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Supply chain analysis under green sensitive consumer
Demand and cost sharing contract

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Supply chain analysis under green sensitive consumer demand and cost sharing contract

Abstract

In this paper, we explore supply chain coordination issues arising out of green supply chain initiatives and explore the impact of cost sharing contract on the key decisions of supply chain players undertaking green initiatives. Our motivation comes from firms conducting pioneering work in the area of carbon footprint reduction in their supply chains through product redesign. Through a game theoretic approach we show how product greening levels, prices and profits are influenced by cost sharing contract within the supply chains. We study two models of cost sharing - one in which the retailer offers a cost sharing contract and the other in which the retailer and manufacturer bargain on the cost sharing contract. We also study the impact of greening costs and consumer sensitivity towards green products. Our key contribution lies in modeling cost sharing contract and analyzing its impact on a green supply chain. Our study contributes to the burgeoning field of green supply chains and collaboration between channel partners.
1 Introduction

While the increasing complexity of supply chains globally, have led most organisations fretting and working towards solving supply chain issues; the growing concerns on the environmental impact of supply chains have added another dimension to this complexity. This has resulted in unique challenges in areas of inventory management, product lifecycle decisions, carbon footprint measurement, reverse logistics systems design, conflict between channel partners and coordination issues among other areas (Klassen and McLaughlin, 1996; O'Brien, 1999; Sroufe, 2003; Corbett and Klassen, 2006; Kumar and Putnam, 2008; Ghosh and Shah, 2012; Swami and Shah, 2013). In this paper, we study supply chain coordination issues arising out of green supply chain initiatives and explore the impact of cost sharing contract on key decisions of supply chain players undertaking green initiatives.

Our research is based on a study of global supply chains conducting pioneering work in the area of measurement of green house gas emissions, reduction of carbon footprint and product redesign. In this study, we explore the case of cost sharing contract in a green supply chain and analyse its impact on key decisions of the supply chain players. Our study aims to understand as to why partners in a green supply chain enter into cost sharing contracts and do cost sharing contracts benefit both the retailer and the manufacturer? What impact do cost sharing contracts have on product greening levels and hence the overall strategy of carbon footprint reduction in supply chains? Further, can an optimal cost sharing contract be designed when players in a green supply chain bargain on the contract? Through the current research work, we aim to extricate cost sharing contract understanding as entered into by global supply chain players and add to the growing literature and managerial interest in understanding supply chain contracts in the green channel context.

While the study has mostly followed practices of large market players; the lessons learnt can be extended and implemented across supply chains extensively. Organisations across the globe, including those in emerging economies have stepped up efforts to incorporate
environmentally friendly practices in their products and supply chains. Our problem is particularly motivated by the initiatives of giant retailer; Walmart which over the last several years has strived hard to sell products that sustain Walmart’s resources and the environment (Plambeck, 2007). Between 2005 (when CEO Lee Scott made the above announcement) and now, Walmart has taken giant strides towards greening its supply chain. Some of its initiatives are encouraging sea food procurement only from suppliers who are certified by the Marine Stewardship Council. Additionally, Walmart has invested heavily into procurement of organic cotton from its suppliers in order to introduce organic cotton clothing through its stores. To achieve this, Walmart provided suppliers of organic cotton with valuable knowledge and process assistance through its strong relationships with the environmental non-profit organisations in its networks. Walmart has also implemented cooperative supply chain management practices to motivate suppliers to reduce the environmental impacts of their products and processes (Plambeck, 2007). Walmart today, mandates its suppliers to participate in carbon disclosure projects and largely sources from suppliers who have undergone requisite environmental certifications. There are several benefits out of these initiatives. In addition to improvement in public image and brand building, it is reported that Walmart receives a price premium for its range of green products from it customers, organic cotton clothing being one such example (Plambeck, 2011). And more importantly, the supply chain strategy to sell organic cotton clothing through its stores has given Walmart unprecedented view into its sourcing activities and supply chain and also brought it way closer to its suppliers (Plambeck, 2011). It can be concluded that in order to take advantage of the green conscious consumer market, Walmart undertook several green initiatives whose success has largely depended on close collaboration with the suppliers. Through contractual mechanisms, Walmart has actively worked with its suppliers to bring green products and process changes. The impact of green sensitive consumers on supply chain players and collaboration between green supply chain partners provides an interesting area of study to researchers and practitioners alike. Both these aspects are captured in our model.

In a different example, a large manufacturer like P&G has also entered into the manufac-
turing and marketing of several environmentally conscious products. To cite examples, products like Ariel Excel Gel, Pampers, Gillette Fusion have been innovatively designed such that they not only consume less water and energy during usage but also during their manufacturing. In addition, P&G has initiated measurement of the environmental footprint of its vendors through its annual supplier sustainability performance rating. The firm also collaborates with its suppliers to bring green products into the market and help suppliers improve on their environmental performance (www.pg.com/sustainability).

One of the largest technological manufacturers of the world, Dell has set an ambitious target of 40% absolute emission reduction by 2015 based on 2007 levels. Dell realised that to achieve this; it has to increasingly collaborate with its suppliers who are an integral part of Dell’s supply chain. By 2007, Dell had asked several of its suppliers to incorporate management of greenhouse gas emissions and included it as an important factor for supplier reviews. By 2009, eighty percent of Dell’s suppliers were meeting its requirements and had been mandated by Dell to participate in Carbon Disclosure project (a first of its kind initiative to measure and disclose carbon emissions of leading global organisations). Dell’s collaboration with its suppliers to reduce carbon footprint of its supply chain and bring in more green products into the market is an important consideration in our model (Carbon Disclosure Project Supply Chain Report, 2011).

In another example, world’s largest beverage company Coca-Cola has made significant efforts in measuring and reducing its carbon footprints. It has teamed up with Carbon Trust (a not-for-dividend company headquartered in U.K. which helps organisations measure and reduce their carbon footprints) to help it measure and understand the carbon footprint from various activities in its supply chain such as sourcing and packaging; processing and transportation; manufacturing distributing and chilling the product (www.carbontrust.com). In emerging economies like India, Coca-Cola has often faced accusations about ground water depletion and contamination due to its operations in the state of Kerala (a state located in the southern part of India). It has faced similar accusations elsewhere, leading to several litigations and focused sustainabil-
ity efforts from the leading soft drink maker. Today, firms like Coca-Cola and Bisleri are tying up with third party recyclers for processing and recycling PET bottle wastes. The firms are innovatively leveraging the network of rag pickers in metro cities in countries like India for collection of waste bottles while incentivising them through monetary pay outs, leading to a first of its kind incentive mechanism for recycling bottle wastes (www.articles.economictimes.indiatimes.com).

While such efforts of large players in supply chain greening and reduction of carbon footprints are laudable, widespread efforts in restructuring supply chains for greening and re-designing of products have been less in number. The primary reason cited is the cost of going green. It has been argued that while large investments are mandated in order to change products and processes to cater to environmental friendly supply chains, the economic benefits generated out of such investments are still unclear (Walley and Whitehead, 1994). Further, for many firms, while quick and easy improvements in the processes have been achieved to comply with the environmental requirements (capturing the low-hanging fruits first), subsequent changes have been costlier and much difficult to achieve. Under such a circumstance, several firms are unsure of the benefits of greening. Further, from the previous examples cited, while large players often ask their suppliers to undertake greening initiatives, the costs of greening fall upon the suppliers making it challenging for the suppliers to undertake those initiatives. To overcome this, players like Walmart have undertaken different supply chain strategies. For example, in case of procurement of organic cotton, Walmart committed to Tier-1 suppliers of procuring organic cotton over a five year period reflecting a long-term sourcing commitment (Plambeck, 2011). In another instance, during the organic cotton growth period, farmers faced increased costs as it takes three years for the farmer to turn the soil from conventional farming to organic farming (Plambeck, 2007). To support framers, Walmart purchased millions of pounds of transitional cotton at the same price as that of organic cotton. In another example, in order to acquire personal computers that were compliant with the EU Restrictions on Hazardous Substances (RoHS) Directive, Walmart made a commitment to Toshiba to buy 12 week’s worth of inventory as opposed to its more typical four-week commitment (Plam-
beck, 2007). Bhaskaran and Krishnan (2009) discuss another case of cost sharing between a large pharmaceutical company and its lab partner, where the large pharmaceutical company shares the investment cost with its lab partner for developing and commercializing innovative class of diabetes drugs. Thomke et al. (1999) discuss Dell’s and Sony’s collaboration as a part of Dell’s product development effort with lithium ion battery technology. Coca-Cola’s incentive structure to make rag pickers participate in the reverse logistics channel is another example of collaboration between vendors and focal firms.

2 Problem Description

From the above discussion, several interesting observations can be made.

1. **Green conscious consumer demand and carbon footprint reduction effort by global firms**: It is observed that several global firms are actively pursuing environmental friendly activities within their supply chains. Both retailers and manufacturers participate in carbon reduction measures and introduce environmental friendly products through their supply chains. Studies have outlined numerous factors which drive firms towards greening. Some of them are environmental regulations, brand value, price premium potential from greening etc (Saunders and McGovern, 1993; Hanna and Newman, 1995; Gilley et.al, 2000; Chen, 2001; Swami and Shah, 2013). Out of these, an important factor which drives greening initiatives of firms is considered in our study; where firms voluntarily undertake greening initiatives to capture the green conscious consumer demand and bring more environmentally friendly products into the market. It is evident that retailers like Walmart or PC manufacturers like Dell foresee an opportunity in their respective industries with changing consumer preferences for environmentally friendly products. Previous green supply chain literature have outlined consumer demand as an important factor for greening (Henriques and Sadorsky,1999; Vachon and Klassen, 2008; Ghosh and Shah, 2012). The impact of greening initiatives on consumer demand has been captured in our work.
2. **Supply Chain Collaboration**: It is also observed, that green supply chain initiatives are achieved through collaboration between supply chain players. Large firms undertaking greening initiatives and running programs for carbon footprint measurement and reduction; realize the need to look outside the boundary of their organisations for success of these efforts and extensively collaborate with their suppliers. Collaborative efforts such as joint decision making on green product design, sharing the burden of greening costs, long term supplier commitments have been discussed above among several others. Out of these, the necessity to understand the impact of greening costs motivates our modeling approach in this paper and hence the focus on cost sharing contracts in a green supply chain. More importantly, while cost sharing contracts may benefit the manufacturer by lowering his burden, do cost sharing contracts also benefit the retailer? Under greening costs why do retailers enter into a cost sharing contract. The study aims to answer these questions.

Thus, in this paper, we aim to answer the following questions:

1. What is the impact of a cost sharing contract on key decisions of green supply chain players?
2. How does bargaining on cost sharing between players in a supply chain impact their decisions?
3. What is the impact of greening costs and green sensitive consumer demand on key decisions and profitability of supply chain players?

We apply a game theoretic approach in a single retailer-manufacturer set up, to answer the above questions. In this set up, the manufacturer incurs the greening cost while both the retailer and manufacturer benefit out of the green sensitive market demand. The structure closely reflects the case of Walmart, Dell and other supply chains where supply chain players stand to benefit out of green sensitive consumer demand but the suppliers in the channel incur the greening cost. Further, ‘greening’ is broadly referred to a measurable attribute of the the product which firms leverage to reduce their carbon footprints. For example, from our previous discussion, the amount of hazardous substance in a personal
computer which Toshiba worked on reducing is an example of greening. Similarly, organic cotton clothing which reflects the production of cotton without the use of any synthetic agricultural chemicals or fertilizers is an example of greening. In order to model the cost sharing mechanism we first study a green channel where each player takes decisions for individual profit maximization. In this set up the manufacturer incurs the cost of greening, determines the product greening level and sets the wholesale price of the green product. The retail price is subsequently determined by the retailer. We name this the Decentralized Channel case. To illustrate the sequence of decision making, the supply chain structure is explained below (Refer Figure 1):

![Supply Chain Structure Considered](image)

Figure 1: Supply Chain Structure Considered

We also compute the key decisions for an Integrated channel for better understanding. By definition, a single agent takes decisions for an integrated supply chain (Cachon, 2003). We develop these initial results to build a clear understanding of the supply chain structure in this paper and subsequently delve into the modeling of cost sharing contract. The game theoretic set up in the paper helps model and analyze both the pricing and greening strategies of each player in the channel. We subsequently, model a cost sharing contract between the channel players and discuss the impact of cost sharing contract on the strategic decisions of the green supply chain players. We study two contexts under the cost sharing contract - one where the retailer offers a cost sharing contract and second, where the players negotiate over the cost sharing contract parameter. Through the two contexts
we aim to closely model situations where supply chain players share greening costs as discussed in the case of Walmart, Coca-Cola, Dell among several others. The paper thus aims to provide insights into how cost sharing measures impact greening decisions, prices of green products and profitability of players who initiate product greening. The paper compares and contrasts the equilibrium levels of product greening and individual player’s profits across sub problems. It analyses the channel profits in each case. A comparison of wholesale and retail prices throws interesting insights into how ‘green’ products are priced.

The rest of the paper is organized as follows. In section 3, relevant background literature of this paper is reviewed. In section 4, we present the model conceptualization and formulation. We illustrate the impact of the cost sharing contract on the supply chain strategic decisions. Section 5 presents numerical analysis. In section 6, we summarize the managerial implications of this work and provide directions for future research.

3 Literature Review

In this paper, we review literature spanning across three streams. The first section addresses literature on greening issues in operations management. The review primarily observes the recent analytical work addressing various issues arising out of greening initiatives of firms. The second section discusses work addressing channel coordination in operations management and marketing streams. The third section discusses cooperative bargaining framework as applied to operations management stream.

3.1 Green Supply Chains

The study of greening initiatives of firms and supply chains throw unique challenges to researchers and practitioners. Several research streams focussing on green supply chain
management, reverse logistics, product recovery etc. have emerged in the past. An in-depth review of green supply chain literature has been conducted by Srivastava (2007). The author has classified literature based on a contextual framework into three dimensions, namely, literature which discuss importance of green supply chain management, secondly, body of work which deal with green design (environmental conscious design taking into account life cycle assessment of product and process) and thirdly, literature dealing with green operations (e.g. network design, remanufacturing and waste management). Among supply chain literature pertaining to green supply chains, several authors have focussed on remanufacturing and reverse logistics as an initiative. Studies pertaining to this research area are vast in number (see Savaskan et al., 2004; Savaskan and Wassenhove, 2006; Geyer et al., 2007; Atasu et al., 2008; Mitra and Webster, 2008). For example, Savaskan and Wassenhove (2006) study the interaction between a manufacturer’s reverse channel choice to collect return products and the strategic product pricing decisions in the forward channel when retailing is competitive. Both direct product collection from customers and indirect product collection via retailers is modeled. The authors examine how the allocation of product collection to retailers impacts their strategic behavior in the product market and also the economic trade-offs the manufacturer faces while choosing an optimal reverse channel structure. When a direct collection system is used, channel profits are driven by the impact of scale of returns on collection effort, whereas in the indirect reverse channel, supply chain profits are driven by the competitive interaction between the retailers. The authors also show that the buy-back payments transferred to the retailers for collection of goods provide a wholesale pricing flexibility that can be used to price discriminate between retailers of different profitability.

In an environmental supply chain set up, Swami and Shah (2013) study the problem of coordination in a manufacturer - retailer vertical supply chain, where the players put in efforts for greening their operations. The authors find that the ratio of optimal greening efforts put in by the manufacturer and retailer is equal to the ratio of their green sensitivity ratios and greening cost ratios. The authors also discuss a two-part tariff contract to coordinate such a supply chain. In another interesting work, Corbett and
DeCroix (2001) discuss the case of a supply chain where the buyer wishes to minimize the consumption of indirect material while the supplier’s profits depend on increasing the volume. An inescapable case of incentive conflict, the authors show that shared saving contracts help increase supply chain profits in such a case but do not necessarily lead to reduced consumption. In another study, Ghosh and Shah (2012) discuss the case of a supply chain where players initiate product greening and study the impact of various supply chain structures on key decisions of the players and supply chain. The authors find that collaboration between supply chain players is beneficial for product greening.

3.2 Channel Coordination

Since our study focusses on challenges arising out of greening costs and green sensitive consumer demand, we focus on operations management literature which have addressed similar issues based out of consumer demand and costs. Among early works Thomas (1970) discusses price and production decisions for a single product with a known deterministic demand function. Under the profit maximisation objective, the author finds optimal pricing decisions and planning horizons. The paper suggests an efficient forward algorithm combining these decisions. In another work, Corbett and Karmarkar (2001) examine the impact of fixed and variable costs on the structure and competitiveness of supply chains with a serial structure and price sensitive linear demand. The authors derive price and production quantity decisions based on the number of entrants at each tier in the supply chain. The authors model competition in supply chain through number of players in each tier. Chen, Federgruen and Zheng (2001) model a two echelon distribution system in which the sales volumes of the retailers are endogenously determined on the basis of known demand functions. The demand of the retail market is assumed to be a decreasing function of the retail price in the market. The authors characterise the centralized channel and the decentralized channel optimal strategies. The authors propose a fixed fee contract and discount schemes through which the channel can be coordinated. In this paper, we consider a demand function which is decreasing in the retail price of the
product in the market and increasing in the greening level of the product.

In the marketing literature, among early works, Jeuland and Shugan (1983) discuss channel coordination problem arising out of individual decision making of players. The authors explore problems inherent in channel coordination and explore various mechanisms through which channel coordination can be achieved. McGuire and Staelin (1983) discuss an exclusive dealer channel where two firms manufacturing differentiated but competing products sell their products through exclusive dealers. The authors discuss three cases: exclusive manufacturer- exclusive retailer; two separate manufacturers owning the stores ; one manufacturer selling through a private retailer and the other selling through a company store. Following this, there were several works in channel coordination area. Coughlan and Wernerfelt (1989) discuss various class of distribution models between manufacturers and competing retailers that have been used in literature. They use price as the strategic decision variable. Our problem is close to the work by Choi (1991) who studies a duopoly model of manufacturers who sell their products through a common independent retailer. The decision variables are whole sale prices for the manufacturers and the retail prices for the retailer. Ingene and Parry (1995) discuss channel coordination by a manufacturer that sells through competing retailers. Each retailer faces a downward sloping demand curve in prices. The authors find that a two part tariff is unable to coordinate the channel whereas a quantity discount schedule coordinates the channel. In this paper, we model and analyse cost sharing contracts to understand the impact on key decisions of players in the supply chain. Lee and Staelin (1997) in their work discuss vertical strategic interaction and implications for channel pricing strategy. Vertical strategic interaction is defined as the direction of a channel member’s reaction to the actions of its channel partner within a given demand structure. The authors analyse optimal strategies of two manufacturers selling competing products both carried by two competing retailers. Padmanabhan and Png (1997) discuss manufacturer’s return policies with uncertain demands, limited shelf life and retail competition. The retailers compete in prices. The authors discuss various cases under which the returns policy is profitable for the manufacturer. Trivedi (1998) discusses various models of distribution channels, one
of them being two competing manufacturers and two competing retailers. Using linear
demand function, the competition is modeled on prices. The author analyses the impact
of competitive intensity on both profits and prices. Iyer (1998) studies price and service
competition between a single manufacturer and two retailer channel. The author repres-
sents individual consumer behaviour in terms of value of service and disutility of travel
and from this derives each retailer’s demand function. The author also discusses various
channel coordination mechanisms. Our work adds to the literature on channel studies and
extensively deals with cost sharing contracts in green supply chain context which previous
literature has not addressed. Further, building on the context of retailer-manufacturer set
up, we study the impact of product greening and analyse inter-firm interaction in such a
context.

3.3 Cooperative Bargaining

Operations management literature in the recent past has resorted to use the tool of co-
operative bargaining as proposed by Nash (1950,1953) to study impact of negotiations
between players in a supply chain context. Nagarajan and Sosic (2008) provide a detailed
review of operations management work which apply cooperative bargaining framework.
The authors suggest that negotiations between players in supply chain can happen over
different terms of trade like wholesale prices, purchase quantity etc. Further, negotiation
frameworks can also be applied to profit allocations and contractual parameters. In order
to understand the impact of negotiations, cooperative bargaining is used. For a two per-
son bargaining game, Nash (1951) used an axiomatic approach to look for a bargaining
solution that is symmetric, feasible, Pareto optimal, preserved under linear transforma-
tions and independent of irrelevant alternatives (Roth 1979). Among relevant literature
pertaining to our work, Bhaskaran and Krishnan (2009) use the bargaining framework
to evaluate its impact on product development under investment sharing and innovation
sharing. In an earlier work Kohli and Park (1989) discuss a bargaining problem in which
a buyer and seller negotiate over the order quantity and average unit price. In this paper
we explore the phenomenon of negotiation between players in a green channel over the cost sharing contract parameter and analyse the impact of cost sharing on players and supply chain as a whole.

The current work focusses on understanding the impact of cost sharing on key decisions of players in a green supply chain. Through various models of cost sharing - one, where the retailer offers the cost sharing contract and the other, where the players negotiate over the cost sharing contract parameter, we aim to build a better understanding of green initiatives and cost sharing as observed in practice. The work aims to contribute to literature on supply chains undergoing greening.

4 The Model

In this section, we first discuss the modeling assumptions and related results for decentralized and integrated channels in order to motivate cost sharing contract framework and analysis. We assume that the demand \( (q) \) faced by the supply chain players is a linear function of retail price \( 'p' \) and product greening improvement level \( '\theta' \), where \( \theta \) is a continuous variable. The demand function is given as :

\[
q = a - bp + \alpha \theta \quad \text{where} \quad a > bp, \alpha, b > 0 \tag{1}
\]

In this equation,

\( a \) = the total market potential

\( b \) = the price sensitivity of consumer demand

\( \alpha \) = consumer sensitivity to greening level improvement

Also,

\( w \) = manufacturer’s wholesale price

\( m \) = retailer’s margins

\( c \) = variable production cost of the manufacturer
Further, $p$, $w$ and $m$ are related as $p = w + m$

Notice, that the demand function reflects a ‘green’ sensitive consumer market where the product demand is linearly decreasing in the product price and increasing in the greening level. We capture the impact of product pricing and greening level improvement on consumer demand in a tractable deterministic linear form which reflects the case of environmental friendly products developed by P&G, organic cotton clothing in Walmart’s case and environmental friendly PC’s of Dell whose product changes were primarily to cater to the green sensitive consumer demand. Linearity and deterministic assumptions with respect to price and non-price variables particularly those involving game theoretic supply chain set ups involving inter-firm interaction have been made in several operations management and marketing literature (Jeuland and Shugan, 1983; Choi, 1991; Ingene and Parry, 1995; Choi, 1996; Lee and Staelin, 1997; Tsay and Agrawal, 2000; Savaskan, Bhattacharya and Wassenhove, 2004).

We also assume that the greening improvement in the product does not affect the manufacturer’s marginal cost of production. The greening improvement refers to a product attribute modification which when implemented renders the older product obsolete. The argument is close to the case of P&G’s eco friendly versions of consumer products and Dell’s green PC’s such that the new greener versions of the products today, have completely substituted the previous versions. In order to model the cost of greening improvement, we have considered an increasing and convex cost structure which reflects our earlier discussion on how green improvements have come about through firms making the initial changes in products and processes easily, with subsequent improvements being more difficult with diminishing returns (Walley and Whitehead, 1994; Porter and van der Linde, 1995; Tsay and Agrawal, 2000; Chen, 2001; Bhaskaran and Krishnan, 2009).

We thus consider the cost of greening to given by $I\theta^2$ where $I$ is the greening investment parameter and is a function of greening improvement in the product (Banker, Khosla and Sinha, 1998). Lastly, in our base model, the cost is borne by the manufacturer.

Based on the above model assumptions, the manufacturer’s (index M), retailer’s (index
R) and the supply chain’s (index SC) profit functions thus, are:

\[ \Pi_M = (w - c)q - I\theta^2 \]  
\[ \Pi_R = (p - w)q \]  
\[ \Pi_{SC} = (p - c)q - I\theta^2 \]

We solve the above expressions for the decentralized channel, integrated channel and cost sharing contract case. The equilibrium results for all sub-problems are derived in the Appendix.

4.1 The Integrated and Decentralized Channel Case

We obtain the equilibrium values for the integrated channel which are tabulated below (Refer Appendix for proof). In the decentralized channel case, the manufacturer chooses the product greening level and wholesale price for his profit maximization using the response function of the retailer. Then, retailer decides the price of the product so as to maximize his profit, given greening level and wholesale price. Given this structure, we obtain equilibrium values of greening improvement, wholesale price, retail price and the retailer’s margins. From the equilibrium values we subsequently derive the retailer’s, manufacturer’s and the supply chain’s profits. The results for decentralized channel structure are shown in Table 1. Similar approach for the integrated and decentralized channels are established in previous works (Choi, 1991) and developed here to motivate cost sharing contract analysis.

4.2 Cost Sharing Contract

In this section, in order to understand cost sharing contracts as entered into by green supply chain players in practice, we model and analyze a cost sharing contract between the players in the green channel. Contract analysis holds importance as in our models,
the manufacturer incurs the complete cost of greening. Hence in order to incentivize the manufacturer to participate in the green channel, cost sharing contract could play an important role. More importantly, we seek to find if retailer benefits out of the cost sharing contract. Our model closely reflects the collaborative nature of product development in supply chains as discussed earlier. To understand the cost sharing contract, we assume the following game structure:

1. The retailer offers to share \( \phi \) proportion of the total cost of greening. The manufacturer accepts or rejects the offer. If, the manufacturer accepts the offer, the retailer shares \( \phi \) proportion of the total cost of greening and the manufacturer incurs \( 1 - \phi \) proportion of the greening costs; \( 0 < \phi \leq 1 \)

2. The manufacturer decides on the greening level (\( \theta \)) and wholesale price (\( w \)) of the product taking the cost sharing proportion and the retailer’s reaction function into consideration.

3. The retailer decides the retail price (\( p \)) of the product taking cost sharing proportion, greening level and wholesale price into consideration.

Under the given game structure, the profit functions of the manufacturer and the retailer

<table>
<thead>
<tr>
<th>Equil</th>
<th>Integrated Channel</th>
<th>Decentralized Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta^* )</td>
<td>( \frac{\alpha (a - bc)}{4Ib - \alpha^2} )</td>
<td>( \frac{\alpha (a - bc)}{8Ib - \alpha^2} )</td>
</tr>
<tr>
<td>( w^* )</td>
<td>( - )</td>
<td>( \frac{4I (a - bc)}{8Ib - \alpha^2} + c )</td>
</tr>
<tr>
<td>( p^* )</td>
<td>( \frac{2I (a - bc)}{4Ib - \alpha^2} + c )</td>
<td>( \frac{6I (a - bc)}{8Ib - \alpha^2} + c )</td>
</tr>
<tr>
<td>( m^* )</td>
<td>( \frac{2I (a - bc)}{4Ib - \alpha^2} )</td>
<td>( \frac{2I (a - bc)}{8Ib - \alpha^2} )</td>
</tr>
<tr>
<td>( \Pi^*_{M} )</td>
<td>( - )</td>
<td>( \frac{I (a - bc)^2}{8Ib - \alpha^2} )</td>
</tr>
<tr>
<td>( \Pi^*_{R} )</td>
<td>( - )</td>
<td>( \frac{4bI^2 (a - bc)^2}{(8Ib - \alpha^2)^2} )</td>
</tr>
<tr>
<td>( \Pi^*_{SC} )</td>
<td>( \frac{I (a - bc)^2}{4Ib - \alpha^2} )</td>
<td>( \frac{I (a - bc)^2 (12Ib - \alpha^2)}{(8Ib - \alpha^2)^2} )</td>
</tr>
</tbody>
</table>

Table 1: Equilibrium Values I
are:

\[
\Pi_M = (w - c)(a - bp + \alpha \theta) - (1 - \phi)I\theta^2
\]  
(5)

\[
\Pi_R = (p - w)(a - bp + \alpha \theta) - \phi I\theta^2
\]  
(6)

Since, we are interested in the case where both the players participate in the cost sharing contract, we solve for the case where the manufacturer accepts the contract. We later show, why the manufacturer accepts the contract. The expressions are solved to derive the equilibrium values of decision variables and the cost sharing contract parameter (Refer Appendix). The equilibrium values derived are shown below:

<table>
<thead>
<tr>
<th>Equil</th>
<th>Cost Sharing Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_{opt})</td>
<td>(\frac{\alpha^2}{16Ib})</td>
</tr>
<tr>
<td>(\theta_{opt})</td>
<td>(\frac{2\alpha(a - bc)}{16Ib - 3\alpha^2})</td>
</tr>
<tr>
<td>(w_{opt})</td>
<td>(\frac{16Ib(a + bc) - \alpha^2(a + 5bc)}{2b(16Ib - 3\alpha^2)})</td>
</tr>
<tr>
<td>(p_{opt})</td>
<td>(\frac{16Ib(3a + b) - 3\alpha^2(a + 3bc)}{4b(16Ib - 3\alpha^2)})</td>
</tr>
<tr>
<td>(m_{opt})</td>
<td>(\frac{(a - bc)(16Ib - \alpha^2)}{4b(16Ib - 3\alpha^2)})</td>
</tr>
<tr>
<td>(\Pi_{opt}^R)</td>
<td>(\frac{(a - bc)^2(16Ib + \alpha^2)}{16b(16Ib - 3\alpha^2)})</td>
</tr>
<tr>
<td>(\Pi_{opt}^M)</td>
<td>(\frac{(a - bc)^2(16Ib - \alpha^2)}{8b(16Ib - 3\alpha^2)})</td>
</tr>
<tr>
<td>(\Pi_{opt}^{SC})</td>
<td>(\frac{(a - bc)^2(48Ib - \alpha^2)}{16b(16Ib - 3\alpha^2)})</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium Values under Cost Sharing Contract

**Proposition 1.** The cost sharing parameter \(\phi_{opt}\) is decreasing in the cost of greening \((I)\) and increasing in the consumer sensitivity towards greening \((\alpha)\).

Proof: The partial derivative of \(\phi_{opt}\) w.r.t \(I\) gives

\[
\frac{\delta\phi_{opt}}{\delta I} = \frac{-16\alpha^2b}{(16Ib)^2} < 0
\]  
(7)
The partial derivative of $\phi_{opt}$ w.r.t $\alpha$ gives

$$\frac{\delta \phi_{opt}}{\delta \alpha} = \frac{\alpha}{8Ib} > 0$$

(8)

The proposition indicates that the cost sharing parameter $\phi$ which the retailer incurs is decreasing in the cost of greening(I). This means that under high costs of greening the retailer would share lower proportion of the costs. This he does to maintain his profitability. However, when the consumer sensitivity to greening ($\alpha$) is high, the retailer offers a higher proportion of cost share. This is because when the consumers are highly sensitive to greening, the retailer can share greening costs as the demand for the green product increases substantially even through a small improvement in greening level of the product which helps the retailer maintain his profits. Thus, cost sharing decision by the retailer is influenced by the greening costs and consumer sensitivity to greening.

**Proposition 2.** The equilibrium values in the cost sharing contract are in the following order in comparison to the decentralized supply chain values: $\theta_{opt} \geq \theta^{*}$; $w_{opt} \geq w^{*}$; $p_{opt} \geq p^{*}$

Proof: The above relationships are derived through algebraic comparison. The result illustrates that the cost sharing contract results in a higher greening improvement than the case of decentralized channel. Thus cost sharing contract is beneficial from a greening perspective. However, higher greening improvement also leads to higher wholesale price and retail price of the green product. Thus, from a consumer perspective, green products cost more to purchase. The manufacturer and the retailer would only participate in a cost sharing contract if the contract results in a higher surplus than the non-contractual case. We understand the impact of cost sharing contract on profitability through the next result.

**Proposition 3.** The equilibrium values of profits in the cost sharing contract are in the following order in comparison to the decentralized supply chain values: $\Pi_{M}^{opt} \geq \Pi_{M}^{*}$; $\Pi_{R}^{opt} \geq \Pi_{R}^{*}$
Proof: The above relationships are derived through algebraic comparison. The interesting result indicates that both the manufacturer and retailer incur higher profits in the cost sharing contract case than the decentralized supply chain case. Clearly, cost sharing with the retailer, helps the manufacturer. That is why the manufacturer participates in the cost sharing contract. The reason for higher profitability of the manufacturer lies in the fact that any share of greening costs, helps improve manufacturer’s profitability. Because the greening costs are lowered for the manufacturer, the manufacturer is also able to provide a higher greening improvement in the product. Importantly, though the retailer shares a portion of the cost of greening, he incurs higher profits than the decentralized case. This result provides answer to the questions that we had raised earlier on why do retailers enter into cost sharing contracts in a green supply chain. Sharing the burden of greening costs with the manufacturer, provides a higher greening improvement in the product which subsequently drives up the market demand. The increase in market demand more than compensates for the cost shared by the retailer. The result illustrates why greening involves increased collaboration between manufacturer and retailer through cost sharing contracts and other mechanisms. The result closely relates to the industry observations discussed previously. It is also a reminder to practitioners and policy makers to seek incentive mechanisms to support suppliers in a green supply chain in markets where the consumer is green sensitive.

4.3 Cost sharing through bargaining

To explore the impact of bargaining, in this section a cost sharing contract is proposed between the players who bargain on the cost sharing parameter, following which the decisions are taken on the greening levels and prices. To model the bargaining game on contractual parameters, we assume that both the players adopt the Nash bargaining process as first proposed by Nash (1950, 1953). In the cost sharing contract, the sequence of decision making is the following:

1. The manufacturer and the retailer bargain on the cost sharing parameter given by $\phi$. 

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Following the bargaining process, the manufacturer shares $\phi$ proportion of the total cost of greening and the retailer incurs $1 - \phi$ proportion of the greening costs; $0 < \phi \leq 1$

2. The manufacturer decides on the greening level ($\theta$) and wholesale price ($w$) of the product taking the cost sharing proportion and the retailer’s reaction function into consideration.

3. The retailer decides the retail price ($p$) of the product taking cost sharing proportion, greening level and wholesale price into consideration.

Under the given assumptions, the profit functions of the manufacturer and the retailer are:

$$\Pi_M = (w - c)(a - bp + \alpha\theta) - \phi I\theta^2$$

$$\Pi_R = (p - w)(a - bp + \alpha\theta) - (1 - \phi)I\theta^2$$

We model the bargaining process by substituting the above values in $\Pi_B$ to obtain the optimal cost sharing parameter ($\phi$). The objective function is

$$\max_{\phi} \Pi_B = \Pi_M \Pi_R$$

All the above expressions are solved to derive the equilibrium values of decision variables and the cost sharing contract parameter (Refer the Appendix).

**Proposition 4.** a). There exists a solution to the Nash bargaining problem for $\phi^* \in [1/2, 4/5]$

b). The equilibrium solution to the Nash bargaining problem is given by

$$\phi^* = \frac{(8Ib - \alpha^2) + \sqrt{(\alpha^2 + 8Ib)^2 - 12Ib\alpha^2}}{20Ib} \quad \text{for} \quad \frac{(5 + \sqrt{33})\alpha^2}{16b} \geq I > \frac{11\alpha^2}{48b}$$

Proof: Refer the Appendix. The proposition explains that the share of the greening costs for the manufacturer always remains higher than 0.5. However cost sharing benefits the manufacturer as we would later prove.
Substituting the equilibrium value of \( \phi^* \) in the expressions of greening, wholesale price, retail price, profit functions of the players in the channel and supply chain profit, we derive the equilibrium values as shown in Table 3.

<table>
<thead>
<tr>
<th>( \text{Equil} )</th>
<th>( \text{Cost Sharing through Bargaining} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta^b )</td>
<td>( \frac{5\alpha(a - bc)}{(16 Ib - 7\alpha^2) + 2\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}} )</td>
</tr>
<tr>
<td>( w^b )</td>
<td>( \frac{(a + bc)(8 Ib + \sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}) - \alpha^2(a + 6bc)}{b(16 Ib - 7\alpha^2) + 2\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}} )</td>
</tr>
<tr>
<td>( p^b )</td>
<td>( \frac{(3a + bc)(8 Ib + \sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}) - \alpha^2(3a + 11bc)}{2b(16 Ib - 7\alpha^2) + 2\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}} )</td>
</tr>
<tr>
<td>( \Pi^b_M )</td>
<td>( \frac{(a - bc)^2[(8 Ib - \alpha^2) + \sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}]}{4b(16 Ib - 7\alpha^2) + 2\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}} )</td>
</tr>
<tr>
<td>( \Pi^b_R )</td>
<td>( \frac{(a - bc)^2[(3a^2 + 16 Ib)(\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}) - 3\alpha^4 + 8 Ib(16 Ib - 9\alpha^2)]}{4b(16 Ib - 7\alpha^2) + 2\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}} )</td>
</tr>
<tr>
<td>( \Pi^b_{SC} )</td>
<td>( \frac{(a - bc)^2[24 Ib - 3\alpha^2)(\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}) + 3\alpha^4 + 4 Ib(48 Ib - 17\alpha^2)]}{2b(16 Ib - 7\alpha^2) + 2\sqrt{(\alpha^2 + 8 Ib)^2 - 12 Ib\alpha^2}} )</td>
</tr>
</tbody>
</table>

Table 3: Equilibrium Values under Cost Sharing through Bargaining

**Proposition 5.** The equilibrium values in the cost sharing contract are in the following order in comparison to the decentralized supply chain values: \( \theta^b \geq \theta^*; w^b \geq w^*; p^b \geq p^*; \Pi^b_M \geq \Pi^*_M \)

The above relationships are derived through algebraic comparisons. The proposition suggests that under the cost sharing contract decision through Nash bargaining process, the greening levels achieved are higher for the product in comparison to the decentralized supply chain case. This is because cost sharing with the retailer lowers the burden of the manufacturer and in the supply chain structure, the manufacturer takes a higher level of greening decision. Further, the wholesale price of the green product is higher in comparison to the decentralized case. The cost sharing contract is favourable to the manufacturer as he is able to charge a higher wholesale price by providing higher greening level of the product. Additionally, the retail price of the green product is higher in comparison to the decentralized case. This is attributed to two factors, one because the
retailer incurs a portion of the greening costs and secondly, he faces a higher wholesale price of the product. Thus, in order to maintain his margins, the retailer charges a higher price for the green product. The consumer surplus may be lower under the cost sharing contract as the consumers have to pay a higher price for the product. A comparison of the manufacturer profit shows that the manufacturer incurs higher profits in the cost sharing contract in comparison to the decentralized case. Thus, the contract benefits the manufacturer significantly through sharing of costs with the retailer. Comparison of retailer and supply chain profits pose some degree of analytical complexity and hence, we resort to numerical computation to analyse them.

5 Numerical Analysis

In this section, we present numerical analysis to explain some of the results obtained above. The following values were assumed, $a = 1000, b = 50, c = 6, \alpha = 40$. The value of $I$ was varied from $9-21$, a range derived from the condition $\alpha^2/(4b) \leq I \leq ((5+\sqrt{33})/16)(\alpha^2/b)$. This ensures that we work within the feasible region. We study the impact of greening cost on the decisions variables and compare and contrast the various values. Following this, we illustrate the impact of consumer sensitivity to greening on the decision variables.

**Impact of Greening Investment** – From the figures it can be seen that greening investment has a decreasing impact on the decision variables. Importantly, greening levels decrease with greening costs which substantiates our understanding on why firms struggle to undertake greening initiatives under the increasing costs of greening (Refer Figure 2). Further, the product greening levels are highest for the integrated channel and lowest for the decentralized channel. We have already established analytically that cost sharing contract leads to higher greening levels than decentralized channel values. Interestingly, the greening levels are higher when the players bargain on the cost sharing contract than the case where the retailer offers a cost sharing contract. The results indicate why
negotiations are beneficial during cost sharing from a greening perspective. The results also show that the retail price of the green product is the highest in the integrated channel followed by the case of cost sharing contract under bargaining (Refer Figure 3). It indicates that under cost sharing contract through bargaining, the supply chain players are able to provide a higher level of product greening and as a result charge higher price for the product. Note however, that the retailer profits in the case of bargaining are lower than the case when retailer offers the cost sharing contract (Refer Figure 4). This is a result of the retailer working on his own profit maximization when he offers the cost sharing parameter vis-a-vis when both players negotiate. As a result, the retailer shares a higher proportion of the greening cost under bargaining than the case of retailer deciding the cost sharing contract parameter (Refer Figure 5). Under higher greening costs, the retailer would ideally like to participate in a cost sharing contract where he can individually decide on the cost share.

The plot of optimal supply chain profits reveals interesting results. The cost sharing contract offered by the retailer or obtained through bargaining results in a higher supply chain surplus than the decentralized channel. Additionally, the cost sharing contract through bargaining results in a higher supply chain surplus than the cost sharing contract offered by the retailer (Refer Figure 6). The result indicates that from a supply chain perspective, contract negotiations help. Bargaining on cost sharing contract results in higher greening levels, which impacts the market demand for green product thus benefiting the supply chain.

**Impact of** $\alpha$ — In the following the impact of consumer sensitivity to greening improvement ($\alpha$) is analysed. The following values were assumed, $a = 1000, b = 50, c = 6, I = 40$. The value of $\alpha$ was varied from $9 - 28$, a range derived from the condition $4/(5 + \sqrt{33})(\sqrt{1b}) \leq \alpha \leq 4/3\sqrt{1b}$. It is seen that $\alpha$ positively impacts greening improvement (Refer Figure 7). Clearly, supply chain players would like to participate in markets where consumers are green conscious. Also, as in the earlier case, the product greening levels are highest in the integrated channel case, followed by cost sharing through
bargaining between players followed by the cost sharing contract offered by the retailer. The product greening levels are lowest for the decentralized channel case. The impact of consumer sensitivity to greening on channel profits and individual profitability of players is positive and can be shown numerically as our earlier case. Clearly, green consumer markets provide better opportunities for supply chain players to undertake crucial green initiatives.

Our analysis thus reveals that from a product greening perspective, cost sharing sharing contract offered by the retailer or obtained through bargaining, leads to a higher greening level in the supply chain. Thus, cost sharing contracts benefit product greening. Also, under the case of cost sharing contract offered by the retailer, we find that both the manufacturer and the retailer benefit out of the contract. Additionally, the cost sharing contract generates higher surplus for the supply chain in comparison to the decentralized case. In summary, the results explain why firms enter into cost sharing contracts under product greening and collaborate with supply chain partners to benefit out of green initiatives.

6 Conclusion

Pioneering efforts of firms to undertake carbon footprint measurement and bring about changes in products and processes are being increasingly studied globally. While these efforts have yielded results, there is a need for clear understanding of impact of green initiatives on firms and supply chains. The current study focuses on an important aspect, namely, cost sharing contracts in the context of green supply chains. The study aims to explain why players in a green supply chain enter into cost sharing contractual mechanisms and provides an in-depth view of the impact of cost sharing on key decisions of supply chain players. It also studies two modes of cost sharing contract development – one, where the retailer offers a cost sharing contract and the other, where players bargain on the cost sharing contract parameter. The study reveals that cost sharing is beneficial to the firms
and supply chain. Cost sharing contracts result in higher greening levels, higher profits for individual firms and an increase in supply chain profits. Further bargaining on cost sharing parameter leads to higher greening levels than the case where the retailer offers cost sharing contract. Bargaining also leads to higher supply chain profits. However the retailer may be unwilling to negotiate cost sharing contract parameter as it results in lower profits than the case where the retailer offers the cost sharing contract. Analyzing the impact of greening costs and sensitivity of consumer demand to greening, on each player’s key decisions serves to better explain inter-firm interactions in green consumer markets. The present research work, seeks to add to the growing literature on green supply chains and aims to provide a detailed understanding of cost sharing contracts in this context. The current study serves as an initial step for future work in incentive design in green supply chains. This work can be extended in several directions — a study of impact of green initiatives on cost reduction, the framework of current analysis could be extended to multiple suppliers and retailers, uncertainty in product demand under green initiatives could also provide further insights.
Figure 2: Optimal Greening Level vs I

Figure 3: Optimal Retail Price vs I
Figure 4: Optimal Retailer’s Profit vs $I$

Figure 5: Cost share proportion of retailer vs $I$
Figure 6: Optimal Supply Chain Profit vs I

Figure 7: Optimal Greening Level vs \( \alpha \)
Appendix

Integrated Channel Case

In an integrated channel, we solve for the supply chain’s profit function,

$$\max_{m, \theta} \Pi_{SC} = m(a - b(c + m) + \alpha \theta) - I \theta^2$$  \hspace{1cm} (10)

The first order conditions

$$\frac{\partial}{\partial m} \Pi_{SC} = a - 2bm - bc + \alpha \theta$$  \hspace{1cm} (11)

and

$$\frac{\partial}{\partial \theta} \Pi_{SC} = m\alpha - 2I \theta$$  \hspace{1cm} (12)

Joint concavity of the profit function w.r.t the decision variables can be established as we show in subsequent results. Thus equating the first order conditions to 0 and solving the equations simultaneously, we get,

$$m^* = \frac{2I(a - bc)}{4Ib - \alpha^2}$$  \hspace{1cm} (13)

$$\theta^* = \frac{\alpha(a - bc)}{4Ib - \alpha^2}$$  \hspace{1cm} (14)

Substituting the values in the expressions for optimal price and supply chain profit function gives us the other results.

Decentralized Channel Case

In this set up, we solve for the retailer’s profit function first.

$$\max_m \Pi_{R}(w, \theta) = m(a - b(w + m) + \alpha \theta)$$  \hspace{1cm} (15)

The first order condition

$$\frac{\partial}{\partial m} \Pi_{R} = a - 2bm - bw + \alpha \theta$$  \hspace{1cm} (16)

The second order condition

$$\frac{\partial^2}{\partial m^2} \Pi_{R} = -2b < 0$$  \hspace{1cm} (17)

Thus the retailer’s profit function is strictly concave in $m$. Equating the first order condition to 0 we get,

$$m(w, \theta) = \frac{a - bw + \alpha \theta}{2b}$$  \hspace{1cm} (18)

Solving for the manufacturer’s profit function

$$\max_{(w, \theta)} \Pi_{M} = (w - c)(a - b(w + m) + \alpha \theta) - I \theta^2$$  \hspace{1cm} (19)

We substitute the value of $m$ into the above equation and derive

$$\max_{(w, \theta)} \Pi_{M} = \frac{(w - c)(a - bw + \alpha \theta)}{2} - I \theta^2$$  \hspace{1cm} (20)
The first order condition
\[ \frac{\partial}{\partial w} \Pi_M = -bw + \frac{a}{2} + \frac{bc}{2} + \frac{\alpha \theta}{2} \]  \hspace{1cm} (21)
\[ \frac{\partial}{\partial \theta} \Pi_M = \frac{(w - c)\alpha}{2} - 2I\theta \]  \hspace{1cm} (22)

The second order condition
\[ \frac{\partial^2}{\partial w^2} \Pi_M = -b < 0 \]  \hspace{1cm} (23)
\[ \frac{\partial^2}{\partial \theta^2} \Pi_M = -2I < 0 \]  \hspace{1cm} (24)
\[ \frac{\partial^2}{\partial w \partial \theta} \Pi_M = \frac{\alpha}{2} \]  \hspace{1cm} (25)

So the determinant is
\[ 2Ib - \frac{\alpha^2}{4} \]  \hspace{1cm} (26)

For
\[ 2Ib - \frac{\alpha^2}{4} > 0 \]  \hspace{1cm} (27)

the Hessian H is negative definite. Thus manufacturer’s profit function is jointly concave in w and \( \theta \). Equating the first order conditions to 0 we get,
\[ w(\theta) = \frac{a + bc + \alpha \theta}{2b} \]  \hspace{1cm} (28)
\[ \theta(w) = \frac{\alpha(w - c)}{4I} \]  \hspace{1cm} (29)

Substituting the value of \( w \) into the value of \( \theta \) we get,
\[ \theta^* = \frac{\alpha(a - bc)}{8Ib - \alpha^2} \]  \hspace{1cm} (30)

Substituting the above value of \( \theta \) into the values of \( m \) and \( w \) we get,
\[ m^* = \frac{2I(a - bc)}{8Ib - \alpha^2} \]  \hspace{1cm} (31)
\[ w^* = \frac{4I(a - bc)}{8Ib - \alpha^2} + c \]  \hspace{1cm} (32)

The optimal retail price is obtained as
\[ p^* = m^* + w^* = \frac{6I(a - bc)}{8Ib - \alpha^2} + c \]  \hspace{1cm} (33)

**Cost Sharing Contract**

When retailer shares \( \phi \) proportion of the cost and the manufacturer shares \( 1 - \phi \) proportion of the greening costs, then the profit functions of the retailer and manufacturer are:
\[ \Pi_R = m(a - b(w + m) + \alpha \theta) - \phi I \theta^2 \]  \hspace{1cm} (34)
\[ \Pi_M = (w - c)((a - b)(w + m) + a\theta) - (1 - \phi)I\theta^2 \]  

Based on the game structure, we solve for the retailer’s profit function first.

\[ \max_m \Pi_R(w, \theta) = m(a - b(w + m) + a\theta) - \phi I\theta^2 \]  

The first order condition

\[ \frac{\partial}{\partial m} \Pi_R = a - 2bm - bw + a\theta \]  

The second order condition

\[ \frac{\partial^2}{\partial m^2} \Pi_R = -2b < 0 \]  

Thus the retailer’s profit function is strictly concave in \( m \). Equating the first order condition to 0 we get,

\[ m(w, \theta) = \frac{a - bw + a\theta}{2b} \]  

Solving for the manufacturer’s profit function

\[ \max_{(w, \theta)} \Pi_M = (w - c)((a - b)(w + m) + a\theta) - (1 - \phi)I\theta^2 \]  

We substitute the value of \( m \) into the above equation and derive

\[ \max_{(w, \theta)} \Pi_M = \frac{(w - c)(a - bw + a\theta)}{2} - (1 - \phi)I\theta^2 \]  

The first order condition

\[ \frac{\partial}{\partial w} \Pi_M = -bw + \frac{a}{2} + \frac{bc}{2} + \frac{a\theta}{2} \]  

\[ \frac{\partial}{\partial \theta} \Pi_M = \frac{(w - c)a}{2} - (1 - \phi)2I\theta \]  

The second order condition

\[ \frac{\partial^2}{\partial w^2} \Pi_M = -b < 0 \]  

\[ \frac{\partial^2}{\partial \theta^2} \Pi_M = -(1 - \phi)2I < 0 \]  

\[ \frac{\partial^2}{\partial w \partial \theta} \Pi_M = \frac{\alpha}{2} \]  

So the determinant is

\[ 2Ib(1 - \phi) - \frac{\alpha^2}{4} \]  

For

\[ 2Ib(1 - \phi) - \frac{\alpha^2}{4} > 0 \]  

the Hessian \( H \) is negative definite. Thus manufacturer’s profit function is jointly concave in \( w \) and \( \theta \). Equating the first order conditions to 0 we get,

\[ w(\theta) = \frac{a + bc + a\theta}{2b} \]
\[ \theta(w) = \frac{\alpha(w - c)}{4I(1 - \phi)} \] (50)

Substituting the value of \( w \) into the value of \( \theta \) we get,

\[ \theta^* = \frac{\alpha(a - bc)}{8bI(1 - \phi) - \alpha^2} \] (51)

Substituting the above value of \( \theta \) into the values of \( m \) and \( w \) we get,

\[ m^* = \frac{2I(a - bc)(1 - \phi)}{8Ib(1 - \phi) - \alpha^2} \] (52)
\[ w^* = \frac{4I(a + bc)(1 - \phi) - \alpha c}{8Ib(1 - \phi) - \alpha^2} \] (53)

The optimal retail price is obtained as

\[ p^* = m^* + w^* = \frac{2I(1 - \phi)(3a + bc) - \alpha^2 c}{8Ib(1 - \phi) - \alpha^2} \] (54)

Substituting the above values in the retailer’s profit function, we get,

\[ \Pi_R = \left[ \frac{4I^2b(1 - \phi)^2(a - bc)^2}{(8Ib(1 - \phi) - \alpha^2)^2} - \phi \right] \left( \frac{I\alpha^2(a - bc)^2}{(8Ib(1 - \phi) - \alpha^2)^2} \right) \] (55)

Solving the retailer’s profit function for optimal cost sharing parameter \( \phi \)

\[ \max_{\phi} \Pi_R(\phi) \] (56)

The first order condition

\[ \frac{\partial}{\partial \phi} \Pi_R = \frac{I\alpha^2(a - bc)^2(\alpha^2 - 16Ib\phi)}{(8Ib(1 - \phi) - \alpha^2)^3} \] (57)

The second order condition

\[ \frac{\partial^2}{\partial \phi^2} \Pi_R = \frac{8I^2\alpha^2b(a - bc)^2(5\alpha^2 - 16Ib(1 + 2\phi))}{(8Ib(1 - \phi) - \alpha^2)^4} \] (58)

The retailer’s profit function is strictly concave in \( \phi \) for \( 16Ib(1 + 2\phi) - 5\alpha^2 > 0 \) Thus, using the first order conditions to obtain the optimal value of \( \phi \), we get,

\[ \phi_{opt} = \frac{\alpha^2}{16Ib} \] (59)

Substituting the value of \( \phi_{opt} \) in the above expressions we get,

\[ \theta_{opt} = \frac{2\alpha(a - bc)}{16Ib - 3\alpha^2} \] (60)
\[ w_{opt} = \frac{16Ib(a + bc) - \alpha^2(a + 5bc)}{2b(16Ib - 3\alpha^2)} \] (61)
\[ m_{opt} = \frac{(a - bc)(16Ib - \alpha^2)}{4b(16Ib - 3\alpha^2)} \] (62)
\[ p_{opt} = \frac{16Ib(3a + bc) - 3\alpha^2(a + 3bc)}{4b(16Ib - 3\alpha^2)} \] (63)
\[ \Pi_R^{opt} = \frac{(a - bc)^2(16Ib + \alpha^2)}{16b(16Ib - 3\alpha^2)} \] (64)
\[ \Pi_{opt}^M = \frac{(a - bc)^2(16Ib - \alpha^2)}{8b(16Ib - 3\alpha^2)} \] \tag{65} \\
\[ \Pi_{opt}^{SC} = \frac{(a - bc)^2(48Ib - \alpha^2)}{16b(16Ib - 3\alpha^2)} \] \tag{66}

**Cost Sharing through Bargaining**

We know

\[ \Pi_M = (w - c)(a - bp + \alpha \theta) - \phi I\theta^2 \] \tag{67} \\
\[ \Pi_R = (p - w)(a - bp + \alpha \theta) - (1 - \phi)I\theta^2 \] \tag{68}

Solving the above using backward induction,

\[ \max_p \Pi_R = (p - w)(a - bp + \alpha \theta) - (1 - \phi)I\theta^2 \] \tag{69}

Solving for the first order condition we get,

\[ \frac{\delta \Pi_R}{\delta p} = (p - w)(-b) + (a - bp + \alpha \theta) \] \tag{70}

Solving for the second order condition we get,

\[ \frac{\delta^2 \Pi_R}{\delta p^2} = -2b < 0 \] \tag{72}

Thus, the profit function of the retailer is concave in retail price (p). Thus equating the first order condition to zero and solving for p gives

\[ p = \frac{a + bw + \alpha \theta}{2b} \] \tag{73}

Now solving for the profit function of the manufacturer we get,

\[ \max_{w, \theta} \Pi_M = (w - c)(a - bp + \alpha \theta) - \phi I\theta^2 \] \tag{74}

Solving for the first order condition and substituting the value of p we get,

\[ \frac{\delta \Pi_M}{\delta w} = \frac{a + bc + \alpha \theta - 2bw}{2} \] \tag{75} \\
\[ \frac{\delta \Pi_M}{\delta \theta} = \frac{(w - c)\alpha - 4I\theta\phi}{2} \] \tag{76}

Solving for the second order condition gives,

\[ \frac{\delta^2 \Pi_M}{\delta w^2} = -b < 0 \] \tag{77} \\
\[ \frac{\delta^2 \Pi_M}{\delta \theta^2} = -2I\phi < 0 \] \tag{78} \\
\[ \frac{\delta^2 \Pi_M}{\delta w \delta \theta} = \frac{\alpha}{2} \] \tag{79}

The Hessian \( H = 8Ib\phi - \alpha^2 \) For \( H > 0 \) we get \( I > \frac{\alpha^2}{8b\phi} \). Under the given assumption, the manufacturer’s profit function is jointly concave in \( w \) and \( \theta \). Thus equating the first order conditions to zero and solving simultaneously for \( w \) and \( \theta \) we get,
\[ w(\phi) = \frac{4Ia\phi + c(4ib\phi - \alpha^2)}{8ib\phi - \alpha^2} \] (80)

\[ \theta(\phi) = \frac{\alpha(a - bc)}{8ib\phi - \alpha^2} \] (81)

Substituting the values of \( w(\phi) \), \( \theta(\phi) \) into retail price \( p \) and the profit functions of the players we get,

\[ p(\phi) = \frac{6Ia\phi + c(2ib\phi - \alpha^2)}{8ib\phi - \alpha^2} \] (82)

\[ \Pi_M(\phi) = \frac{I\phi(a - bc)^2}{8ib\phi - \alpha^2} \] (83)

\[ \Pi_R(\phi) = \frac{I(a - bc)^2(4ib\phi^2 - \alpha^2 + \alpha^2\phi)}{(8ib\phi - \alpha^2)^2} \] (84)

We model the bargaining process by substituting the above values in \( \Pi_B \) to obtain the optimal cost sharing parameter \( (\phi) \). The objective function is

\[ \max_{\phi} \Pi_B = \Pi_M \Pi_R \] (85)

The first order condition w.r.t \( \phi \) gives

\[ \frac{\delta\Pi_B}{\delta\phi} = -\frac{I^2\alpha^2(a - bc)^4(20ib\phi^2 - \alpha^2 + 2\alpha^2\phi - 16ib\phi)}{(8ib\phi - \alpha^2)^4} \] (86)

The second order condition gives

\[ \frac{\delta^2\Pi_B}{\delta\phi^2} = \frac{2I^2\alpha^2(a - bc)^4(160I^2b^2\phi^2 - 24Iba^2 + 44Iba^2\phi + \alpha^4 - 192I^2b^2\phi)}{(8ib\phi - \alpha^2)^5} \] (87)

The second order condition is negative for \( \phi < T \)

where \( T = \frac{(48ib - 11\alpha^2) + \sqrt{(9\alpha^2)^2 + (48ib)^2} - 96ib\alpha^2}{80ib} \)

Now \( 0 < \phi \leq 1 \), so subjecting the bound \( T \) to the limits \( 0 < T \leq 1 \) gives the condition

\[ \frac{(5 + \sqrt{33})\alpha^2}{16b} \geq I > \frac{\alpha^2}{24b} \] (88)

Thus, \( \Pi_B \) is concave in \( \phi \) for the above condition. Now solving the first order condition for \( \phi \) gives

\[ \phi^* = \begin{cases} 
\frac{(8ib - \alpha^2) + \sqrt{(\alpha^2 + 8ib)^2 - 12Iba^2}}{20ib} \\
\frac{(8ib - \alpha^2) - \sqrt{(\alpha^2 + 8ib)^2 - 12Iba^2}}{20ib}
\end{cases} \] (89)

Verifying if \( (8ib - \alpha^2) > \sqrt{(\alpha^2 + 8ib)^2 - 12Iba^2} \) Squaring both sides, we derive \( (8ib - \alpha^2)^2 > (\alpha^2 + 8ib)^2 - 12Iba^2 \)

On simplification, we derive \(-16Iba^2 > 4Iba^2 \Rightarrow 20Iba^2 < 0 \) which is not true because \( I, b, \alpha > 0 \).

Thus, the equilibrium \( \phi^* = \frac{(8ib - \alpha^2) + \sqrt{(\alpha^2 + 8ib)^2 - 12Iba^2}}{20ib} \)

**Proof (Proposition 2):**

\[ \theta^{opt} \geq \theta^* \Rightarrow (16ib - 2\alpha^2) \geq (16ib - 3\alpha^2) \]
⇒ \( \alpha^2 \geq 0 \)

\[ w^{opt} \geq w^* \Rightarrow (8Ib - \alpha^2)(16Ib(a + bc) - \alpha^2(a + 5bc)) \geq 2b(16Ib - 3\alpha^2)(4I(a - bc) + c(8Ib - \alpha^2)) \]

⇒ \( \alpha^4(a - bc) \geq 0 \)

\[ p^{opt} \geq p^* \Rightarrow (8Ib - \alpha^2)(16Ib(3a + bc) - 3\alpha^2(a + 3bc)) \geq (64Ib^2 - 12\alpha^2b)(6Ia + 2Ibc - \alpha^2c) \]

⇒ \( 3\alpha^4(a - bc) \geq 0 \)

**Proof (Proposition 3):**

\[ \Pi_{opt}^R \geq \Pi^*_R \Rightarrow (8Ib - \alpha^2)(16Ib + \alpha^2) \geq 64I^2b^2(16Ib - 3\alpha^2) \]

⇒ \( \alpha^6 \geq 0 \)

\[ \Pi_{opt}^M \geq \Pi^*_M \Rightarrow (8Ib - \alpha^2)(16Ib - \alpha^2) \geq 8IB(16Ib - 3\alpha^2) \]

⇒ \( \alpha^4 \geq 0 \)

**Proposition 4**

a). There exists a solution to the Nash bargaining problem such that \( \phi^* \in [1/2, 4/5] \)

b). The equilibrium solution to the Nash bargaining problem is given by

\[ \phi^* = \frac{(8Ib - \alpha^2) + \sqrt{(\alpha^2 + 8Ib)^2 - 12Ib\alpha^2}}{20Ib} \quad \text{for} \quad \frac{(5 + \sqrt{33})\alpha^2}{16b} \geq I > \frac{\alpha^2}{24b} \]

Proof: Note that the first order condition of \( \Pi_B \) w.r.t \( \phi \) gives

\[ \frac{\delta \Pi_B}{\delta \phi} = \frac{-I^2\alpha^2(a - bc)^4(20Ib\phi^2 - \alpha^2 + 2\alpha^2\phi - 16Ib\phi)}{(8Ib\phi - \alpha^2)^4} \quad (88) \]

which is always negative for \( \phi \geq 4/5 \) and always positive for \( \phi \leq 1/2 \). As \( \Pi_B \) is continuous in \( \phi \), this implies there should exist a solution in the range \([1/2, 4/5]\). The proof for Part b has been shown above.

**Proof (Proposition 5):**

a) \( \theta^b \geq \theta^* \Rightarrow (12Ib + \alpha^2) \geq \sqrt{(\alpha^2 + 8Ib)^2 - 12Ib\alpha^2} \)

On squaring and simplification,

⇒ \( 20Ib(4Ib + \alpha^2) \geq 0 \)

b) \( w^b \geq w^* \Rightarrow 20Ib(4Ib + \alpha^2) \geq 0 \)

c) \( p^b \geq p^* \Rightarrow 20Ib(4Ib + \alpha^2) \geq 0 \)

d) \( \Pi^b _M \geq \Pi^*_M \Rightarrow 20Ib(4Ib + \alpha^2) \geq 0 \)
References


