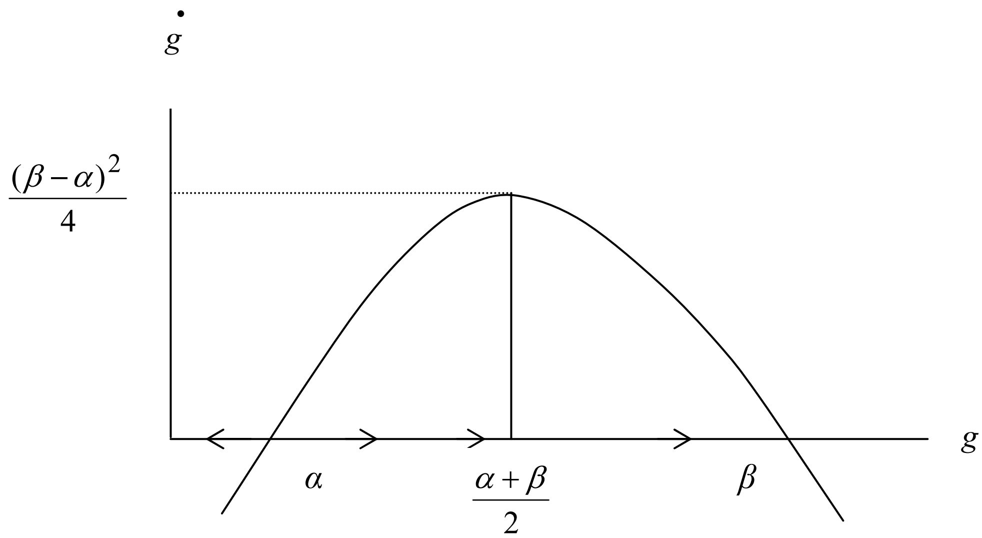

Fig 9: The dynamics of growth in generalised Harrod-Domar model

3.2.1. Properties of Growth in the Generalized Harrod-Domar model

Property VII

The economy grows, i.e. $\dot{g}(t) > 0$ provided that $\beta > \alpha$. Growth is invariably investment-driven.

Property VIII

If $g(0) > \alpha$ and $g(0) \neq \beta$ then $\dot{g}(t) > 0$ for all $t \geq 0$ and $\lim_{t\to\infty} g(t) = \beta$. The long-run rate of growth is the Harrod-Domar rate.

If $g(0) < \alpha$, then $g(t) \to -\infty$ as $t \to \infty$. If $g(0) = \alpha$, then $g(t) = \alpha$ for all $t \geq 0$. If $g(0) = \beta$ then $g(t) = \beta$ for all $t \geq 0$.

Property IX

An increase in α reduces $\dot{g}(t)$ in the short-run. For all $t < \infty$, $g(t)$ is lower for all t as well, but the long-run growth rate is unaffected. An increase in β increases both the short-run as well as the long-run growth rates.

Property X

The ratio (I/C) is proportional to the growth rate. The behaviour of (I/C) does not help identify the growth driver.

3.2.2. Discussions

Similar to the previous model of pure demand growth, here also a rise in the average rate of exponential growth does not necessary signal a ‘structural break’. The growth rate may rise on its own – except when it is too low initially – simply because it is lower than the Harrod-Domar level. In a sense the growth rate rises because initially autonomous consumption is too high relative to the capital stock.

It is crucial that the growth rate at any date must be more than α, otherwise the economy would experience a falling growth rate and hit zero growth rate in finite time. Consider a one-shot discrete jump in α to α', at some date τ. Then $\dot{g}(t)$ is lower for all $t > 0$. This means that $g(t)$ is lower for all $t > 0$ as well. If it so happens that $g(\tau) < \alpha'$, the growth rate starts falling and will eventually become negative. If $g(\tau) > \alpha'$, then the long-run rate of growth remains unaffected.

In this model, growth in the short run and may be also in the long run, is adversely affected by an increase in the rate of growth of autonomous consumption. If the jump in α is drastic, that may result in a perpetual fall in the growth rate and make it negative. Otherwise, for small up-thrusts in α, the long-run rate of growth will be maintained. This offers an interesting perspective on the growth process. Provided that α is raised by small amounts over time long run consumption growth would increase in the long-run, though the short run growth rate takes a hit.

Thus the rate of growth of autonomous consumption acts as a threshold for growth to be positive.

This result is at sharp variance with the model of pure demand determined growth. In that model a rise in α in investment-driven growth regime causes the short-run growth rate to go up though it does not affect the long-run rate of growth. With excess capacity, a rise in any component of autonomous demand is helpful. However, in capacity constrained models like the Harrod-Domar model a rise in consumption demand must reduce investment since output is given. Hence the growth rate must go down in the short run. Moreover there is a possibility of the growth rate becoming *negative* – in the demand determined model the growth rate is never negative for positive α and β – so that quantum of change in α assumes importance.

3.2.3. Solutions and Proofs

See Appendix B

4. Conclusions

4.1. The Question of Structural Breaks

The results obtained can now be used to comment on the large and growing literature on Indian growth. The Indian economy has been growing at a faster pace in recent decades than it did in the first few decades after independence. Two discernible phases of economic growth in India since independence have been recognised: 1950 to 1980 and 1980 to 2008. Compared to the preceding 30 years, there was a distinct step-up in rates of growth for GDP and GDP per capita. The earlier period has been commonly described as ‘the Hindu growth rate era’ in which the average growth rate was 3.5 percent. During the period from 1980 to 2008 the average growth rate has gone up to 6 percent. Econometric exercises, such as those of Wallack [16]; Hausman, Pritchett and Rodrik [4]; Rodrik and Subramaniam [12] and Virmani [15], suggest that the structural break in the trend rate of growth takes place around the early 1980s.

Models developed in this paper throw up growth rates that change *endogenously* over time. When time is periodized, a change in the observed average rate of exponential growth does not necessarily signal a 'structural break'. In the pure demand determined model changes in growth rate occur whenever the autonomous consumption and investment growth rates differ from each other. In the generalised Harrod-Domar model, a positive growth rate of GDP is ensured whenever investment growth rate exceeds the autonomous consumption growth rate.

To capture 'structural breaks' the model has to be capable of generating alternative regimes. Thus if we amalgamate the two models of this paper and show that growth can take the economy from one regime (say, excess capacity) to another (say, full capacity), then the intersection of the two regimes would indeed signal a 'structural break'.

4.2. The Drivers of Growth

There is an emerging literature which seeks to pin down the central cause of the rapid economic growth in India in the last two decades. Several arguments have been advanced to explain this recent surge in growth rate in the Indian economy. The reasons broadly cover three aspects: demand side considerations, supply side factors and institutional reforms. It has been suggested that increased growth during the 1980's was generated by the expansion of aggregate demand mostly through a rapid increase in public expenditure on investment and consumption (see Nayyar [10], and Joshi and Little [5]).

Nayyar [9] documents the following potential factors behind the high growth rate. First, expansionary macroeconomic policies led to an increase in aggregate demand which stimulated an increase in the rate of growth of output. Second, there was a sustained increase in the investment-GDP ratio in 1980s, contributing to the step-up in economic growth during the 1980s. Third, there was also a significant increase in public investment which was sustained through the 1980s. While this contributed to the increase in aggregate demand, by creating new infrastructure or improving existing infrastructure it could have stimulated growth in output by alleviating supply constraints.

There are other studies (see Acharya [1], Mohan [7], The Economic Survey [14]) that have recognised the role of either rising investment or consumption demand in sustaining the high growth in India. A survey by NCAER [8] and a report by McKinsey Global Institute [6] noted the growing importance of the Indian middle class in driving the consumption leading to a situation of 'consumption boom'. In a recent paper Patnaik [11] has looked into the question as to why rising share of economic surplus in output has not created any serious realisation problem and hence any consequent tendency towards stagnation in the Indian economy in the recent years. The reason adduced is that the rising share of economic surplus in output has been accompanied by greater consumption by the surplus earners themselves and also by greater investment that has been stimulated by such consumption. Thus Patnaik also considers the 'consumption boom' as one of the driving factors for recent Indian growth.

The findings of these studies have been culminated into a recent debate: Is the recent surge in Indian growth rate ‘consumption driven’ or ‘investment driven’? However, mostly the discussion has taken the form of informal observations through the lens of Indian macro data with occasional employment of econometric methods. Moreover, there has been inadequate focus on the *composition* of aggregate demand and its drivers. There is hardly any study that uses an analytical framework to study the implications for growth of distinguishing between rising investment and rising consumption demands. Often the differences between outcomes in a demand constrained situation from a capacity constrained one have not been taken into account. The models dealt with in this paper make the distinction explicit.

Another inadequacy is the absence of a proper dynamic framework when commenting on the nature and features of Indian growth. Most commentators rely on a ‘comparative statics’ framework. Thus while citing the behaviour of indicators like saving-GDP ratio or investment-GDP ratio or investment-consumption ratio for fostering (or hindering) growth, the conditions under which the claim is valid are hardly mentioned. The present paper tries to fill up this lacuna by proposing explicit dynamic models that can be used to analyze observable macro-data that can identify the drivers of growth. In doing so, this paper proposes formal definitions of consumption-led and investment-led growth and in the process seeks to clarify the analytical distinction between these two types of growth. This issue had remained somewhat unclear in the existing literature.

Appendix A

Proofs of Properties I & II

The differential equation (3) is a Bernoulli equation that has a well-known closed form solution.

$$\mu(t) = \frac{1}{1+\phi\, e^{-(\beta-\alpha)t}} \qquad (10)$$

where $\phi = \dfrac{1}{\mu(0)} - 1$

From (10) it follows that,

(i) when $\alpha > \beta$, i.e. $\theta < 0$, as $t \to \infty$, $e^{-(\beta-\alpha)t} \to \infty$; hence $\mu \to 0$

(ii) when $\alpha < \beta$, i.e. $\theta > 0$, as $t \to \infty$, $e^{-(\beta-\alpha)t} \to 0$;hence $\mu \to 1$

For the solution of $g(t)$ note that $g(t) = \alpha + \mu(\beta - \alpha)$

Therefore, using the solution for μ, we get

$$g(t) = \alpha + \frac{\beta-\alpha}{1+\phi\, e^{-(\beta-\alpha)t}} \qquad (11)$$

From (11) it follows that,

(i) when $\alpha > \beta$, i.e. $\theta < 0$, as $t \rightarrow \infty$, $e^{-(\beta-\alpha)t} \rightarrow \infty$; hence $g(t) \rightarrow \alpha$

(ii) when $\alpha < \beta$, i.e. $\theta > 0$, as $t \rightarrow \infty$, $e^{-(\beta-\alpha)t} \rightarrow 0$; hence $g(t) \rightarrow \beta$

(iii) Clearly $max[\alpha, \beta] \geq g(t) \geq min[\alpha, \beta]$ and $\lim_{t\rightarrow\infty} g(t) = max[\alpha, \beta]$

(iv) $g(0) = \alpha + \dfrac{\beta-\alpha}{1+\phi} = \mu(0)\beta + [1-\mu(0)]\alpha$; since $0 < \mu(0) < 1$, we have

(a)when $\alpha > \beta$, $g(0) > \beta$ and

(b) when $\alpha < \beta$, $g(0) > \alpha$

(v) When $\beta > \alpha$, $g(0) > \alpha$ and since $\dot{g}$ is positive when the economy starts between α and β, it must be that $\alpha < g(0) < \beta$. Similarly when $\beta < \alpha$, $g(0) < \alpha$, and $\beta < g(0) < \alpha$.

Property III

From (4), $\dot{g} = (\beta-\alpha)\,\dot{\mu} = \mu\,(1-\mu)\,(\beta-\alpha)^2$. Let $\beta > \alpha$. A discrete increase in α causes a discrete reduction in $\dot{g}(t)$ for $t < \infty$, given $\mu(0)$. What happens to the level of $g(t)$? Note that $g(0) = \mu(0)\beta + [1-\mu(0)]\alpha$. Hence a jump in α will shift $g(0)$ up. This causes the $g(t)$ path to shift up at all subsequent dates. However long-run $g(t)$ is unchanged and the rate of increase in $g(t)$ over time is reduced:

$\lim_{t\rightarrow\infty} g(t) = max[\alpha, \beta] = \beta$, independent of α.

On the other hand with $\beta > \alpha$, a jump in β induces a jump in $\dot{g}$ in the short-run. Also $g(0)$ jumps up. This causes the $g(t)$ path to shift up at all subsequent dates. In this case the rate of increase of $g(t)$ is raised (compare this conclusion with the shift in α discussed above). A rise in β increases both the short-run and the long-run growth rates.

Let α_1 and β_1 denote the new levels and α_0 and β_0 the initial levels.

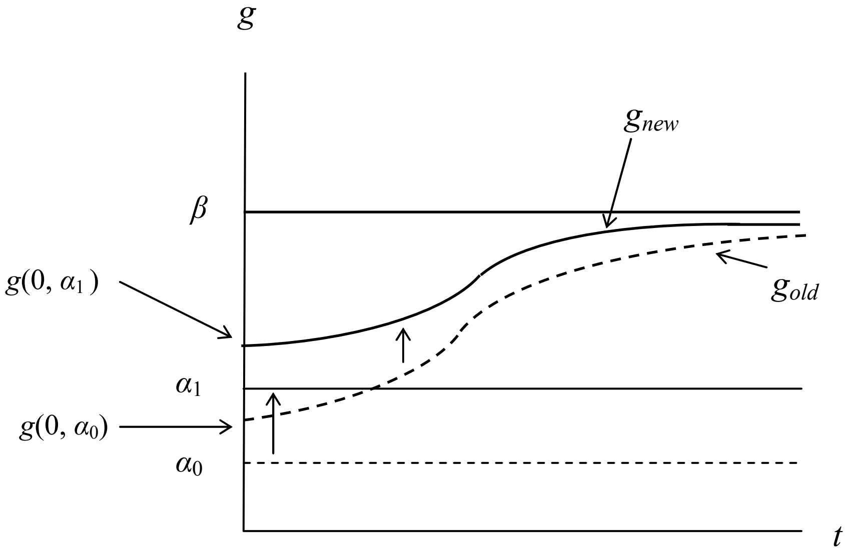

Fig 10 (a): The effect of changes in α on the growth rate

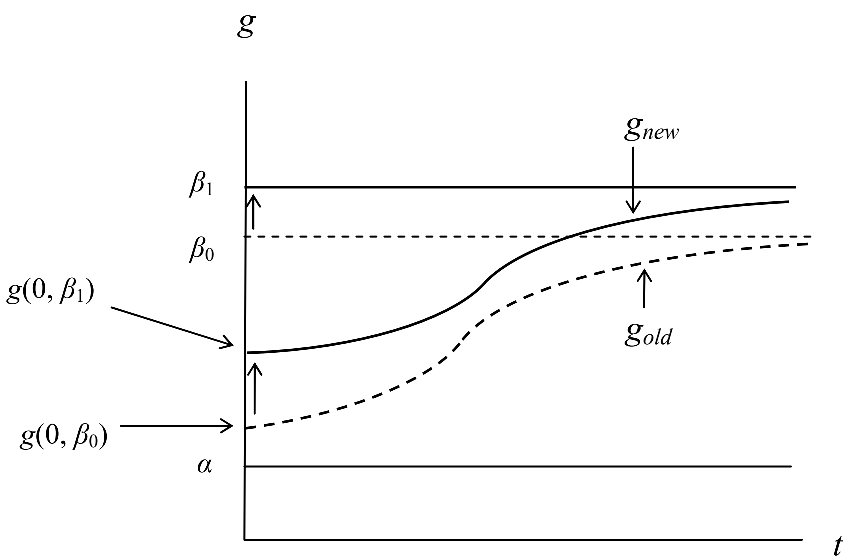

Fig 10 (b): The effect of changes in β on the growth rate

Property IV

We focus on g_C, the rate of growth of 'total' consumption C (autonomous plus induced). We know how the two components are growing – the autonomous at the rate α and the induced at the rate $g(t)$.

Differentiating the consumption function, $C = \overline{C} + cY$, we get

$$g_C = (\alpha - \beta)\,(\frac{\bar{C}}{C}) + g$$

i.e., $g_C - g = (\alpha - \beta)\,(\frac{\bar{C}}{C})$ (12)

Moreover we know that

$$g(t) = \alpha + \mu(t)\,(\beta - \alpha) \tag{13}$$

Thus to figure out g_C we need to figure out ($\bar{C}/C$). This can be done as follows.

$$C \;=\; \bar{C} \;+\; cY \Rightarrow \frac{\bar{C}}{C} = 1 - c\,(\frac{Y}{A})\,(\frac{A}{\bar{C}})\,(\frac{\bar{C}}{C})$$

$$= 1 - \;(\frac{c}{1-c})\,(\frac{1}{1-\mu})\,\frac{\bar{C}}{C}, \text{ using the multiplier and the definition of } \mu.$$

$$\frac{\bar{C}}{C} = \frac{1}{1 + \frac{\delta}{1-\mu}} \text{ where } \delta \equiv \frac{c}{1-c} \tag{14}$$

This shows that ($\bar{C}/C$) and μ are inversely related.

<u>Property IV(i)</u>

Consider first the case $\alpha > \beta$. From (13), $\alpha > g$. Since g is a weighted average of α and β, we have $\alpha > g > \beta$ in the short-run.

From (12), $g_C > g$. When $\alpha > \beta$, μ falls over time to 0 and from (14) [$C/\bar{C}$] rises over time to $(1 - c)$. Thus $\alpha > g_C$; hence $\alpha > g_C > g > \beta$. Now $g_C > g$ implies that (C/Y) rises over time and $g_C > \beta$ implies that (C/I) rises over time. In the long-run g and g_C both converge to α.

<u>Property IV(ii)</u>

Next consider the case $\beta > \alpha$. In this case, from (13), $\alpha < g$. Since g is the weighted average of α and β, $\alpha < g < \beta$ in the short-run. From (12), $g_C < g$. When $\alpha < \beta$, μ rises over time to 1and from (14) [$\bar{C}/C$] falls over time to 0. Hence $\alpha < g_C$ and $\alpha < g_C < g < \beta$. Now $\beta > g$ implies that (I/Y) rises over time. Since $\beta > g_C$, (I/C) rises over time as well. In the long-run both g and g_C converge to β.

Appendix B

The solution to the differential equation (9) is given as follows.

$$\dot{g} = (\beta - g)(g - \alpha) \tag{15}$$

$$\frac{1}{\beta-\alpha}\int_{g(0)}^{g(t)}[\frac{1}{g-\alpha}+\frac{1}{\beta-g}]dx=\int_0^t dt$$

$$i.e. \ln\frac{\beta-g(t)}{g(t)-\alpha}-\ln\frac{\beta-g(0)}{g(0)-\alpha}=-(\beta-\alpha)t$$

$$i.e. \ln\frac{\beta-g(t)}{g(t)-\alpha}-\ln\phi=-(\beta-\alpha)t$$

$$\text{So, } g(t)=\alpha+\frac{\beta-\alpha}{1+\phi\, e^{-(\beta-\alpha)t}} \qquad (16)$$

Where $\phi=\dfrac{\beta-g(0)}{g(0)-\alpha}$ is a constant.

<u>Property VI & VII</u>

From (15) it is clear that $\dot{g}$ will be positive, i.e. the economy will grow only if $\alpha < g(t) < \beta$ or $\beta < g(t) < \alpha$. For positive $\overline{C}$ from (6) $g(t) < \beta$. Hence we must have $\alpha < g(t) < \beta$, i.e. $\beta > \alpha$.

Further from (16), as $t \to\infty$, for $\beta > \alpha$, $\lim_{t\to\infty} g(t) = \beta$ from below. This shows that the long-run rate is always the Harrod-Domar rate. Moreover from (15) if $g(t) < \alpha$ with $\beta > \alpha$, then as $t \to\infty$, $g(t) \to -\infty$, i.e. $g(t)$ will become zero in finite time.

It is also evident that If $g(0) = \alpha$, then $g(t) = \alpha$ for all $t \geq 0$. If $g(0) = \beta$ then $g(t) = \beta$ for all $t \geq 0$.

For $g(t) > \alpha$ from (10), we must have $(\beta - \overline{C}/K) > \alpha$. This imposes a condition on the initial level of $\dfrac{\overline{C}}{K(0)}$. The condition is given by $\dfrac{\overline{C}}{K(0)} < (\beta - \alpha)$. This shows that the economy would grow provided the initial level of autonomous consumption and growth of it are not too high.

<u>Property VIII</u>

From (15) it can be seen that a rise in α reduces $\dot{g}$ in the short-run. Since $g(0) = \beta - \overline{C}/K(0)$ from (6), a change in α does not affect $g(0)$. This means that $g(t)$ is lower in the short-run as well and $g(t)$ increases slower than before. Thus a rise in α reduces the short-run rate of growth. However as $\lim_{t\to\infty} g(t) = \beta$ always, the long-run growth rate remains unaffected.

From (15), an increase in the β increases $\dot{g}$ in the short-run and also raises $g(0)$. This means that the new path of $g(t)$ will lie everywhere above the old one. As $\lim_{t\to\infty} g(t) = \beta$, the long-run growth rate rises as well.

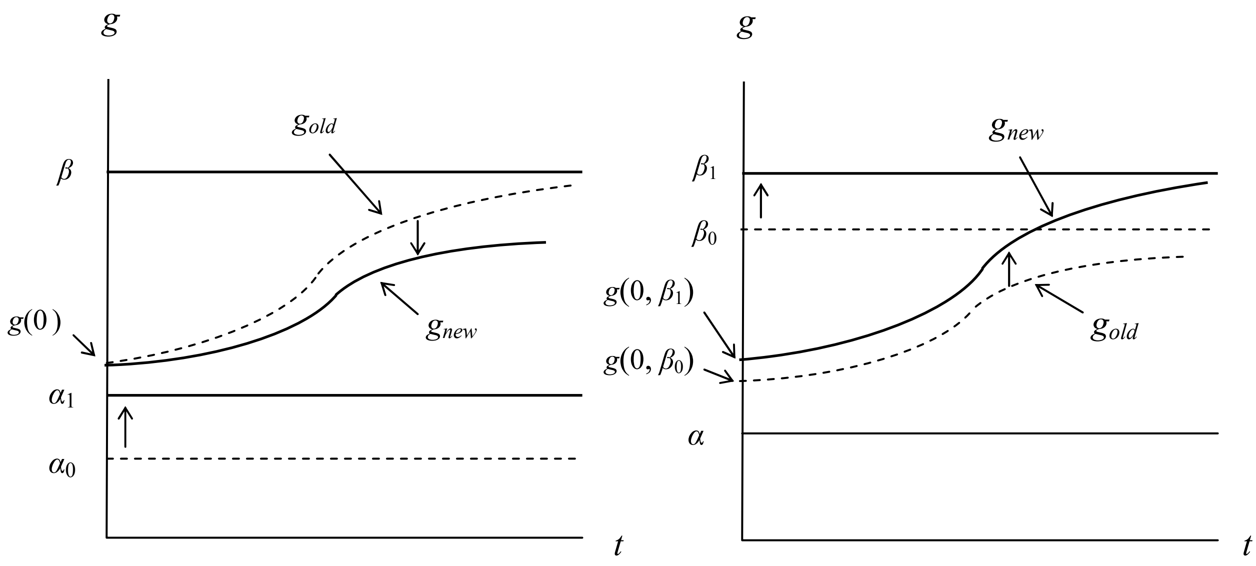

Fig 11: The effect of rise in α (left panel) and β (right panel) on the growth path

The quantum of discrete change in α is important. After the one shot rise in α at date τ, $g(\tau)$ will fall but should remain higher than α'. Otherwise the growth rate will eventually become negative. After elapse of sufficient time from $t = 0$, a rise in α would probably cause no problem in terms of degeneration of the economy, though the short-run growth rates falls. If the quantum of jump in α is drastic, the condition $g(\tau) > \alpha'$ is unlikely to hold.

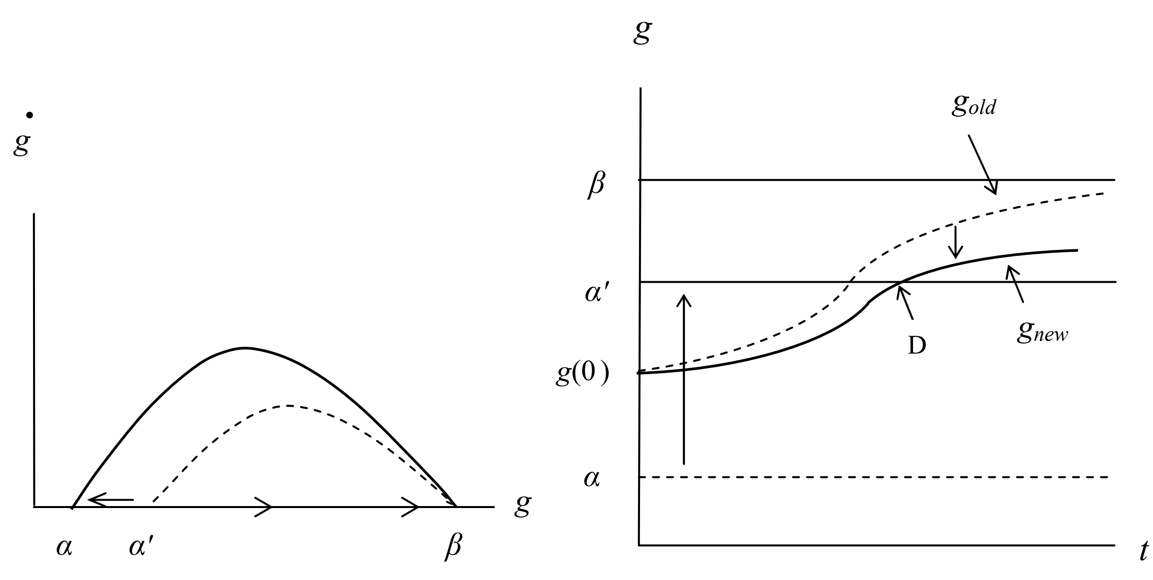

Fig 12: Dynamics of growth rate for different α

From the right hand panel of the above diagram, it is seen that $g(t)$ will continue to increase if after the increase in α, the economy is to the right of point D (the point of degeneration). Otherwise $g(t)$ will fall continuously and eventually becomes negative.

<u>Property X</u>

We have, $C = \overline{C} + (1-s)Y = \overline{C} + (1-s)BK$ and $I = Kg_K \equiv gK$

Therfore $\dfrac{I}{C} = \dfrac{gK}{\overline{C} + (1-s)BK} = \dfrac{g}{\dfrac{\overline{C}}{K} + (1-s)B}$

$$= \frac{g}{sB - g + (1-s)B} = \frac{g}{B-g} = \frac{s}{1-s} \qquad (17)$$

Thus (I/C) is directly proportional to $g(t)$. So whenever g is observed to rise, it must be the case that (I/C) rises. We have shown that when α rises, g would be lower at every date and also rises at a reduced rate. Thus when autonomous consumption growth rate increases it is the case that the path of (I/C) is pushed down, and there is a one time discrete fall in (I/C) or a discrete jump in (C/I). A one shot rise in α makes (I/C) to be lower at each g which is again lower at each date. However the economy continues to grow just because $g(t) < \beta$, so that (I/C) eventually rises once again along the new path though slower than before, i.e. slope of (I/C) is lower at each date now (see Fig. 13). Thus a one time rise in (C/I) does not signal that the growth is consumption-driven.

Moreover, in a large number of cases (I/C) will rise since the growth rate rises simply because it is lower than the Harrod-Domar level. But it would not be correct to attribute this rise to an increase in the investment rate of growth. Further, when $g(t) < \alpha$, $g(t)$ and hence (I/C) falls over time. Clearly this cannot be a case of consumption-driven growth.

However if the marginal propensity to save s rises (and β rises) with constant α, then g will rise at each date, causing a discrete jump in (I/C) and thereafter a higher (I/C) at each subsequent date. Thus this change in the ratio (I/C) may be interpreted as an investment-driven change.

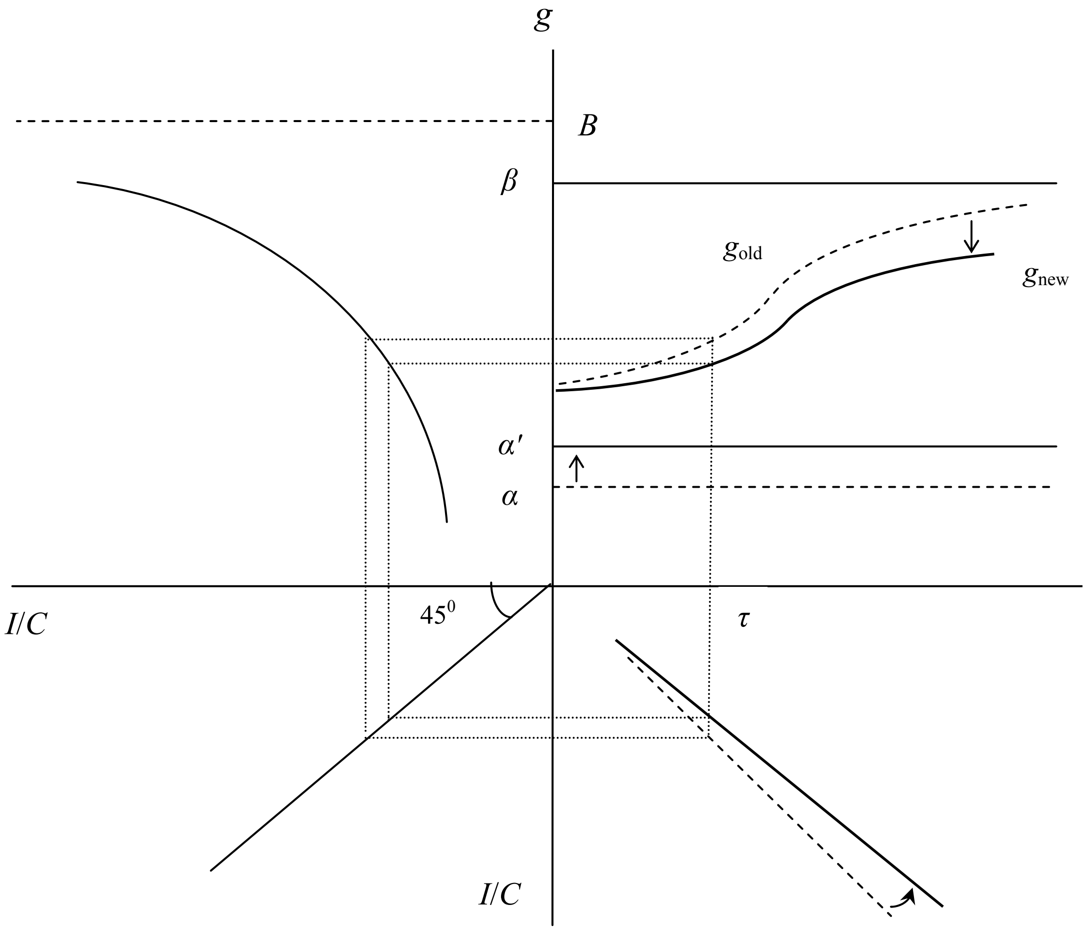

Fig 13: Behaviour of the ratio *I*/*C* for different *α*

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