## **Decision Rules for Timing the Purchase of a Stock**

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#### Abstract

The problem of timing the purchase and sale of stocks has attracted the attention of practitioners and empirical researchers in finance since the beginning of the present century. However, the popular trading rules like the filter rules, the relative strength rules, and the moving average methods have been proved to be inferior to a naïve buy-and-hold strategy by various empirical studies. Indeed, the Efficient Market Hypothesis, which states that prices of securities reflect the available information in the stock market, precludes the possibility of a class of trading rules being more effective than buy-and-hold. This class, called the "one security and cash" class, postulated that at a point of time, an investor can hold the security or hold cash, he is not permitted to invest in any other security.

In this dissertation, we formulate some decision rules for timing the purchase of a stock and a portfolio of stocks. The stock prices are assumed to follow a certain Markov stochastic process called Geometric Brownian Motion. The investor following our decision rules is assumed to invest his initial cash in a riskless asset and to buy the stock whenever the stocks price comes down to or "hits" a pre-determined threshold value. The threshold values keep changing with time, giving rise to a time varying "trajectory" of threshold values. Two such decision rules are formulated in this thesis.

The first one is a simple decision rule, in which, the investor buys the stock when the stock's price  $S_t$  at time t, t = 0 hits the trajectory T  $S_0 e^{rt}$ , where  $S_0$  denotes the stock's price at time 0, r is the riskless rate of return, and T is a real number between 0 and 1. The strategy is shown to beat a buy-and-hold strategy on the average, under certain closed form conditions on the stock's expected return and volatility parameters. We obtain mathematically, the performance figures for the strategy, like the probability of "success" of the investor, i.e. the probability that the investor is able to buy the stock within a fixed time horizon; and the expected terminal wealth of the investor at the end of a time interval of length T. A method of obtaining a local optimum within this family of timing strategies, by choosing T so as to maximize the investor's expected terminal wealth, has been derived. Estimation of the strategy has also been pursued. Since the "one security and cash" constraint has been relaxed, success of the decision rule does not amount to

"beating the market" in a strict sense. However, it is argued that permitting the investor to invest in a riskless asset is consistent with the common sense principle of minimizing the present value of cash outflow in timing the purchase.

The second decision rule is an optimal static one wherein, the objective of maximizing the investor's expected terminal wealth is achieved by dynamic programming following Karlin (1962). The threshold trajectory in this case is not obtained in a closed form, but as a solution to a particular differential equation derived here. The expected terminal wealth yielded by the strategy, and a lower bound on the probability of success of the investor have also been arrived at.

The problem of timing the purchase of a portfolio of stocks is attempted to be solved by applying the simple decision rule to a portfolio in two ways: (i) Purchasing the portfolio in one shot when the value  $p_t$  of the portfolio hits the trajectory ?  $p_0 e^{rt}$ , where  $p_t$  denotes the value of the portfolio at time t, and ? is, as before, a real number between 0 and 1, and (ii) Buying the component stocks separately, i.e. buying requisite number of shares of the i-th stock when its price  $S_t^i$  hits the i-th stock's trajectory  $T_i S_0 e^{rt}$ . For the first strategy, the approach to obtaining an optimum ? is derived. Conditions are also determined under which any one of the above strategies is better than the other.

The Fixed Investment Proportion Maintenance (FIPM) strategy proposed by Evans (1968), whereby an investor keeps constant proportion of wealth invested in each of the stocks in his portfolio, is shown to be inferior, in terms of expected terminal wealth, to buy-and-hold strategy under our assumptions, thus disproving a conjecture by Evans. Application of the simple decision rule on an FIPM portfolio is shown to be almost identical to its application on a single stock.

Empirical tests carried out on the stocks comprising the Bombay Stock Exchange Sensitive Index reveal that the decision rules would have generated returns significantly higher than that of a buy-and-hold policy during the period 1995-96. The robustness of the decision rules is tested by applying them on a stream of stock process generated from a process different from GBM. The prices are generated from a discrete mixture of normal distributions. Though the theoretically calculated performance figures like probability of success and expected terminal wealth turn out to be inaccurate, the broad goal of beating a buy-and-hold policy is achieved even in this case.

The dissertation is closed with a few concluding remarks about possible extensions and some related problems.

### **Related Publications**

Raychaudhuri, M. and Basu, S. K., *Timing the Purchase of a Stock*, WPS-270, Working Paper Series, IIM Calcutta, 1996.

Raychaudhuri, M. and Basu, S. K., *An Optimal State Policy for Timing the Purchase of a Stock*, WPS-278, Working Paper Series, IIM Calcutta, 1996.