Joint replenishment of multi retailer with variable replenishment cycle under VMI

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ABSTRACT
In the recent article, Darwish and Odah [1] develop a scheme that allows for identical replenishment cycles for all the retailers, in the context of a single vendor supplying a group of retailers under VMI partnership. This paper proposes an alternative replenishment scheme allowing for different replenishment cycles for each retailer. An example has been shown to illustrate the cost savings under the proposed model.

KEYWORDS
(D) Supply chain management, Multi-Item Joint Replenishment, Vendor Managed Inventory, Inventory Management

1. INTRODUCTION
The benefits of Vendor managed Inventory in terms of reduced cost and improved service are clearly stated by Waller, Johnson and Davis[5]. Since then many authors have tried to come up with single vendor multi retailer replenishment schemes. Viswanathan and Piplani [4] propose a scheme where the replenishment cycle for supplier is fixed and retailers may order at those intervals only. Zhanga et al. [6] present a model wherein the vendor has constant production cycle and retailers can have different ordering cycles. Hariga et al. [2] take unequal reorder intervals for vendor and retailers can receive more than one shipment in each vendor cycle. Recently in their paper entitled “Vendor managed inventory model for single-vendor multi-retailer supply chains”, Darwish & Odah [1] present a mathematical model for retailer replenishment and provide an optimal solution for the same. In developing the model, they consider a policy of replenishing all retailers at the same time. They assume that each retailer gets replenished every T periods and the supplier sets up every nT period, where n is an integer greater than equal to 1. In this note, we show that changing the replenishment policy may lead to cost savings. Specifically, we assume that the retailers are not replenished simultaneously, instead, a retailer $i$ is replenished every $m_i T$ period, where, $T$ is the base replenishment cycle, and
is an integer. The solution methodology ensures that at least one retailer is replenished every period and the others may be replenished every $T$, $2T$, $3T$ etc. periods. We further assume that the supplier sets up every $T$ periods and unlike Darwish & Odah [1] supplier does not carry any inventory. Thus $n$ is assumed to be 1 in our case. This replenishment policy does not force every retailer to get replenished every cycle. Our proposed policy can be seen as a generalization of the policy adopted by Darwish & Odah [1], to the extent that, for $m_i = 1$, for all $i$, all retailers would get replenished every $T$ period. Our proposed policy can be found in Silver [3], who has used it in the context of joint replenishment of items.

2. MATHEMATICAL MODEL

The relevant factors involved in the model are given below:

- $T$: Base replenishment cycle
- $m_i$: Integer variable for $i^{th}$ retailer
- $A$: Supplier setup cost ($)
- $h_s$: Inventory carrying cost of supplier ($/unit/unit time)
- $a_i$: Order cost for $i^{th}$ retailer($)
- $h_i$: Inventory carrying cost of $i^{th}$ retailer($/unit/unit time)
- $D_i$: Annual demand for the $i^{th}$ retailer.
- $D$: Annual demand for supplier. ($\sum D_i$)
- $U_i$: Upper limit set by the retailer.
- $P_i$: Penalty for $i^{th}$ retailer for exceeding the upper limit $U_i$ ($/unit$)
- $X_i$: Quantity by which the upper limit is exceeded

The Total Relevant Cost (TRC) can be expressed as follows:

$$TRC = \sum (a_i + h_i D_i + P_i X_i)$$
Minimize :-

\[
\frac{A}{T} + \sum_{i=1}^{n} \frac{a_i}{m_i T} + \frac{1}{2} \sum_{i=1}^{n} D_i m_i T h_i + \frac{1}{2} \sum_{i=1}^{n} P_i m_i T D_i X_i^2
\]

Subject to \( D_i m_i T - U_i \leq X_i \)

\( X_i \geq 0, \ m_i \in I \) (Set of all integers)

The factor that controls the cycle time of a retailer is \( a_i/D_i h_i \). The higher this ratio, the higher is the value of \( m_i \), and vice-versa. We identify the retailer with the lowest \( a_i/D_i h_i \) ratio and set the value of \( m_i \) of the retailer equal to 1. This ensures that at least one retailer is replenished every cycle. We have used the software ‘Lingo 13.0’ to solve for values of other \( m_i \)’s and \( T \). The replenishment quantities for retailers are calculated as \( Q_i = D_i m_i T \).

The example considered by Darwish and Odah [1] is such that the \( (a_i/D_i h_i) \) ratio for all the retailers are very close, suggesting that the replenishment cycle for the items should be similar and in turn all the \( m_i \)’s come out to be 1. This is shown in the table given below:

<table>
<thead>
<tr>
<th>Retailer</th>
<th>( D_i )</th>
<th>( a_i )</th>
<th>( h_i )</th>
<th>( a_i/D_i h_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>2300</td>
<td>45</td>
<td>7.5</td>
<td>.0026</td>
</tr>
<tr>
<td>R2</td>
<td>1200</td>
<td>30</td>
<td>8.5</td>
<td>.0029</td>
</tr>
<tr>
<td>R3</td>
<td>3000</td>
<td>60</td>
<td>7</td>
<td>.0028</td>
</tr>
<tr>
<td>R4</td>
<td>1800</td>
<td>35</td>
<td>8</td>
<td>.0024</td>
</tr>
<tr>
<td>R5</td>
<td>800</td>
<td>25</td>
<td>9</td>
<td>.0034</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supplier</th>
<th>( D )</th>
<th>( A )</th>
<th>( h_s )</th>
<th>( A/Dh_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sum D_i = 9100 )</td>
<td>300</td>
<td>.75</td>
<td>.044</td>
</tr>
</tbody>
</table>

As mentioned, the retailer with the lowest value of \( a_i/D_i h_i \) (say \( R_{\text{lowest}} \)) will get replenished every cycle. In a similar way the supplier’s \( A/Dh_s \) ratio controls its replenishment cycle. A lower value of \( A/Dh_s \) ratio should lead to a lower value of \( n \). Moreover, \( n \) will take a value of more than 1, only in the case where \( A/Dh_s \) ratio of supplier is relatively much higher as compared to \( a_i/D_i h_i \) ratio of \( R_{\text{lowest}} \). Such a case arises in the context of single supplier – single retailer situation, as the setup cost of the supplier is relatively higher compared to the ordering cost of the retailer.
However in the context of multiple retailers, the supplier’s $A/Dh_s$ ratio is either close to or less than $a_i/D_ih_i$ ratio of $R_{\text{lowest}}$ (since $D$ is now the cumulative demand of all retailers taken together) and hence the assumption of $n=1$ is justifiable. In the example considered by Darwish and Odah [1] $A/Dh_s$ has been taken as 0.044, which is more than $a_i/D_ih_i$ of any of the retailer suggesting lower frequency of setups and hence supporting $n$ greater than 1.

The example and the resulting cost savings are shown in the next section.

3. EXAMPLE

Consider a case involving four retailers and one supplier with the input data shown in Table 1:

Table 1: Data corresponding to the case of Multiple Retailers with different values of $a_i/D_ih_i$

<table>
<thead>
<tr>
<th>RETAILER</th>
<th>ANNUAL DEMAND</th>
<th>$a_i$</th>
<th>$h_i$</th>
<th>$a_i/D_ih_i$</th>
<th>EOQ</th>
<th>Upper limit ($U_i$)</th>
<th>Penalty $P_i$ (per extra unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>400</td>
<td>40</td>
<td>.8</td>
<td>.125</td>
<td>200</td>
<td>250</td>
<td>2</td>
</tr>
<tr>
<td>R2</td>
<td>1000</td>
<td>45</td>
<td>.8</td>
<td>.0563</td>
<td>336</td>
<td>400</td>
<td>1.5</td>
</tr>
<tr>
<td>R3</td>
<td>8000</td>
<td>50</td>
<td>1</td>
<td>.0063</td>
<td>895</td>
<td>1050</td>
<td>2</td>
</tr>
<tr>
<td>R4</td>
<td>18000</td>
<td>60</td>
<td>1</td>
<td>.0033</td>
<td>1470</td>
<td>1750</td>
<td>1</td>
</tr>
<tr>
<td>Supplier</td>
<td>$D$</td>
<td>$A$</td>
<td>$h_s$</td>
<td>$A/Dh_s$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\Sigma D_i = 27400$</td>
<td>120</td>
<td>.75</td>
<td>.0058</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The value for $m_4$ (Retailer 4 has lowest value for $a_i/D_ih_i$) is set to 1 and then the other values are obtained using LINGO 13.0, as shown in Table 2.

Table 2: Results corresponding to the proposed model.

<table>
<thead>
<tr>
<th>RETAILERS</th>
<th>$m_i$</th>
<th>REPLENISHMENT QTY</th>
<th>TRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>4</td>
<td>192</td>
<td>3944.64</td>
</tr>
</tbody>
</table>
For the same example, using the model and algorithm suggested by Darwish & Odah [1], the value of \( n \) came out to be 1; the other results are given in the table below:

**Table 3: Results corresponding to the model by Darwish & Odah [1]**

<table>
<thead>
<tr>
<th>RETAILERS</th>
<th>REPLACEMENT QUANTITIES</th>
<th>TRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>55.6</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>139</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>1112</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>2503</td>
<td>4267.38</td>
</tr>
</tbody>
</table>

We can clearly see the reduction in the total relevant cost.

**4. CONCLUSION**

The factor \( \frac{a_i}{D_i h_i} \) plays an important role in realizing the benefit of different replenishment cycle for retailers. We show that allowing different replenishment cycles for the retailers results in the reduction of total relevant cost as compared to the scenario where all the retailers are restricted to have the same replenishment cycle. In the context where the retailers are heterogeneous with respect to the factors such as demand, ordering cost, and inventory carrying cost the proposed model will perform better as compared to the model suggested by Darwish & Odah [1].

**REFERENCES**


