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Competitive Dynamics between Traditional and Online Retailing under Customer Showrooming Behaviour and Strategies to Counter Showrooming

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Abstract

Showrooming is a phenomenon when a customer views a product at a physical store, but buys it online from a competitor's website. In this paper, we develop economic models of price-competition between a traditional and an online retailer under customer showrooming behaviour. Our results indicate that showrooming hurts the traditional retailer and benefits the online retailer in terms of sales volumes and profits. The combined offline and online market expands under showrooming. We consider two strategies – effort/investment made and online entry by the traditional retailer – to counter showrooming. Either strategy makes the traditional retailer better off, and the online retailer worse off, in terms of sales volumes and profits; also, the overall market, including offline and online sales, contracts. Moreover, when the traditional retailer makes an online entry, although its offline sales decrease, its total offline and online sales increase; also, although the overall market contracts, total online sales and the online price increase. We consider two scenarios of simultaneous and sequential moves made by the retailers to set their prices. We observe that both the retailers benefit under sequential moves than in the simultaneous move; however, the overall market demand is lower in sequential moves than in the simultaneous move. We conclude the paper by highlighting the managerial implications of this research and providing possible directions for future research.

Keywords: Retailing; Showrooming; Pricing; Competitive Dynamics; Economic Models

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Competitive Dynamics between Traditional and Online Retailing under Customer Showrooming Behaviour and Strategies to Counter Showrooming

1. Introduction

Showrooming is the phenomenon when a customer visits a traditional or brick-and-mortar retail store to view and experience a product, but instead of buying the product from the store, she buys it online from a competitor's website. It is as if the physical store acts as a showroom for the online sales channel. The term 'showrooming' became popular when there were talks in the US media that the electronics chain Best Buy had become a 'showroom for Amazon' (Goodfellow, 2012; Quint et al., 2013). Traditional retailers[2] consider showrooming a serious threat to their sales potential. The growing availability of smartphones and easy accessibility of the internet have further fueled customer showrooming behaviour and added to the concern of traditional retailers. Surveys have shown that showrooming can vary from about 40% to 60%, and can be as high as 70% if only shoppers using smartphones in-store are taken into consideration (Zimmerman, 2012b; Quint et al., 2013; Balakrishnan et al., 2014; Rapp et al., 2015; Gensler et al., 2017; Rejon-Guardia and Luna-Nevarez, 2017; Kuksov and Liao, 2018). Research shows that for certain product categories, such as electronics and appliances, 83% of shoppers practice showrooming (Teixeira and Gupta, 2015). Shoppers, who use smartphones, look for price comparison, product information and customer review on the websites and apps of traditional and online retailers before making a purchasing decision. Products such as electronics, appliances, sporting goods, clothing, shoes, books and furniture, which are 'non-digital' in nature (Balakrishnan et al., 2014; Bell et al., 2014; Mehra et al., 2018) and for which shoppers value in-store view-touch-feel-and-fit experience, are more prone to showrooming than other types of products that are generic in nature such as groceries (Quint et al., 2013; Rejon-Guardia and Luna-Nevarez, 2017; Jing, 2018).

The primary reasons for showrooming cited by shoppers, who engage in showrooming, are lower online prices (Amazon's prices for consumer electronics were 11% and 8% lower than Walmart's and Best Buy's in-store prices, respectively; also, Amazon's prices were 14% below

[2]Traditional retailers sell through physical or brick-and-mortar retail stores.

Target's prices (Zimmerman, 2012b)), a desire to experience the product at a physical store before purchasing it online, shopping and delivery convenience of online purchase, and product unavailability at physical stores (Quint et al., 2013; Teixeira and Watkins, 2015). Showrooming has pretty badly affected large traditional US retailers. Some of the scaring news items published in the literature are as follows. Walmart lost $20 billion in market cap in one day (Mohammed, 2015). Target's sales were flat. Sales at Best Buy stores opened in the previous year had fallen by more than 4%. At J.C. Penney, same-store sales dropped by 26% compared to the same period the year before (Teixeira and Watkins, 2015). Benjy's quarterly loss mounted to $700 million (Teixeira and Gupta, 2015).While the traditional retailers were experiencing a sharp decline in sales, the online retail market was growing at 17% per year (Teixeira and Watkins, 2015). According to another estimate, online retail sales grew 23% in 2015 while Amazon became the largest online retailer accounting for 26% of total online retail sales (Sopadjieva et al., 2017). Although the decline in sales at physical stores and growth of online retail cannot be entirely attributed to the phenomenon of showrooming, it is now evident that showrooming does play a significant role in weaning away shoppers from physical stores to the online marketplace.

1.1 Strategies adopted by traditional retailers to counter showrooming

Traditional retailers adopt various strategies to counter showrooming. Among the defensive strategies are high discounts and online price matching, creating barriers/disincentives for shoppers to engage in showrooming, forcing manufacturers to impose minimum advertised prices on online retailers, and charging manufacturers for preferential display of their products on store shelves (Teixeira and Gupta, 2015). For price-sensitive shoppers, who do not look beyond better price deals, the online price-matching strategy is the best strategy (e.g. Target and Best Buy). However, it is doubtful whether this strategy would be sustainable for traditional retailers in the long run since they generally incur higher overhead and operational costs than online retailers. Shoppers may be discouraged to engage in showrooming in physical stores by imposing an 'entry fee' or 'looking fee' (Moran, 2013), which may be adjusted against purchase during check-out; otherwise, they run the risk of forfeiting the fee collected from them for using stores as showrooms only. Traditional retailers may also mask product information and make it difficult for shoppers to engage in in-store price comparison by scanning barcodes and taking photographs. The above tactics may reduce the incidence of showrooming; however, they may also drive away genuine customers, who might have preferred to purchase from physical stores. Traditional retailers may ask manufacturers to set a floor price for every product for both offline and online sales. They may also charge manufacturers a display fee to use their stores as showrooms which may compensate for their loss due to showrooming and online price matching. Although this tactic is not new in retail, this may actually drive away manufacturers to other retailers where they need not pay a display fee.

Besides these defensive strategies to counter showrooming, traditional retailers may actually realize that showrooming is an unavoidable phenomenon and they have to live with it (Wohlsen, 2012), and, therefore, instead of taking showrooming head-on, they may consider developing tactics that would differentiate shopping in physical stores from the online shopping experience and create value for shoppers of physical stores (Freeman, 2014; Sit et al., 2018). For example, traditional retailers may focus on the factors that put online shopping at a disadvantage against shopping in physical stores, such as touch-and-feel experience, personal encounter with sales people, response to product-related queries,customized in-store service, instant gratification, delivery, installation and maintenance of products at home, after-sales service and contact information of service personnel, and ease of return in case of defects and product dissatisfaction (Quint et al., 2013). Many shoppers would like to view and experience a product, and have face-to-face interactions with sales staff to get detailed product information and advice before deciding to purchase. With proper information and guidance, shoppers would be willing to pay a few extra dollars for products they really care about (Vossoughi, 2014). Therefore, it is imperative for traditional retailers to redesign the store layout and make it more attractive and visually appealing with a prominent display of products and installation of touchpads for easy dissemination of product-related information. Store sales people should be knowledgeable enough to address any shopper query and properly trained to be courteous and friendly with shoppers to provide them with a feel-good in-store experience (Quint et al., 2013; Sawhney et al., 2017). Rapp et al. (2015) find negative relationships between perceived showrooming behaviour and salesperson self-efficacy and performance, which are positively moderated by salesperson coping strategies and cross-selling strategies. The authors note that the negative effects of showrooming can be mitigated by moderating the approach and behaviour of salespersons, which again emphasizes the need for training.Gensler et al. (2017), on the other hand, contend that increasing the number of in-store sales personnel, instead of providing more training to the existing staff, is an effective way to curtail showrooming.

Shoppers may be made to feel the advantage of instant gratification in case of in-store purchase unlike in online shopping where shoppers need to wait during the delivery period (Teixeira and Gupta, 2015). Stores may house service desks that can arrange delivery and installation of products at home, and for do-it-yourself products, may assist shoppers in self-installation. They also get in touch with service personnel for maintenance and repair at home, and when products need to be brought to stores for repair, both during the warranty period and thereafter. However, in case of products purchased online, there are no intermediaries and shoppers need to contact manufacturers and arrange for servicing and repair all by themselves.Also, it is easier for shoppers to return and exchange products if purchased in stores than if purchased online. Overall, traditional retailers may emphasize the service associated with selling a product and focus on 'servicization' of products and providing complete solutions (Sawhney et al., 2017), rather than merely looking at selling as a transactional activity.In a similar vein, large-format retail stores may rent out excess floor space for setting up ATMs, cafeterias and entertainment zones to enrich shopping experience that may lead to an increase in footfall and thereby a rise in stores' sales (Vossoughi, 2014; D'Andrea, 2018). Moreover, in-store product assortment may be relooked into and stores may showcase exclusive products with special arrangements with manufacturers (e.g. Macy's has exclusive tie-ups with Tommy Hilfiger and Martha Stewart) or create store brands that are only available for sale in-store and not available online (Brynjolfsson et al., 2013; Mehra et al., 2018). In January 2012, Target's CEO wrote a letter to its suppliers seeking assistance in creating products that would onlybe available at Target (Zimmerman, 2012a).Traditional retailers may also create switching costsfor in-store shoppers with personalized service, additional discounts, promotional events, and rewards and loyalty programmes (Zimmerman, 2012b; Brynjolfsson et al., 2013; Quint et al., 2013; Mohammed, 2015). Finally, traditional retailers may disseminate detailed product-related information and may also sell online through their websites and mobile apps to provide shoppers with a seamless shopping experience through omnichannel retailing (Brynjolfsson et al., 2013; Bell et al., 2014; Teixeira and Gupta, 2015; Gao and Su, 2017; Sopadjieva et al., 2017; Wiener et al., 2018; He et al., 2020). Walmart was able to increase its online sales by 12% for shoppers, who used the Walmart mobile app in-store (Wohlsen, 2012).

1.2 Literature review

Although the extant literature is rich in multichannel competition in retail, literature on competition between traditional and online retailers in the presence of showrooming is scarce (Mehra et al., 2018). In multichannel retail, the literature is replete with the competitive dynamics between a manufacturer and a traditional retailer when the manufacturer decides to sell directly through an online sales channel besides selling through the brick-and-mortar retail store (See, for example, Chiang et al., 2003; Tsay and Agrawal, 2004; Cattani et al., 2006; Chen et al., 2008; Cai et al., 2009; Hua et al., 2010; Dan et al., 2012; Xiong et al., 2012; Xu et al., 2012; Xiao et al., 2014; Rodriguez and Aydin, 2015; Ding et al., 2016; Matsui, 2016, 2017; Xiao and Shi, 2016; Chen et al., 2017; Wang et al., 2018; Yan et al., 2018; Feng et al., 2019; Modak and Kelly, 2019; Xu et al., 2019; Zhang et al., 2019). There is also a vast amount of literature on click-and-mortar, i.e. when a traditional retailer creates an online channel and sells both offline and online (See, for example, Zettelmeyer, 2000; Bernstein et al., 2008; Huang and Swaminathan, 2009; Mahar et al., 2009; Zhang, 2009; Ofek et al., 2011; Chen and Chen, 2017; Zhang and Wang, 2017; Zhang et al., 2017; Jiang et al., 2018; Radhi and Zhang, 2019; Zhou et al., 2019). Brynjolfsson and Smith (2000) and Li et al. (2015) study the dynamics among traditional retailers that sell through physical stores only, online retailers that sell online only and 'hybrid' retailers that sell both offline and online. Brynjolfsson et al. (2009) and Abhishek et al. (2016) analyze the competitive dynamics between traditional and online retailers. Agatz et al. (2008) present a literature review on the integration of e-fulfilment with multiple alternative distribution channels or bricks-and-clicks.

None of the above papers considers showrooming in multichannel retail. There is, of course, literature on freeriding where customers free-ride information on one channel and buy on another channel. Wu et al. (2004) study the free riding phenomenon among two groups of online retailers where one group of retailers provides informational services while the other group does not. Customers may free-ride information provided by the former group of retailers and buy at a lower price from the latter group of retailers. The authors find that an online retailer has to provide informational services to make positive profits even if there is free riding and retailers cannot make positive profits by free riding all the time.Shin (2007), in the context of two traditional retailers – one service-providing and the other free-riding – shows that free riding benefits not only the free-riding retailer, but also the retailer that provides service. Free riding not only reduces the intensity of price competition, but also enables the service-providing retailer to charge a higher price and make positive profits.

Xing and Liu (2012) consider a manufacturer selling through a traditional and an online retailer. The online retailer free-rides the sales effort put up by the traditional retailer which reduces the effort of the traditional retailer, thereby affecting the manufacturer's profit and overall supply chain performance. The authors discuss the role of various contracts in coordinating the sales effort of the traditional retailer and improving the supply chain efficiency.Zhou et al. (2018) consider a manufacturer selling through a direct online sales channel and a traditional retailer. The manufacturer's online sales channel free-rides the pre-sales informational services provided by the traditional retailer by sharing its cost of service. The authors investigate how free riding affects the pricing/service strategies and profits of the dual channels.

Showrooming, as defined earlier, is a special kind of service free riding in retail (Gensler et al., 2017; Jing, 2018). According to Balakrishnan et al. (2014), showrooming intensifies competition between a traditional and an online retailer, reducing profits for both the firms. Basak et al. (2017) also observe that profits for both the retailers decrease as showrooming increases. Therefore, reduced showrooming is not only beneficial for the traditional retailer, but also desirable from the point of view of the online retailer. However, a high level of showrooming benefits customers by reducing retail prices.

Mehra et al. (2018) consider competition between a traditional and an online retailer under showrooming. They show that showrooming is detrimental to the profit of the traditional retailer. They analyze two strategies for the traditional retailer to counter showrooming, namely price matching and exclusivity of product assortment through arrangements with known brands and creation of store brands. While price matching is proposed to be a short-term strategy, exclusivity of product assortment is considered to be a long-term strategy. The authors show that price matching is more effective under showrooming than when there is no showrooming, and implementing product exclusivity through the store-brand strategy is better than exclusivity through the known-brand strategy under showrooming while the opposite is true when there is no showrooming.Kuksov and Liao (2018) consider the role of contracts between a manufacturer and a traditional retailer and show that the traditional retailer's profit may actually increase under showrooming. The authors state that the overall demand increases under showrooming and hence the manufacturer may incentivize the traditional retailer for providing informational services either through a lower wholesale price or through direct compensation. Manufacturer incentives may, therefore, increase the traditional retailer's profit even if it invests in improving the store service level. The authors have developed their model under some strong assumptions such as a single manufacturer has been considered who is selling both offline and online and hence inter-brand competition has been ignored; all shoppers, irrespective of whether they would ultimately buy from a physical store or online, visit the physical store which may not be true in practice, and some shopperswould never visit the physical store and always buy online; and shoppers' valuation of products online is lower than that in a physical store which, again, may not be always true, especially for 'non-digital' products. The authors do, of course, admit that their model also shows the possibility that showrooming could be detrimental to the traditional retailer's profit.

Jing (2018) considers competition between a traditional and an online retailer in the presence of showrooming. The author shows that under low product match uncertainty, showrooming intensifies competition and decreases the profits of both the retailers, thus supporting the retailers' recent strategy to stock more exclusive products. However, the author concludes that under high product match uncertainty, showrooming may have different effects on competition and may very likely increase the online retailer's profit. Zhang and Zhang (2020) consider offline entry by a supplier that sells through an online channel. Their study focuses on the online retailer's demand information sharing strategy with the supplier under the agency selling and reselling agreements. The authors observe that the online retailer may be better off with supplier offline entry when there is showrooming and is always worse off when there is no showrooming. When the supplier makes an offline entry, showrooming enables customers to get information offline and buy online which may bring additional online revenues and benefit the online retailer. For a low level of showrooming, the loss from channel competition due to supplier offline entry dominates the benefits of showrooming and hence it hurts the online retailer while for a high level of showrooming, the benefits of showrooming outweigh the loss from channel competition and the online retailer benefits by supplier offline entry.

1.3 Motivation and contribution

This paper develops economic models of price-competition between a traditional and an online retailer under customer showrooming behaviour. The models are innovative and practical, and they differ from the existing models available in the related literature. Modeling assumptions and their differences with existing models are highlighted in a subsequent section. Two scenarios have been considered for setting prices, namely when the retailers simultaneously set their prices, and when one of the retailers acts as the Stackelberg leader, and the other follower, to sequentially set their prices. The justification of considering two scenarios is also provided in a subsequent section. Further, when the retailers simultaneously decide on their prices, the competitive dynamics in the presence of the traditional retailer's strategies, namely the effort/investment made by the traditional retailer and the traditional retailer's online entry, to counter showrooming is analyzed.

The extant literature, as presented in the previous section on literature review, shows contradictory results as to the benefits/losses accrued to the traditional and online retailers in the presence of showrooming. Moreover, either simultaneous or sequential decision-making on prices by the retailers has been considered, and no comparison of these two decision-making processes has been made with respect to their relative performances. The objective of this paper is to fill this research gap by investigating the movement of prices, sales and profits of the two retailers under showrooming and also when the traditional retailer adopts the above-mentioned strategies to counter showrooming under some practical assumptions made in the paper. It is intended to compare the assumptions and findings of the current paper with those in the extant literature, and observe under what conditions the results corroborate or conflict with each other.Another objective of this paper is to compare the simultaneous and sequential decision-making processes of setting prices by the retailers in terms of sales, profits and market sizes, which has not been addressed in the literature so far. In particular, the following questions have been addressed in this paper:

a) Does showrooming benefit, or hurt, the traditional and online retailers?
b) Does showrooming increase, or decrease, the overall market demand?

c) How do the strategies adopted by the traditional retailer to counter showrooming alter the competitive dynamics and affect the overall market demand?
d) How do the simultaneous and sequential moves by the retailers to set prices compare in terms of prices, sales volumes and profits of the retailers and the overall market demand?

The significant findings of this paper are as follows:

When the retailers simultaneously set their prices,

a) Prices, sales and profits decrease for the traditional retailer and increase for the online retailer under showrooming.
b) The combined offline and online demand increases with showrooming, indicating customer benefits due to showrooming.
c) When the traditional retailer puts in effort/makes an investment to counter showrooming, the price, sales and profit of the traditional retailer increase while the same decrease for the online retailer. Also, the combined offline and online demand decreases, hurting customers in the process.
d) When the traditional retailer makes an online entry to counter showrooming, its price and profit increase. As far as the sales volume is concerned, the traditional retailer's offline sales volume decreases; however, its total sales volume, including offline and online sales, increases. On the other hand, for the online retailer, while the price increases, its sales volume and profit decrease. From the customers' point of view, while online sales, including the sales of the online arm of the traditional retailer and the online retailer, increase, total offline and online sales decrease.This result points to the fact that upon online entry by the traditional retailer, although the online market expands, the overall market, including offline and online, contracts.

When the retailers sequentially set their prices, in comparison to when the retailers simultaneously set their prices,

a) Irrespective of which retailer acts as the leader, prices and profits of both the retailers increase. However, the price charged by a retailer is the highest when it acts as the leader while the profit made by a retailer is the highest when it acts as a follower.

b) When the traditional retailer acts as the leader, offline sales decrease and online sales increase.
c) When the online retailer acts as the leader, offline sales increase and online sales decrease.
d) Irrespective of which retailer acts as the leader, total offline and online sales decrease, and the combined sales volume is the lowest when the traditional retailer acts as the leader.

Almost all the proofs (except one) in this paper are parameter-independent, i.e. they hold for the entire ranges of parameter values and not for specific ranges, thereby making the findings of this paper robust.

The rest of the paper is organized as follows. Section 2 describes the problem and modeling assumptions. Section 3 presents the economic models. In Section 3.1, the demand and profit functions for the retailers are explained. Section 3.2 presents the economic models when the retailers simultaneously set their prices. Models for investments made and online entry by the traditional retailer to counter showrooming are derived in Sections 3.2.1 and 3.2.2, respectively. Section 3.3 presents the economic models when the retailers sequentially set their prices. Models when the traditional retailer acts as the leader and when the online retailer acts as the leader are derived in Sections 3.3.1 and 3.3.2, respectively. Section 4 presents a summary of results and highlights the managerial implications of this research. Finally, Section 5 presents concluding remarks and directions for future research.

2. Problem description and modeling assumptions

We consider price-competition between a traditional and an online retailer, who sell an identical product, under customer showrooming behaviour. First we develop economic models when the retailers simultaneously decide on their prices, in line with Balakrishnan et al. (2014) and Mehra et al. (2019). We assume that the selling period is short and the retailers simultaneously announce their prices based on the best response function of the other retailer. Next we consider two strategies – effort/investment made and online entry by the traditional retailer – to counter showrooming and extend the economic models. Finally, we develop economic models for the scenario when one of the retailers acts as the Stackelberg leader, and the other follower, to sequentially set their prices, in line with Basak et al. (2017). The sequential decision-making process is relevant when the selling period is sufficiently long and one of the retailers is dominant and more powerful than the other retailer. In this scenario, the leader first sets its price based on the best response function of the follower and then the follower sets its price by observing the price set by the leader.

Following are the practical assumptions made in the models and their justifications:

a) The market potential of the traditional retailer is greater than that of the online retailer. Although online retail sales are growing faster than offline retail sales, according to the latest US Census Bureau report, online retail sales are still about 10% of total retail sales in the US (U.S. Census Bureau News, 2019). Therefore, models developed in this paper ensure that the traditional retailer's offline sales volume always exceeds the online retailer's sales volume. This is in contrast to the assumptions made by Basak et al. (2017) and Zhou et al. (2018), who either have allowed the market potential of the online retailer to exceed that of the traditional retailer or have considered equal market potential for offline and online sales.
b) The own- and cross-price sensitivities of the online demand are greater than the respective own- and cross-price sensitivities of the offline demand, which leads to the offline retail price being greater than the online retail price in every model developed in this paper. As mentioned earlier, one of the primary reasons for shopping online is that online retail prices are generally lower than offline retail prices for most of the products, and the showrooming phenomenon bears testimony to this fact wherein price-sensitive shoppers check products in traditional stores, but prefer to buy online because of lower prices. This is, again, in contrast to the assumptions made by Basak et al. (2017) and Zhou et al. (2018), who consider the same own- and cross-price sensitivities of the offline and online demand.
c) Customer showrooming behaviour will have an impact only on the market potential of a retailer, thereby affecting the demand faced by the retailer. The market potential of the traditional retailer can be assumed as the footfall, i.e. the number of shoppers that set foot in the traditional store. When some of these shoppers showroom, i.e. they check the product in the traditional store, but buy online, the market potential of the traditional store decreases and the market potential of the online store increases by the same number. This is contrary to the assumption made by Basak et al. (2017) and Zhang and Zhang (2020), who define the shift in demand as a function of sales effort put in by the traditional retailer. However, their

assumption does not take into consideration the fact that showrooming is a behavioural phenomenon that is influenced by internal and external factors such as demographics, price-consciousness, previous showrooming experience, social pressure and accepted social norm, availability of smartphones and other mobile devices, accessibility to high-speed internet and a continued upsurge of internet retailers (Rejon-Guardia and Luna-Nevarez, 2017; Dahana et al., 2018; Sit et al., 2018). There is no empirical evidence that increasing sales effort by the traditional retailer will increase the demand faced by the online retailer due to showrooming. On the contrary, the extant literature highlights increasing sales effort by the traditional retailer as a strategy to counter showrooming. Therefore, in this paper, we have modeled customer showrooming behaviour as an exogenous parameter that affects the market potential of either retailer.

3. Model development

In this section, first the notations used in model development are listed. Then the demand and profit functions for the traditional and online retailers are explained. Next, a game-theoretic model of the system when the retailers simultaneously decide on their respective prices under showrooming is derived, followed by model development for two common strategies of the traditional retailer to counter showrooming, namely additional investment and entry into the online market. Finally, another game-theoretic model has been developed for the system when the retailers move sequentially to decide on their respective prices under showrooming, one of them being the leader and the other being the follower.

Notations:

Index

i {1: Traditional retailer; 2: Online retailer}

Parameters

α Market potential of online sales as a fraction of the market potential of offline sales

β, γ, θ Parameters representing sensitivity of demand functions to prices *s* Parameter representing the showrooming behaviour of customers as a fraction of the market potential of offline sales

Variables

$p_1(p_2)$ Price charged by the traditional (online) retailer

$q_1(q_2)$ Demand/sales volume for the traditional (online) retailer

$\Pi_1(\Pi_2)$ Profit of the traditional (online) retailer

3.1 Demand and profit functions for the retailers

The normalized demand functions for the traditional and online retailers can be written as follows:

$$q_1 = (1-s) - p_1 + \beta\theta p_2$$
$$q_2 = (\alpha + s) - \theta p_2 + \gamma p_1$$

We assume that the showrooming behaviour of customers affects the market potential of offline and online sales. Accordingly, the expressions for market potential in the demand functions reflect the showrooming effect.Hereα, β and γ are parameters such that $0 < \alpha, \beta, \gamma < 1$. Since it is assumed that online customers are more price-sensitive than offline customers, it follows $\theta > 1$. Since it is also assumed that online customers are more likely to switch to offline purchase in case of an increase in online price than offline customers, who prefer to shop at traditional stores and may not be as tech-savvy as online customers to make a move to online stores when the offline price increases, it follows $\beta > \gamma$. According to the assumption that the market potential of online sales is lower than the market potential for offline sales, it follows $\alpha + s < 1 - s$ or $s < \frac{1-\alpha}{2}$.

Therefore, assuming that the variable cost is the same for online and offline sales and normalizing it to zero, the profit functions for the traditional and online retailers can be written as follows:

$$\Pi_1 = p_1 q_1 = p_1[(1-s) - p_1 + \beta\theta p_2]$$
$$\Pi_2 = p_2 q_2 = p_2[(\alpha+s) - \theta p_2 + \gamma p_1]$$

3.2 Simultaneous move by the retailers to set prices

Here the traditional and online retailers move simultaneously to decide on their respective prices not knowing what the pricing strategy of the other retailer would be. Assuming that both the retailers are rational, they settle for Nash equilibrium prices, as commonly found in the literature.

Therefore, to obtain Nash equilibrium prices, we partially differentiate the profit functions with respect to their corresponding prices and equate them to zero, which leads us to

$$\frac{\partial \Pi_1}{\partial p_1} = 0 \quad \Rightarrow \quad 2p_1 - \beta\theta p_2 = 1 - s \tag{1}$$

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \quad \Rightarrow \quad -\gamma p_1 + 2\theta p_2 = \alpha + s \tag{2}$$

Solving Eqs. (1) and (2), we obtain the following Nash equilibrium prices:

$$p_1 = \frac{\beta(\alpha+s) + 2(1-s)}{4-\beta\gamma} \tag{3}$$

$$p_2 = \frac{2(\alpha+s) + \gamma(1-s)}{\theta(4-\beta\gamma)} \tag{4}$$

The expressions for Nash equilibrium q_1, q_2, Π_1 and Π_2 are obtained as follows:

$$\begin{aligned} q_1 &= (1-s) - p_1 + \beta\theta p_2 = (1-s) - \frac{\beta(\alpha+s) + 2(1-s)}{4-\beta\gamma} + \frac{\beta[2(\alpha+s) + \gamma(1-s)]}{4-\beta\gamma} \\ &= \frac{\beta(\alpha+s) + 2(1-s)}{4-\beta\gamma} = p_1 \end{aligned} \tag{5}$$

$$\begin{aligned} q_2 &= (\alpha+s) - \theta p_2 + \gamma p_1 = (\alpha+s) - \frac{2(\alpha+s) + \gamma(1-s)}{4-\beta\gamma} + \frac{\gamma[\beta(\alpha+s) + 2(1-s)]}{4-\beta\gamma} \\ &= \frac{2(\alpha+s) + \gamma(1-s)}{4-\beta\gamma} = \theta p_2 \end{aligned} \tag{6}$$

$$\Pi_1 = p_1 q_1 = \frac{[\beta(\alpha + s) + 2(1 - s)]^2}{(4 - \beta\gamma)^2} \tag{7}$$

$$\Pi_2 = p_2 q_2 = \frac{[2(\alpha + s) + \gamma(1 - s)]^2}{\theta(4 - \beta\gamma)^2} \tag{8}$$

Proposition 1: The following will hold under showrooming:

a) $p_1 > p_2$

b) $q_1 > q_2$

c) $\Pi_1 > \Pi_2$

Proofs of propositions and corollaries are given in the Appendix.

Proposition (1) shows that the price, sales volume and profit of the traditional retailer are higher than those for the online retailer under showrooming. Corollary (1) shows that the price, sales volume and profit of the traditional retailer are higher than those for the online retailer even if there is no showrooming, i.e. $s = 0$.

Corollary 1: The following will hold even if there is no showrooming, i.e. $s = 0$:

a) $p_1 > p_2$

b) $q_1 > q_2$

c) $\Pi_1 > \Pi_2$

Therefore, Proposition (1) and Corollary (1) show that the relationships between prices, sales volumes and profits of the traditional and online retailers remain unchanged under customer showrooming behaviour.This is especially true under the assumption that the market potential of offline sales is higher than that of online sales.

Further, it can be easily shown from Eqs. (3) – (8) that the Nash equilibrium prices, sales volumes and profits of both the retailers assume fractional values under normalized demand functions.

Proposition 2: p_1, q_1 and Π_1 decrease with s for the traditional retailer while p_2, q_2 and Π_2 increase with s for the online retailer.

As expected, Proposition (2) shows that under customer showrooming behaviour, the online retailer benefits at the expense of the traditional retailer.

Proposition 3: The combined offline and online demand, $q_1 + q_2$ increases with s.

Proposition (3) shows that although the traditional retailer loses market share, the gain in the market share of the online retailer results in an overall increase in demand under customer showrooming behaviour. This result is in line with the observation made by Bell et al. (2018).

Proposition 4: While prices, sales volumes and profits of both the retailers increase with the parameters, α, β and γ, the online retailer's price and profit decrease with the parameter, θ.

Proposition (4) indicates that increasing marketing potential of the online channel and cross-price sensitivity parameters benefit both the retailers. On the other hand, increasing own-price sensitivity parameter for the online retailer expectedly decreases its price and profit while the traditional retailer remains unaffected.

3.2.1 Effort/investment made by the traditional retailer to counter showrooming

Suppose the traditional retailer puts in effort by way of rearranging the layout and display at the showroom to make it more attractive to customers, providing better in-store experiences, introducing loyalty/rewards programmes for customer retention, investing in technology such as mobile apps and in-store wi-fi, investing in inventory so that items never go out of stock, and so on to mitigate customer showrooming behaviour. Let the normalized level of effort put in by the traditional retailer be represented by ε such that $0 < \varepsilon < 1$. Also, let the associated normalized investment made by the traditional retailer be ε^2 such that the investment required increases quadratically with the level of effort. Although Basak et al. (2017) consider a linear cost function, we consider an increasing and convex cost function in line with Tsay and Agrawal (2004), Xing and Liu (2012), Kuksov and Liao (2018), Zhou et al. (2018), and Zhang and Zhang (2020). It is assumed that with a level of effort of ε, the showrooming parameter, s reduces to

$s(1-\varepsilon)$. Then the normalized demand and profit functions for the retailers can be written as follows:

$$q_1 = \{1-s(1-\varepsilon)\} - p_1 + \beta\theta p_2$$
$$q_2 = \{\alpha + s(1-\varepsilon)\} - \theta p_2 + \gamma p_1$$

$$\Pi_1 = p_1 q_1 - \varepsilon^2 = p_1\left[\{1-s(1-\varepsilon)\} - p_1 + \beta\theta p_2\right] - \varepsilon^2$$
$$\Pi_2 = p_2 q_2 = p_2\left[\{\alpha + s(1-\varepsilon)\} - \theta p_2 + \gamma p_1\right]$$

It may be noted that for the traditional retailer, ε is an additional variable in this model. Therefore, partially differentiating Π_1 with respect to $p_1$and ε, partially differentiating Π_2 with respect to p_2, and equating them to zero, we get

$$\frac{\partial \Pi_1}{\partial p_1} = 0 \quad \Rightarrow \quad 2p_1 - \beta\theta p_2 - s\varepsilon = 1 - s \tag{9}$$

$$\frac{\partial \Pi_1}{\partial \varepsilon} = 0 \quad \Rightarrow \quad sp_1 - 2\varepsilon = 0 \tag{10}$$

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \quad \Rightarrow \quad -\gamma p_1 + 2\theta p_2 + s\varepsilon = \alpha + s \tag{11}$$

Solving Eqs. (9) – (11), we get the following:

$$p_1 = \frac{2[\beta(\alpha+s) + 2(1-s)]}{2(4-\beta\gamma) - s^2(2-\beta)} \tag{12}$$

$$p_2 = \frac{(4-s^2)(\alpha+s) + (2\gamma - s^2)(1-s)}{\theta[2(4-\beta\gamma) - s^2(2-\beta)]} \tag{13}$$

$$\varepsilon = \frac{s[\beta(\alpha+s) + 2(1-s)]}{2(4-\beta\gamma) - s^2(2-\beta)} \tag{14}$$

The expressions for q_1, q_2, Π_1 and Π_2 are obtained as follows:

$$q_1 = 1 - s\left[1 - \frac{s\{\beta(\alpha+s)+2(1-s)\}}{2(4-\beta\gamma)-s^2(2-\beta)}\right] - \frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} + \frac{\beta[(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)}$$
$$= \frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} = p_1 \quad (15)$$

$$q_2 = \alpha + s\left[1 - \frac{s\{\beta(\alpha+s)+2(1-s)\}}{2(4-\beta\gamma)-s^2(2-\beta)}\right] - \frac{(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)}{2(4-\beta\gamma)-s^2(2-\beta)} + \frac{2\gamma[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)}$$
$$= \frac{(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)}{2(4-\beta\gamma)-s^2(2-\beta)} = \theta p_2 \quad (16)$$

$$\Pi_1 = p_1 q_1 - \varepsilon^2 = \frac{(4-s^2)[\beta(\alpha+s)+2(1-s)]^2}{[2(4-\beta\gamma)-s^2(2-\beta)]^2} \quad (17)$$

$$\Pi_2 = p_2 q_2 = \frac{[(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)]^2}{\theta[2(4-\beta\gamma)-s^2(2-\beta)]^2} \quad (18)$$

Proposition 5: The following will still hold:

a) $p_1 > p_2$

b) $q_1 > q_2$

c) $\Pi_1 > \Pi_2$

Corollary (2) checks if, as before, prices, sales volumes and profits of both the retailers assume fractional values under normalized demand functions. Corollary (2) checks if $p_1 < 1$. Then the rest of the proof follows. Corollary (2) also checks if $\varepsilon < 1$ since by definition, $0 < \varepsilon < 1$.

Corollary 2: The following will hold:

a) $p_1 < 1$

b) $\varepsilon < 1$

Since Eqs. (15) and (17) show $q_1 = p_1$ and $\Pi_1 = p_1 q_1 - \varepsilon^2$, it follows from Corollary (2) that $q_1, \Pi_1 < 1$. Also, since Proposition (5) shows $p_1 > p_2$, $q_1 > q_2$ and $\Pi_1 > \Pi_2$, it follows directly that $p_2, q_2, \Pi_2 < 1$.

Proposition (6) checks if the price, demand/sales volume and profit of the traditional retailer increase upon investment made to counter showrooming.

Proposition 6: The following hold for the traditional retailer:

a) Price charged increases.
b) Demand/sales volume increases.
c) Profit increases given $\gamma > 0.064$.

Part (c) of Proposition (6)highlights the fact that the profit of the traditional retailer may not always increase upon investment made to counter showrooming. This is clear from Eq. (17) that although the price and demand/sales volume of the traditional retailer, and hence the revenue,increase post-investment, as shown in parts (a) and (b) of Proposition (6), the investment made may actually bring down the profit below the pre-investment level depending on the choice of parameter values. Therefore, the traditional retailer has to weigh options before making any investment to counter showrooming, and would wish to invest only when the benefits outweigh the cost. Part (c) of Proposition (6) shows thatwhen $\gamma > 0.064$, for any combination of other parameter values, the profit of the traditional retailer always increases post-investment.

Proposition 7: The following hold for the online retailer post-investment by the traditional retailer:

a) Price charged decreases.
b) Demand/sales volume decreases.
c) Profit decreases.

Proposition (7) shows that the online retailer loses upon the traditional retailer's investment to counter showrooming.

Proposition 8: The combined offline and online demand, $q_1 + q_2$ decreases post-investment by the traditional retailer.

Proposition (8) shows that although the traditional retailer benefits at the cost of the online retailer by making an investment to counter showrooming, the combined offline and online customer demand/sales volume falls below the pre-investment level.

3.2.2 Online entry by the traditional retailer to counter showrooming

The traditional retailer can make a foray into the online market to counter the effect of showrooming. Therefore, the traditional retailer sells both offline and online. While it sets the offline price, the online price is set by its online arm along with the online retailer, and the prices charged by the online arm of the traditional retailer and the online retailer are the same, thereby creating an undifferentiated online marketplace for the online customer. This is in line with the assumption made by Balakrishnan et al. (2014). Hence, we have the following offline and online demand functions, respectively:

$$q_1 = (1 - s) - p_1 + \beta\theta p_2$$
$$q_2 = (\alpha + s) - \theta p_2 + \gamma p_1$$

While the offline demand can be attributed solely to the traditional retailer, the online demand has to be apportioned to the traditional and online retailers. Let $\lambda q_2\ (0 < \lambda < 1)$ of the online demand be attributed to the online arm of the traditional retailer and $(1-\lambda)q_2$ of the online demand be attributed to the online retailer. This is also in line with the assumption made by Balakrishnan et al. (2014).

Therefore, offline and online sales of the traditional retailer = $q_1 + \lambda q_2$ and online sales of the online retailer = $(1-\lambda)q_2$.

The profit functions for the traditional and online retailers can be written as follows, respectively:

$$\Pi_1 = p_1 q_1 + \lambda p_2 q_2 = p_1[(1-s) - p_1 + \beta\theta p_2] + \lambda p_2[(\alpha + s) - \theta p_2 + \gamma p_1]$$
$$\Pi_2 = (1-\lambda)p_2 q_2 = (1-\lambda)p_2[(\alpha + s) - \theta p_2 + \gamma p_1]$$

The fixed cost/investment for setting up an online arm by the traditional retailer has not been included in its profit function. It is assumed that the traditional retailer will consider setting up an online arm only if the benefits outweigh the fixed cost/investment.

Now, partially differentiating the profit functions with respect to their prices and equating them to zero, we get

$$\frac{\partial \Pi_1}{\partial p_1} = 0 \quad \Rightarrow \quad 2p_1 - (\beta\theta + \lambda\gamma)p_2 = 1 - s \tag{19}$$

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \quad \Rightarrow \quad -\gamma p_1 + 2\theta p_2 = \alpha + s \tag{20}$$

Solving Eqs. (19) and (20), we get the following:

$$p_1 = \frac{(\beta\theta + \lambda\gamma)(\alpha + s) + 2\theta(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} \tag{21}$$

$$p_2 = \frac{2(\alpha + s) + \gamma(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} \tag{22}$$

Following are the expressions for q_1 andq_2:

$$\begin{aligned} q_1 &= (1 - s) - \frac{(\beta\theta + \lambda\gamma)(\alpha + s) + 2\theta(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} + \beta\theta \frac{2(\alpha + s) + \gamma(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} \\ &= \frac{(\beta\theta - \lambda\gamma)(\alpha + s) + (2\theta - \lambda\gamma^2)(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} \end{aligned} \tag{23}$$

$$\begin{aligned} q_2 &= (\alpha + s) - \theta \frac{2(\alpha + s) + \gamma(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} + \gamma \frac{(\beta\theta + \lambda\gamma)(\alpha + s) + 2\theta(1 - s)}{\theta(4 - \beta\gamma) - \lambda\gamma^2} \\ &= \frac{\theta[2(\alpha + s) + \gamma(1 - s)]}{\theta(4 - \beta\gamma) - \lambda\gamma^2} \end{aligned} \tag{24}$$

Proposition (9) shows that the price, sales volume (offline + online) and profit of the traditional retailer are higher than those of the online retailer, respectively.

Proposition 9: The following will hold:

a) $p_1 > p_2$

b) $q_1 + \lambda q_2 > (1-\lambda)q_2$

c) $\Pi_1 > \Pi_2$

Proposition 10: The following will hold for the traditional retailer post its entry into the online market:

a) Price charged increases.
b) Offline sales volume decreases. However, total sales (offline and online) increase.
c) Profit increases.

Proposition (10) shows that upon online entry by the traditional retailer, even if its profit from offline sales may decrease, its total profit from offline and online sales will increase. This observation is similar to that made by Bernstein et al. (2008). Also, Gao and Su (2017) note that if showrooming customers are persuaded to purchase from the traditional retailer's online channel, it may benefit the traditional retailer.

Proposition 11: The following will hold for the online retailer post the traditional retailer's entry into the online market:

a) Price charged increases.
b) Sales volume decreases.
c) Profit decreases.

The results of Propositions (10) and (11) are in line with the observations made by Balakrishnan et al. (2014).

Proposition 12: The following will hold post the traditional retailer's entry into the online market:

a) Total online sales of the online arm of the traditional retailer and online retailer increase.
b) Total offline and online sales decrease.

Proposition (12) shows that upon the traditional retailer's entry into the online market, although the online market expands, the total market size, including offline and online sales, contracts.

3.3 Sequential move by the retailers to set prices

In this game, one of the retailers acts as the Stackelberg leader and the other acts as the follower. The leader moves first and sets its price. The follower then makes its move and sets its price based on the price set by the leader. To reach equilibrium, the leader derives the follower's best response function and incorporates it into its profit function to determine its price. Subsequently, the follower determines its price by observing the price set by the leader.

3.3.1Traditional retailer as the leader and online retailer as the follower

Given a price, p_1 set by the traditional retailer, we get the best price for the online retailer from Eq. (2)as $p_2 = \frac{\alpha + s + \gamma p_1}{2\theta}$. Incorporating the expression for p_2 in the profit function for the traditional retailer as given in Section 3.1, we get

$$\Pi_1 = p_1\left[(1-s) - p_1 + \frac{\beta}{2}(\alpha + s + \gamma p_1)\right]$$

Now, $\frac{d\Pi_1}{dp_1} = 0 \quad \Rightarrow \quad p_1 = \frac{\beta(\alpha + s) + 2(1-s)}{2(2-\beta\gamma)}$ (25)

It can be easily shown that Π_1 is concave in p_1. Therefore, p_1, as obtained in Eq. (25), maximizes Π_1.

Also, the following expressions may be obtained:

$$p_2 = \frac{(4-\beta\gamma)(\alpha + s) + 2\gamma(1-s)}{4\theta(2-\beta\gamma)} \quad (26)$$

$$q_1 = \frac{\beta(\alpha + s) + 2(1-s)}{4} \quad (27)$$

$$q_2 = \frac{(4-\beta\gamma)(\alpha + s) + 2\gamma(1-s)}{4(2-\beta\gamma)} \quad (28)$$

$$\Pi_1 = \frac{[\beta(\alpha + s) + 2(1-s)]^2}{8(2-\beta\gamma)} \quad (29)$$

$$\Pi_2 = \frac{[(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)]^2}{\theta[4(2-\beta\gamma)]^2} \tag{30}$$

Proposition 13: The following will hold when the traditional retailer acts as the leader and the online retailer acts as the follower, in comparison to when both the retailers move simultaneously to set their respective prices:

a) Prices of both the retailers increase.
b) While offline sales decrease, online sales increase.
c) Profits of both the retailers increase.
d) Total offline and online sales decrease.

3.3.2Online retailer as the leader and traditional retailer as the follower

Given a price, p_2 set by the online retailer, we get the best price for the traditional retailer from Eq. (1) as $p_1 = \frac{1-s+\beta\theta p_2}{2}$. Incorporating the expression for p_1 in the profit function for the online retailer as given in Section 3.1, we get

$$\Pi_2 = p_2\left[(\alpha+s)-\theta p_2+\frac{\gamma}{2}(1-s+\beta\theta p_2)\right]$$

Now, $\frac{d\Pi_2}{dp_2}=0 \quad \Rightarrow \quad p_2 = \frac{2(\alpha+s)+\gamma(1-s)}{2\theta(2-\beta\gamma)}$ (31)

It can be easily shown that Π_2 is concave in p_2. Therefore, p_2, as obtained in Eq. (31), maximizes Π_2.

Also, the following expressions may be obtained:

$$p_1 = \frac{2\beta(\alpha+s)+(4-\beta\gamma)(1-s)}{4(2-\beta\gamma)} \tag{32}$$

$$q_1 = \frac{2\beta(\alpha + s) + (4 - \beta\gamma)(1 - s)}{4(2 - \beta\gamma)} \quad (33)$$

$$q_2 = \frac{2(\alpha + s) + \gamma(1 - s)}{4} \quad (34)$$

$$\Pi_1 = \frac{[2\beta(\alpha + s) + (4 - \beta\gamma)(1 - s)]^2}{[4(2 - \beta\gamma)]^2} \quad (35)$$

$$\Pi_2 = \frac{[2(\alpha + s) + \gamma(1 - s)]^2}{8\theta(2 - \beta\gamma)} \quad (36)$$

Proposition 14: The following will hold when theonline retailer acts as the leader and the traditional retailer acts as the follower, in comparison to when both the retailers move simultaneously to set their respective prices:

a) Prices of both the retailers increase.
b) While offline sales increase, online sales decrease.
c) Profits of both the retailers increase.
d) Total offline and online sales decrease.

Proposition (15) shows the relationships between prices, sales volumes and profits of each of the retailers under simultaneous and sequential moves.

Proposition 15: Let the superscript 'Sim' denote the game when the retailers make simultaneous moves. Also, let the superscripts 'Seq (TR = L)' and 'Seq (OR = L)' denote the games when the traditional retailer (TR) is the leader (L) and when the online retailer (OR) is the leader (L) under sequential moves, respectively. Then the following will hold:

a) $p_1^{Seq(TR=L)} > p_1^{Seq(OR=L)} > p_1^{Sim}$
b) $q_1^{Seq(OR=L)} > q_1^{Sim} > q_1^{Seq(TR=L)}$
c) $\Pi_1^{Seq(OR=L)} > \Pi_1^{Seq(TR=L)} > \Pi_1^{Sim}$
d) $p_2^{Seq(OR=L)} > p_2^{Seq(TR=L)} > p_2^{Sim}$
e) $q_2^{Seq(TR=L)} > q_2^{Sim} > q_2^{Seq(OR=L)}$

f) $\Pi_2^{Seq(TR=L)} > \Pi_2^{Seq(OR=L)} > \Pi_2^{Sim}$

g) $(q_1 + q_2)^{Sim} > (q_1 + q_2)^{Seq(OR=L)} > (q_1 + q_2)^{Seq(TR=L)}$

The above are standard results for upward sloping reaction curves (See, for example, Gal-Or, 1985).

4. Summary of results and managerial implications

The important results of this research that provide significant managerial insights are the following:

a) Showrooming hurts the traditional retailer and benefits the online retailer by decreasing the sales volume and profit for the former and increasing the same for the latter.
b) The overall market demand, including offline and online sales, increases under showrooming.
c) The traditional retailer is better off, and the online retailer is worse off, when the traditional retailer adopts a counter-strategy to mitigate the ill effects of showrooming.
d) The combined offline and online sales decrease post adoption of a strategy by the traditional retailer to counter showrooming.
e) When the traditional retailer makes an online entry, its offline sales decrease, but its total offline and online sales increase.
f) When the traditional retailer makes an online entry, total online sales and the online retail price increase.
g) Both the traditional and online retailers achieve higher sales volumes in sequential moves when the other retailer acts as the leader than in the simultaneous move.
h) Both the traditional and online retailers make higher profits in sequential moves than in the simultaneous move and the higher profit made by a retailer in sequential moves is when the other retailer acts as the leader.
i) The combined offline and online sales decrease in sequential moves than in the simultaneous move and the overall market demand is the lowest when the traditional retailer acts as the leader.

The major learning for managers of traditional stores from this research is that instead of taking defensive strategies to counter showrooming such as price matching, charging a fee for showrooming, not allowing mobile devices or disabling wi-fi and internet in store, which are short-term and would drive shoppers away from stores, they should accept showrooming as a natural phenomenon and leverage the opportunity to reap benefits for their stores. As shown in the paper, improved in-store shopping experience has the potential to not only convert some confirmed showroomers into in-store buyers, but also increase the overall market potential of traditional stores. By focusing more on pre- and post-sale value-added services, traditional stores may differentiate themselves from online stores and provide shoppers with more value-for-time and value-for-money. They should focus more on services than on products, i.e. 'servicization' of products, and sell a complete package of which products are only a part. The other strategy discussed in this paper is the traditional retailer's online entry, i.e. omnichannel retailing, which has also been found effective. When the traditional retailer sets up an online store, besides its physical store, showroomers have an option to check products in the physical store and buy from the traditional retailer's online store, which although brings down offline sales, boosts the combined offlineand online sales. The various pro-active strategies that managers of traditional stores may adopt to mitigate the adverse impact of showrooming have already been mentioned in detail in Section 1.1. The essence of the findings of this paper points to the fact that traditional stores will be better off if they embrace showrooming as an unavoidable phenomenon and adopt innovative strategies to mitigate the illeffects of showrooming rather than taking a defensive approach to counter it.

5. Conclusions and directions for future research

In this paper, we have developed innovative economic models for price-competition between a traditional and an online retailer under customer showrooming behaviour. We have shown that showrooming hurts the traditional retailer and benefits the online retailer. However, the overall market demand, including offline and online sales, increases under showrooming. We have also considered two strategies – effort/investment made and online entry by the traditional retailer – to counter showrooming, and observed that while the strategies benefit the traditional retailer and hurt the online retailer, the overall market demand declines. In particular, when the traditional retailer makes an online entry, although its physical sales decrease, its total sales increase, and also, although the sales of the online retailer decrease, the online market expands and the online retail price increases.We have developed economic models under both simultaneous and sequential moves made by the retailers. It has been observed that both the retailers make higher profits under sequential moves than in the simultaneous move. However, the overall market demand is lower under sequential moves than in the simultaneous move. We comment that the traditional retailer is better off if it accepts showrooming as an inevitable phenomenon and adopts innovative strategies to mitigate its negative impacts, rather than taking short-term, defensive strategies to counter it. Almost all the proofs (except one), shown in this paper, hold for the entire ranges of the parameter values, thereby making the results robust and not dependent on specific parameter ranges.

In this paper, we have considered showrooming. One possible direction for future research is to consider the reverse of showrooming, i.e. webrooming where shoppers search for product information and compare prices on the internet and then visit and buy from a physical store. A related extension could be cross-channel free riding, i.e. simultaneous existence of showrooming and webrooming (Chen et al., 2018). A more general research direction would be to consider omnichannel retailing that provides shoppers with a seamless shopping experience through multiple channels such as physical and online stores, mobile devices, social media, desktops, televisions, telephones, catalogues and so on. In omnichannel retailing, shoppers may search for product information on one channel, experience products on a different channel, place purchase orders on another channel, pick up the orders from yet another channel or get them delivered to home. The implication for traditional retailers in omnichannel retailing is that they can fulfil online orders either from their distribution centres or from their physical stores. The implication for online retailers in omnichannel retailing is that they can open showrooms where shoppers may experience or try on products before purchasing them online. There has been some recent research on webrooming and omnichannel retailing (See, for example, Jing, 2018 and Zhang and Zhang, 2020 for webrooming, and Verhoef et al.,2015;Gao and Su, 2017; Bell etal., 2018; Chen et al., 2018;von Briel, 2018; Wiener et al., 2018 and Gupta et al., 2019 for omnichannel retailing). However, we feel there are still ample opportunities for research in these areas. Finally, in this paper, we have not considered product returns. Generally, product returns are higher in online retail than in offline retail. The effect of omnichannel retailing on product returns (See, for example, He et al., 2020) could also be an interesting direction for future research.

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Appendix

Proof of Proposition 1:

a) Suppose the inequality holds. Then

$$p_1 > p_2 \quad \Rightarrow \quad \frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma} > \frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)}$$
$$\Rightarrow \quad \theta[\beta(\alpha+s)+2(1-s)] > 2(\alpha+s)+\gamma(1-s) \quad (\because 4-\beta\gamma>0)$$

Since $\theta>1$, it would suffice to show $\beta(\alpha+s)+2(1-s) > 2(\alpha+s)+\gamma(1-s)$

This follows $(2-\gamma)(1-s) > (2-\beta)(\alpha+s)$. Since by assumption $1-s>\alpha+s$ and $2-\gamma > 2-\beta \ (\because \beta>\gamma)$, the inequality holds[3] and $p_1 > p_2$.

b) Suppose the inequality holds. Then

$$q_1 > q_2 \quad \Rightarrow \quad \frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma} > \frac{2(\alpha+s)+\gamma(1-s)}{4-\beta\gamma}$$
$$\Rightarrow \quad \beta(\alpha+s)+2(1-s) > 2(\alpha+s)+\gamma(1-s) \quad (\because 4-\beta\gamma>0)$$

As shown in part (a), the inequality holds and $q_1 > q_2$.

c) Suppose the inequality holds. Then

$$\Pi_1 > \Pi_2 \quad \Rightarrow \quad p_1 q_1 > p_2 q_2 \quad \Rightarrow \quad q_1^2 > \frac{q_2^2}{\theta} \quad \Rightarrow \quad q_1 > \frac{q_2}{\sqrt{\theta}}$$

Since in part (b), it is already shown $q_1 > q_2$, it follows $q_1 > q_2 > \frac{q_2}{\sqrt{\theta}} \quad (\because \theta > 1)$.

Therefore, the inequality holds and $\Pi_1 > \Pi_2$.

Proof of Corollary 1:

a) Suppose the inequality holds. Then

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[3] The logic for the proof of this and subsequent similar inequalities is as follows: If x_i and y_i $(i=1,2)$ are mathematical expressions such that $x_1>0$ and $y_1>y_2>0$, then $x_1>x_2 \Rightarrow x_1y_1>x_2y_2$.

$$p_1 > p_2 \quad \Rightarrow \quad \frac{2+\alpha\beta}{4-\beta\gamma} > \frac{2\alpha+\gamma}{\theta(4-\beta\gamma)}$$

$$\Rightarrow \quad \theta(2+\alpha\beta) > 2\alpha+\gamma \quad (\because 4-\beta\gamma > 0)$$

Since $\theta > 1$, it would suffice to show $2+\alpha\beta > 2\alpha+\gamma$

This follows $2-\gamma > \alpha(2-\beta)$. Since $\beta > \gamma$ and $\alpha < 1$, the inequality holds and $p_1 > p_2$.

b) Suppose the inequality holds. Then

$$q_1 > q_2 \quad \Rightarrow \quad \frac{2+\alpha\beta}{4-\beta\gamma} > \frac{2\alpha+\gamma}{4-\beta\gamma}$$

$$\Rightarrow \quad 2+\alpha\beta > 2\alpha+\gamma \quad (\because 4-\beta\gamma > 0)$$

As shown in part (a), the inequality holds and $q_1 > q_2$.

c) As shown in part (c) of Proposition (1), the inequality holds and $\Pi_1 > \Pi_2$.

Proof of Proposition 2:

Eq. (3) can be rewritten as follows:

$$p_1 = \frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma} = \frac{2+\alpha\beta-(2-\beta)s}{4-\beta\gamma}$$

Since $\beta < 1$, p_1 decreases with s.

Since from Eq. (5), $q_1 = p_1$, q_1 also decreases with s.

Also, since $\Pi_1 = p_1 q_1$, Π_1 decreases with s.

Eq. (4) can be rewritten as follows:

$$p_2 = \frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)} = \frac{2\alpha+\gamma+(2-\gamma)s}{\theta(4-\beta\gamma)}$$

Since $\gamma < 1$, p_2 increases with s.

Since from Eq. (6), $q_2 = \theta p_2$, q_2 also increases with s.

Also, since $\Pi_2 = p_2 q_2$, Π_2 increases with s.

Proof of Proposition 3:

From Eqs. (5) and (6), we can write

$$q_1 + q_2 = \frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma} + \frac{2(\alpha+s)+\gamma(1-s)}{4-\beta\gamma} = \frac{2+\alpha\beta+2\alpha+\gamma+(\beta-\gamma)s}{4-\beta\gamma}$$

Since $\beta>\gamma$, q_1+q_2 increases with s.

Proof of Proposition 4:

It is clear from Eqs. (3) – (8) that prices, sales volumes and profits of both the retailers increase with the parameters, α, β and γ. From Eqs. (3), (5) and (7), it may be seen that the price, sales volume and profit of the traditional retailer remain unaffected with change in the parameter, θ. Eqs. (4), (6) and (8) show that for the online retailer, while the sales volume remains unaffected, the price and profit decrease with the parameter, θ.

Proof of Proposition 5:

a) Suppose the inequality holds. Then

$$p_1 > p_2 \quad \Rightarrow \quad \frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} > \frac{(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)}{\theta[2(4-\beta\gamma)-s^2(2-\beta)]}$$

$$\Rightarrow \quad 2\theta[\beta(\alpha+s)+2(1-s)] > (4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)$$

Since $\theta > 1$, it would suffice to show $2[\beta(\alpha+s)+2(1-s)] > (4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)$

This follows $(4-2\gamma+s^2)(1-s) > (4-2\beta-s^2)(\alpha+s)$. Since by assumption $1-s > \alpha+s$, it would suffice to show $4-2\gamma+s^2 > 4-2\beta-s^2$ or $2(\beta-\gamma+s^2) > 0$, which is true since $\beta > \gamma$. Hence the inequality holds and $p_1 > p_2$.

b) Suppose the inequality holds. Then

$$q_1 > q_2 \quad \Rightarrow \quad \frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} > \frac{(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)}{2(4-\beta\gamma)-s^2(2-\beta)}$$
$$\Rightarrow \quad 2[\beta(\alpha+s)+2(1-s)] > (4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)$$

As shown in part (a), the inequality holds and $q_1 > q_2$.

c) Suppose the inequality holds. Then

$$\Pi_1 > \Pi_2 \quad \Rightarrow \quad \frac{(4-s^2)[\beta(\alpha+s)+2(1-s)]^2}{[2(4-\beta\gamma)-s^2(2-\beta)]^2} > \frac{[(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)]^2}{\theta[2(4-\beta\gamma)-s^2(2-\beta)]^2}$$
$$\Rightarrow \theta\left[(4-s^2)\{\beta(\alpha+s)+2(1-s)\}^2\right] > [(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)]^2$$

Since $\theta > 1$, it would suffice to show $(4-s^2)[\beta(\alpha+s)+2(1-s)]^2 > [(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)]^2$

Suppose the above inequality holds. Then

$$(4-s^2)[\beta(\alpha+s)+2(1-s)]^2 > [(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)]^2$$
$$\Rightarrow$$
$$(4-s^2)\left[\beta^2(\alpha+s)^2+4(1-s)^2+4\beta(\alpha+s)(1-s)\right] >$$
$$(4-s^2)^2(\alpha+s)^2+(2\gamma-s^2)^2(1-s)^2+2(4-s^2)(2\gamma-s^2)(\alpha+s)(1-s)$$
$$\Rightarrow$$
$$\left[4(4-s^2)-(2\gamma-s^2)^2\right](1-s)^2+2(4-s^2)\left[2(\beta-\gamma)+s^2\right](\alpha+s)(1-s) >$$
$$(4-s^2)(4-s^2-\beta^2)(\alpha+s)^2$$

Now since $s < \frac{1-\alpha}{2}$, $\alpha < 1$ and $\beta > \gamma$, the coefficient of $(\alpha+s)(1-s)$ in the left hand side of the above expression is positive and $2(4-s^2)\left[2(\beta-\gamma)+s^2\right](\alpha+s)(1-s) > 0$. Therefore, it remains to show $\left[4(4-s^2)-(2\gamma-s^2)^2\right](1-s)^2 > (4-s^2)(4-s^2-\beta^2)(\alpha+s)^2$

Since $1-s > \alpha+s$, it would suffice to show $4(4-s^2)-(2\gamma-s^2)^2 > (4-s^2)(4-s^2-\beta^2)$

which implies

$$(4-s^2)(\beta^2+s^2) > (2\gamma-s^2)^2 \quad \Rightarrow \quad 4(\beta^2-\gamma^2)+s^2\left[4-(\beta^2+2s^2)+4\gamma\right] > 0$$

Since $\beta > \gamma$ and $\beta^2 + 2s^2 < 4$, the above inequality holds and hence $\Pi_1 > \Pi_2$.

Proof of Corollary 2:

a) To prove, we have to show $\frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} < 1$, with reference to Eq. (12).

Suppose the inequality holds. Then $\frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} < 1$ implies

$$2(2+\alpha\beta)-2s(2-\beta) < 2(4-\beta\gamma)-s^2(2-\beta) \quad \Rightarrow 2(2+\alpha\beta) < 2(4-\beta\gamma)+s(2-s)(2-\beta)$$

Since $0 < \beta, s < 1$, $s(2-s)(2-\beta) > 0$. Therefore, it would suffice to show $2(2+\alpha\beta) < 2(4-\beta\gamma)$ or $\beta(\alpha+\gamma) < 2$ which is true given $\alpha, \beta, \gamma < 1$. Hence the inequality holds and $p_1 < 1$.

b) From Eqs. (12) and (14), we see $\varepsilon = \frac{sp_1}{2}$. Since $s< 1$ and part (a) shows $p_1 < 1$, it follows $\varepsilon < 1$.

Proof of Proposition 6:

a) To prove, we have to show $\frac{2[\beta(\alpha+s)+2(1-s)]}{2(4-\beta\gamma)-s^2(2-\beta)} > \frac{\beta(\alpha+s)+2(1-s)}{(4-\beta\gamma)}$, with reference to Eqs. (3) and (12), which implies $s^2(2-\beta) > 0$. This is true since $s> 0$ and $\beta< 1$. Therefore, price charged by the traditional retailer increases upon investment made to counter showrooming.

b) Since $q_1 = p_1$ for both the scenarios, with reference to Eqs. (5) and (15), the proof follows from part (a).

c) To prove, we have to show $\frac{(4-s^2)[\beta(\alpha+s)+2(1-s)]^2}{[2(4-\beta\gamma)-s^2(2-\beta)]^2} > \frac{[\beta(\alpha+s)+2(1-s)]^2}{(4-\beta\gamma)^2}$, with reference to Eqs. (7) and (17), which implies

$$(4-s^2)(4-\beta\gamma)^2 > [2(4-\beta\gamma)-s^2(2-\beta)]^2$$
$$\Rightarrow \quad (4-s^2)(4-\beta\gamma)^2 > 4(4-\beta\gamma)^2 + s^4(2-\beta)^2 - 4s^2(4-\beta\gamma)(2-\beta)$$
$$\Rightarrow \quad (4-s^2)(2-\beta)^2 > \beta^2(2-\gamma)^2$$

Since β< 1, $(2-\beta)^2 > \beta^2$. Therefore, it is to be shown $(2-\gamma)^2 < 4-s^2$.

Now, by assumption $s < \frac{1-\alpha}{2}$ and $0 < \alpha < 1$. Therefore, $0 < s < 0.5$. Now, putting the upper bound on s in the inequality $(2-\gamma)^2 < 4-s^2$, we get $(2-\gamma)^2 < 4-0.5^2 = 3.75$ which implies $|2-\gamma| < \sqrt{3.75} = 1.936$, i.e. $0.064 < \gamma < 3.936$. Since $\gamma < 1$ by definition, profit of the traditional retailer increases upon investment made to counter showrooming, given $\gamma > 0.064$.

Proof of Proposition 7:

a) To prove, we have to show $\frac{(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)}{\theta[2(4-\beta\gamma)-s^2(2-\beta)]} < \frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)}$, with reference to Eqs. (4) and (13). Suppose the inequality holds. Then

$$\frac{(4-s^2)(\alpha+s)+(2\gamma-s^2)(1-s)}{\theta[2(4-\beta\gamma)-s^2(2-\beta)]} < \frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)}$$
$$\Rightarrow \quad (4-s^2)(4-\beta\gamma)(\alpha+s)+(2\gamma-s^2)(4-\beta\gamma)(1-s) <$$
$$2[2(4-\beta\gamma)-s^2(2-\beta)](\alpha+s)+\gamma[2(4-\beta\gamma)-s^2(2-\beta)](1-s)$$
$$\Rightarrow \quad -\beta(2-\gamma)(\alpha+s) < 2(2-\gamma)(1-s)$$
$$\Rightarrow \quad (2-\gamma)[\beta(\alpha+s)+2(1-s)] > 0$$

Since $\gamma < 1$, the above is true. Hence, the inequality holds and the price charged by the online retailer decreases post-investment by the traditional retailer.

b) Since $q_2 = \theta p_2$ for both the scenarios, with reference to Eqs. (6) and (16), the proof follows from part (a).

c) Since $\Pi_2 = p_2 q_2$ for both the scenarios, with reference to Eqs. (8) and (18), and both p_2 and q_2 decrease, as shown in parts (a) and (b), the profit of the online retailer also decreases post-investment by the traditional retailer.

Proof of Proposition 8:

From Eqs. (5) and (6), we get the pre-investment combined offline and online demand $= \frac{(2+\beta)(\alpha+s)+(2+\lambda)(1-s)}{4-\beta\gamma}$. Also, from Eqs. (15) and (16), we get the post-investment combined offline and online demand $= \frac{(4-s^2+2\beta)(\alpha+s)+(4-s^2+2\gamma)(1-s)}{2(4-\beta\gamma)-s^2(2-\beta)}$. It is to be shown $\frac{(4-s^2+2\beta)(\alpha+s)+(4-s^2+2\gamma)(1-s)}{2(4-\beta\gamma)-s^2(2-\beta)} < \frac{(2+\beta)(\alpha+s)+(2+\gamma)(1-s)}{4-\beta\gamma}$

Suppose the inequality holds. Then it implies

$$\left[2(2+\beta)-s^2\right](4-\beta\gamma)(\alpha+s)+\left[2(2+\gamma)-s^2\right](4-\beta\gamma)(1-s) < (2+\beta)\left[2(4-\beta\gamma)-s^2(2-\beta)\right](\alpha+s) + (2+\gamma)\left[2(4-\beta\gamma)-s^2(2-\beta)\right](1-s)$$

$$\Rightarrow$$

$$-\beta(\beta-\gamma)(\alpha+s) < 2(\beta-\gamma)(1-s)$$

Since $1-s > \alpha+s$ and $\beta > \gamma$, the inequality holds and q_1+q_2 decreases post-investment by the traditional retailer.

Proof of Proposition 9:

a) Suppose the inequality holds. Then, with reference to Eqs. (21) and (22)

$$\frac{(\beta\theta+\lambda\gamma)(\alpha+s)+2\theta(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2} > \frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}$$
$$\Rightarrow \quad (\beta\theta+\lambda\gamma)(\alpha+s)+2\theta(1-s) > 2(\alpha+s)+\gamma(1-s)$$
$$\Rightarrow \quad (2\theta-\gamma)(1-s) > (2-\beta\theta-\lambda\gamma)(\alpha+s)$$

Since $1-s>\alpha+s$, it would suffice to prove $2\theta-\gamma>2-\beta\theta-\lambda\gamma$ or $(2+\beta)\theta>2+\gamma-\lambda\gamma$. Since $\beta>\lambda$ and $\theta>1$, $(2+\beta)\theta>2+\gamma$. Hence, the inequality holds and $p_1>p_2$.

b) The inequality holds for $\lambda\geq 0.5$. However, we have to provide a general proof.

Suppose the inequality holds. Then $q_1+\lambda q_2>(1-\lambda)q_2$ implies $q_1>(1-2\lambda)q_2$ or with reference to Eqs. (23) and (24)

$$\frac{(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}>(1-2\lambda)\frac{\theta[2(\alpha+s)+\gamma(1-s)]}{\theta(4-\beta\gamma)-\lambda\gamma^2}$$
$$\Rightarrow\quad [2\theta-\lambda\gamma^2-\gamma\theta(1-2\lambda)](1-s)>[2\theta(1-2\lambda)-\beta\theta+\lambda\gamma](\alpha+s)$$

Since $1-s>\alpha+s$, we have to show $2\theta-\lambda\gamma^2-\gamma\theta(1-2\lambda)>2\theta(1-2\lambda)-\beta\theta+\lambda\gamma$ or $\theta[(\beta-\gamma)+2\lambda(2+\gamma)]>\lambda\gamma(1+\gamma)$. Since $\beta>\lambda$ and $\theta>1$, it would suffice to show $2\lambda(2+\gamma)>\lambda\gamma(1+\gamma)$ which is true since $\gamma<1$. Hence the inequality holds and $q_1+\lambda q_2>(1-\lambda)q_2$.

c) Suppose the inequality holds. Then $p_1q_1+\lambda p_2q_2>(1-\lambda)p_2q_2$ or $p_1q_1>(1-2\lambda)p_2q_2$. The inequality holds for $\lambda\geq 0.5$. To provide a general proof, from Eqs. (21) – (24), we may write

$$\frac{(\beta\theta+\lambda\gamma)(\alpha+s)+2\theta(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}\times\frac{(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}>$$
$$(1-2\lambda)\frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}\times\frac{\theta[2(\alpha+s)+\gamma(1-s)]}{\theta(4-\beta\gamma)-\lambda\gamma^2}$$
$$\Rightarrow$$
$$[(\beta\theta+\lambda\gamma)(\alpha+s)+2\theta(1-s)]\times[(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)]>$$
$$\theta(1-2\lambda)[2(\alpha+s)+\gamma(1-s)]^2$$

Now, the coefficient of $(\alpha+s)(1-s)$ in the left hand side of the inequality is $(\beta\theta+\lambda\gamma)(2\theta-\lambda\gamma^2)+2\theta(\beta\theta-\lambda\gamma)=4\beta\theta^2-\lambda\gamma^2(\beta\theta+\lambda\gamma)$. Also, the coefficient of $(\alpha+s)(1-s)$ in the right hand side of the inequality is $4\gamma\theta(1-2\lambda)$.

Suppose the inequality $4\beta\theta^2-\lambda\gamma^2(\beta\theta+\lambda\gamma)>4\gamma\theta(1-2\lambda)$ holds. Then it implies $4\theta(\beta\theta-\gamma)+\lambda\gamma[(8-\beta\gamma)\theta-\lambda\gamma^2]>0$. Since $0<\lambda,\beta,\gamma<1$, $\theta>1$ and $\beta>\gamma$, the inequality holds.

Therefore, it would suffice to take the square terms and show

$$[2\theta(2\theta-\lambda\gamma^2)-\theta\gamma^2(1-2\lambda)](1-s)^2>[4\theta(1-2\lambda)-\beta^2\theta^2+\lambda^2\gamma^2](\alpha+s)^2$$

Since $1-s>\alpha+s$, we have to show

$$2\theta(2\theta-\lambda\gamma^2)-\theta\gamma^2(1-2\lambda)>4\theta(1-2\lambda)-\beta^2\theta^2+\lambda^2\gamma^2$$
$$\Rightarrow\quad (4+\beta^2)\theta^2>(4+\gamma^2)\theta-\lambda(8\theta-\lambda\gamma^2)$$

Since $0<\lambda,\beta,\gamma<1$, $\theta>1$ and $\beta>\gamma$, the inequality holds and $\Pi_1>\Pi_2$.

Proof of Proposition 10:

a) Comparing Eqs. (3) and (21), it is straightforward to show that the price charged increases.

b) To show that the offline sales volume decreases, we have to check if $\frac{(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}<\frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma}$, with reference to Eqs. (5) and (23). Suppose the inequality holds. Then it implies

$$[(4-\beta\gamma)(\beta\theta-\lambda\gamma)-\beta\{\theta(4-\beta\gamma)-\lambda\gamma^2\}](\alpha+s)<$$
$$[2\{\theta(4-\beta\gamma)-\lambda\gamma^2\}-(4-\beta\gamma)(2\theta-\lambda\gamma^2)](1-s)$$

Since $1-s>\alpha+s$, it would suffice to show

$$(4-\beta\gamma)(\beta\theta-\lambda\gamma)-\beta\{\theta(4-\beta\gamma)-\lambda\gamma^2\}<2\{\theta(4-\beta\gamma)-\lambda\gamma^2\}-(4-\beta\gamma)(2\theta-\lambda\gamma^2)$$

which implies

$-2\lambda\gamma(2-\beta\gamma) < \lambda\gamma^2(2-\beta\gamma) \;\; \Rightarrow \;\; \lambda\gamma(2+\gamma)(2-\beta\gamma) > 0$ which is true since $\beta, \gamma < 1$. Hence, the inequality holds and the offline sales volume decreases.

To show that total sales (offline and online) increase, we have to check if

$$\frac{(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}+\lambda\frac{\theta[2(\alpha+s)+\gamma(1-s)]}{\theta(4-\beta\gamma)-\lambda\gamma^2} > \frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma},$$

with reference to Eqs. (5), (23) and (24).

Suppose the inequality holds. Then it implies

$$\frac{(\beta\theta-\lambda\gamma+2\lambda\theta)(\alpha+s)+(2\theta-\lambda\gamma^2+\lambda\gamma\theta)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2} > \frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma} \;\; \Rightarrow$$

$$(\beta\theta-\lambda\gamma+2\lambda\theta)(4-\beta\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2+\lambda\gamma\theta)(4-\beta\gamma)(1-s) >$$

$$\beta[\theta(4-\beta\gamma)-\lambda\gamma^2](\alpha+s)+2[\theta(4-\beta\gamma)-\lambda\gamma^2](1-s) \;\; \Rightarrow$$

$$[(-\lambda\gamma^2+\lambda\gamma\theta)(4-\beta\gamma)+2\lambda\gamma^2](1-s) > [-\lambda\beta\gamma^2-(-\lambda\gamma+2\lambda\theta)(4-\beta\gamma)](\alpha+s)$$

Since $1-s > \alpha+s$, it would suffice to show

$$(-\lambda\gamma^2+\lambda\gamma\theta)(4-\beta\gamma)+2\lambda\gamma^2 > -\lambda\beta\gamma^2-(-\lambda\gamma+2\lambda\theta)(4-\beta\gamma)$$

which implies $\lambda[\{\theta(2+\gamma)-\gamma(1+\gamma)\}(4-\beta\gamma)+\gamma^2(2+\beta)] > 0$. This is, however, true since $\lambda > 0$, $\theta > \gamma$ and $0 < \beta, \gamma < 1$. Therefore, the inequality holds and total sales (offline and online) increase.

c) To show that the profit increases, we have to check if, with reference to Eqs. (7) and (21) – (24), the following inequality holds:

$$\frac{(\beta\theta+\lambda\gamma)(\alpha+s)+2\theta(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}\times\frac{(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}+$$

$$\lambda\frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}\times\frac{\theta[2(\alpha+s)+\gamma(1-s)]}{\theta(4-\beta\gamma)-\lambda\gamma^2} > \frac{[\beta(\alpha+s)+2(1-s)]^2}{(4-\beta\gamma)^2}$$

Suppose the inequality holds. Then it implies

$$\frac{[(\beta\theta+\lambda\gamma)(\alpha+s)+2\theta(1-s)][(\beta\theta-\lambda\gamma)(\alpha+s)+(2\theta-\lambda\gamma^2)(1-s)]}{[\theta(4-\beta\gamma)-\lambda\gamma^2]^2}+$$
$$\lambda\theta\frac{[2(\alpha+s)+\gamma(1-s)]^2}{[\theta(4-\beta\gamma)-\lambda\gamma^2]^2}>\frac{[\beta(\alpha+s)+2(1-s)]^2}{(4-\beta\gamma)^2}$$

Left hand side of the above inequality =

$$\frac{[\beta^2\theta^2+\lambda(4\theta-\lambda\gamma^2)](\alpha+s)^2+\theta(4\theta-\lambda\gamma^2)(1-s)^2+(\beta\theta+\lambda\gamma)(4\theta-\lambda\gamma^2)(\alpha+s)(1-s)}{[\theta(4-\beta\gamma)-\lambda\gamma^2]^2}$$

Right hand side of the above inequality =

$$\frac{\beta^2(\alpha+s)^2+4(1-s)^2+4\beta(\alpha+s)(1-s)}{(4-\beta\gamma)^2}$$

Therefore, rearranging the terms on the left and right hand sides of the above inequality, we get the following:

$$\left[\theta(4\theta-\lambda\gamma^2)(4-\beta\gamma)^2-4\{\theta(4-\beta\gamma)-\lambda\gamma^2\}^2\right](1-s)^2+$$
$$\left[(\beta\theta+\lambda\gamma)(4\theta-\lambda\gamma^2)(4-\beta\gamma)^2-4\beta\{\theta(4-\beta\gamma)-\lambda\gamma^2\}^2\right](\alpha+s)(1-s)>$$
$$\left[\beta^2\{\theta(4-\beta\gamma)-\lambda\gamma^2\}^2-\{\beta^2\theta^2+\lambda(4\theta-\lambda\gamma^2)\}(4-\beta\gamma)^2\right](\alpha+s)^2$$

Now, the coefficient of $(\alpha+s)(1-s)$ on the left hand side of the above inequality =

$$(\beta\theta+\lambda\gamma)(4\theta-\lambda\gamma^2)(4-\beta\gamma)^2-4\beta\{\theta(4-\beta\gamma)-\lambda\gamma^2\}^2=$$
$$[(\beta\theta+\lambda\gamma)(4\theta-\lambda\gamma^2)-4\beta\theta^2](4-\beta\gamma)^2+8\lambda\beta\theta\gamma^2(4-\beta\gamma)-4\lambda^2\beta\gamma^4=$$
$$\lambda\gamma[\theta(4-\beta\gamma)-\lambda\gamma^2](4-\beta\gamma)^2+4\lambda\beta\gamma^2[2\theta(4-\beta\gamma)-\lambda\gamma^2]$$

Since $0<\lambda,\beta,\gamma<1$ and $\theta>1$, the coefficient of $(\alpha+s)(1-s)$ on the left hand side of the above inequality is positive. Also, since $1-s>\alpha+s$, it would suffice to show

$$\theta(4\theta-\lambda\gamma^2)(4-\beta\gamma)^2-4[\theta(4-\beta\gamma)-\lambda\gamma^2]^2>$$
$$\beta^2[\theta(4-\beta\gamma)-\lambda\gamma^2]^2-[\beta^2\theta^2+\lambda(4\theta-\lambda\gamma^2)](4-\beta\gamma)^2$$

If the above inequality holds, then

$$-\lambda\theta\gamma^2(4-\beta\gamma)^2-4\lambda^2\gamma^4+8\lambda\theta\gamma^2(4-\beta\gamma)>$$
$$\beta^2\left[\lambda^2\gamma^4-2\lambda\theta\gamma^2(4-\beta\gamma)\right]-\lambda\left(4\theta-\lambda\gamma^2\right)(4-\beta\gamma)^2 \quad \Rightarrow$$
$$\lambda\left[\left\{\theta\left(4-\gamma^2\right)-\lambda\gamma^2\right\}(4-\beta\gamma)^2+\gamma^2\left\{2\theta(4-\beta\gamma)-\lambda\gamma^2\right\}\left(4+\beta^2\right)\right]>0$$

Since $0<\lambda,\beta,\gamma<1$ and $\theta>1$, the above inequality holds and hence the profit increases.

Proof of Proposition 11:

a) Comparing Eqs. (4) and (22), it is straightforward to show that the price charged increases.

b) To show that the sales volume decreases, we have to check if $(1-\lambda)\dfrac{\theta[2(\alpha+s)+\gamma(1-s)]}{\theta(4-\beta\gamma)-\lambda\gamma^2}<\dfrac{2(\alpha+s)+\gamma(1-s)}{4-\beta\gamma}$, with reference to Eqs. (6) and (24).

   Suppose the inequality holds. Then the above inequality implies

   $$(1-\lambda)\theta(4-\beta\gamma)<\theta(4-\beta\gamma)-\lambda\gamma^2 \quad \Rightarrow \quad \lambda\left[\theta(4-\beta\gamma)-\gamma^2\right]>0$$

   Since $0<\lambda,\beta,\gamma<1$ and $\theta>1$, the above inequality holds and hence the sales volume decreases.

c) To show that the profit decreases, we have to check if $(1-\lambda)\theta\dfrac{[2(\alpha+s)+\gamma(1-s)]^2}{\left[\theta(4-\beta\gamma)-\lambda\gamma^2\right]^2}<\dfrac{[2(\alpha+s)+\gamma(1-s)]^2}{\theta(4-\beta\gamma)^2}$, with reference to Eqs. (8), (22) and (24).

   Suppose the inequality holds. Then it implies

   $$(1-\lambda)\theta^2(4-\beta\gamma)^2<\left[\theta(4-\beta\gamma)-\lambda\gamma^2\right]^2 \quad \Rightarrow \quad \lambda\left[\theta(4-\beta\gamma)\left\{\theta(4-\beta\gamma)-2\gamma^2\right\}+\lambda\gamma^4\right]>0$$

   Since $0<\lambda,\beta,\gamma<1$ and $\theta>1$, the above inequality holds and hence the profit decreases.

Proof of Proposition 12:

a) The proof is straightforward by comparing Eqs. (6) and (24).

b) To prove, we have to show, with reference to Equations (5), (6), (23) and (24)

$$\frac{(\beta\theta-\lambda\gamma+2\theta)(\alpha+s)+(2\theta-\lambda\gamma^2+\gamma\theta)(1-s)}{\theta(4-\beta\gamma)-\lambda\gamma^2}<\frac{(2+\beta)(\alpha+s)+(2+\gamma)(1-s)}{4-\beta\gamma}$$

Suppose the inequality holds. Then it implies

$$\begin{aligned}&[(2+\beta)\theta-\lambda\gamma](4-\beta\gamma)(\alpha+s)+[(2+\gamma)\theta-\lambda\gamma^2](4-\beta\gamma)(1-s)<\\&[\theta(4-\beta\gamma)-\lambda\gamma^2][(2+\beta)(\alpha+s)+(2+\gamma)(1-s)] \quad \Rightarrow\\&\lambda\gamma[2\gamma(1+\beta)-4](\alpha+s)<\lambda\gamma^2[2-\gamma(1+\beta)](1-s)\end{aligned}$$

Since $1-s>\alpha+s$, it would suffice to show $\lambda\gamma[2\gamma(1+\beta)-4]<\lambda\gamma^2[2-\gamma(1+\beta)]$ which implies $\lambda\gamma[4-2\beta\gamma-\gamma^2(1+\beta)]>0$.

Since $0<\lambda,\beta,\gamma<1$, the above inequality holds and hence it proves that total offline and online sales decrease.

Proof of Proposition 13:

a) By comparing Eqs. (3) and (25), it is straightforward to show that the price charged by the traditional retailer increases. To show that the price charged by the online retailer also increases, we have to prove the following, with reference to Eqs. (4) and (26):

$$\frac{(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)}{4\theta(2-\beta\gamma)}>\frac{2(\alpha+s)+\gamma(1-s)}{\theta(4-\beta\gamma)}$$

Supposing the above inequality is true, it can be rearranged to write the following:

$$[2\gamma(4-\beta\gamma)-4\gamma(2-\beta\gamma)](1-s)>[8(2-\beta\gamma)-(4-\beta\gamma)^2](\alpha+s)$$

Since $1-s>\alpha+s$, it would suffice to show $2\gamma(4-\beta\gamma)-4\gamma(2-\beta\gamma)>8(2-\beta\gamma)-(4-\beta\gamma)^2$ which implies $2\beta\gamma^2>-\beta^2\gamma^2$

However, the above is true, and hence it shows that the price charged by the online retailer increases.

b) By comparing Eqs. (5) and (27), it can be easily shown that offline sales decrease. To show that online sales increase, we have to prove the following, with reference to Eqs. (6) and (28):

$$\frac{(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)}{4(2-\beta\gamma)} > \frac{2(\alpha+s)+\gamma(1-s)}{4-\beta\gamma}$$

However, it is already shown in part (a) that the above inequality holds. Hence, it follows that online sales increase.

c) A comparison of Eqs. (7) and (29) shows that the profit of the traditional retailer increases. To show that the profit of the online retailer also increases, the following must hold, with reference to Eqs. (8) and (30):

$$\frac{[(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)]^2}{\theta[4(2-\beta\gamma)]^2} > \frac{[2(\alpha+s)+\gamma(1-s)]^2}{\theta(4-\beta\gamma)^2}$$

The proof follows part (a). Also, since the price charged by the online retailer and online sales increase, it is intuitively true that the profit of the online retailer increases.

d) The expression for total offline and online sales is the following, with reference to Eqs. (27) and (28):

$$\begin{aligned} q_1+q_2 &= \frac{\beta(\alpha+s)+2(1-s)}{4} + \frac{(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)}{4(2-\beta\gamma)} \\ &= \frac{[\beta(2-\beta\gamma)+4-\beta\gamma](\alpha+s)+[2(2-\beta\gamma)+2\gamma](1-s)}{4(2-\beta\gamma)} \end{aligned}$$

To show that total offline and online sales decrease, we have to show the following, with reference to Eqs. (5) and (6):

$$\frac{[\beta(2-\beta\gamma)+4-\beta\gamma](\alpha+s)+[2(2-\beta\gamma)+2\gamma](1-s)}{4(2-\beta\gamma)} < \frac{(2+\beta)(\alpha+s)+(2+\gamma)(1-s)}{4-\beta\gamma}$$

Rearranging the above inequality, we can write the following:

$$[\{\beta(2-\beta\gamma)+4-\beta\gamma\}(4-\beta\gamma)-4(2-\beta\gamma)(2+\beta)](\alpha+s)<$$
$$[4(2-\beta\gamma)(2+\gamma)-\{2(2-\beta\gamma)+2\gamma\}(4-\beta\gamma)](1-s)$$

which upon simplification gives

$$-\beta^2\gamma(2-\gamma-\beta\gamma)(\alpha+s)<2\beta\gamma(2-\gamma-\beta\gamma)(1-s)$$

Now, since $0<\beta,\gamma<1$, $2-\gamma-\beta\gamma>0$. Also, $1-s>\alpha+s$. Therefore, the above inequality holds and it follows that total offline and online sales decrease.

Proof of Proposition 14:

a) While it is straightforward to show that the price charged by the online retailer increases by comparing Eqs. (4) and (31), to show that the price charged by the traditional retailer also increases, we have to show that the following inequality holds, with reference to Eqs. (3) and (32):

$$\frac{2\beta(\alpha+s)+(4-\beta\gamma)(1-s)}{4(2-\beta\gamma)}>\frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma}$$

Suppose the above inequality holds. Then rearranging the terms, we may write

$$[(4-\beta\gamma)^2-8(2-\beta\gamma)](1-s)>[4\beta(2-\beta\gamma)-2\beta(4-\beta\gamma)](\alpha+s)$$

which upon simplification gives

$$\beta^2\gamma^2(1-s)>-2\beta^2\gamma(\alpha+s)$$

However, since $1-s>\alpha+s$, the above inequality holds, and hence the price charged by the traditional retailer increases.

b) While it is easy to show that online sales decrease by comparing Eqs. (6) and (34), to show that offline sales increase, we have to show the following, with reference to Eqs. (5) and (33):

$$\frac{2\beta(\alpha+s)+(4-\beta\gamma)(1-s)}{4(2-\beta\gamma)}>\frac{\beta(\alpha+s)+2(1-s)}{4-\beta\gamma}$$

However, in part (a), it is shown that the above inequality holds. Hence, offline sales increase.

c) Since both the price charged by the traditional retailer and offline sales increase, the profit of the traditional retailer increases. To show that the profit of the online retailer also increases, we have to compare Eqs. (8) and (36). Since the denominator of Eq. (8) is larger than the denominator of Eq. (36), while their numerators are the same, it is intuitive that the profit of the online retailer increases.

d) Total offline and online sales can be obtained from Eqs. (33) and (34) as follows:

$$q_1 + q_2 = \frac{2\beta(\alpha+s)+(4-\beta\gamma)(1-s)}{4(2-\beta\gamma)} + \frac{2(\alpha+s)+\gamma(1-s)}{4}$$
$$= \frac{[2\beta+2(2-\beta\gamma)](\alpha+s)+[4-\beta\gamma+\gamma(2-\beta\gamma)](1-s)}{4(2-\beta\gamma)}$$

If total offline and online sales decrease, the following inequality must hold, with reference to Eqs. (5) and (6):

$$\frac{[2\beta+2(2-\beta\gamma)](\alpha+s)+[4-\beta\gamma+\gamma(2-\beta\gamma)](1-s)}{4(2-\beta\gamma)} < \frac{(2+\beta)(\alpha+s)+(2+\gamma)(1-s)}{4-\beta\gamma}$$

Supposing the above inequality holds, rearranging the terms, we may write

$$[\{2\beta+2(2-\beta\gamma)\}(4-\beta\gamma)-4(2-\beta\gamma)(2+\beta)](\alpha+s) <$$
$$[4(2-\beta\gamma)(2+\gamma)-\{4-\beta\gamma+\gamma(2-\beta\gamma)\}(4-\beta\gamma)](1-s)$$

which upon simplification gives

$$-2\beta\gamma(2-\beta-\beta\gamma)(\alpha+s) < \beta\gamma^2(2-\beta-\beta\gamma)(1-s)$$

Now, since $0<\beta,\gamma<1$, $2-\beta-\beta\gamma>0$. Also, $1-s>\alpha+s$. Therefore, the above inequality holds and it follows that total offline and online sales decrease.

Proof of Proposition 15:

a) Since in Propositions (13) and (14), it is shown that the price of the traditional retailer increases under sequential moves, it is evident that p_1^{Sim} will be the lowest. To show $p_1^{Seq(TR=L)} > p_1^{Seq(OR=L)}$, we have to prove the following inequality, with reference to Eqs. (25) and (32):

$$\frac{\beta(\alpha+s)+2(1-s)}{2(2-\beta\gamma)} > \frac{2\beta(\alpha+s)+(4-\beta\gamma)(1-s)}{4(2-\beta\gamma)}$$

It can be easily shown that the above inequality holds by rearranging the terms. Hence, the proposition holds.

b) While Proposition (13) shows that offline sales decrease, Proposition (14) shows that offline sales increase. Hence, the proof is straightforward.

c) Propositions (13) and (14) show that the profit of the traditional retailer increases under sequential moves. Therefore, it is evident that Π_1^{Sim} will be the lowest. To show $\Pi_1^{Seq(OR=L)} > \Pi_1^{Seq(TR=L)}$, we have to prove the following inequality, with reference to Eqs. (29) and (35):

$$\frac{[2\beta(\alpha+s)+(4-\beta\gamma)(1-s)]^2}{[4(2-\beta\gamma)]^2} > \frac{[\beta(\alpha+s)+2(1-s)]^2}{8(2-\beta\gamma)}$$

Supposing the inequality holds, the following is obtained upon simplification:

$\beta^2\gamma^2(1-s)^2 + 4\beta^2\gamma(\alpha+s)(1-s) + 2\beta^3\gamma(\alpha+s)^2 > 0$ which is true.

Therefore, the inequality holds, and hence the proof.

d) Since Propositions (13) and (14) show that the price of the online retailer increases under sequential moves, it is clear that p_2^{Sim} will be the lowest. To prove $p_2^{Seq(OR=L)} > p_2^{Seq(TR=L)}$, we have to show that the following inequality holds, with reference to Eqs. (26) and (31):

$$\frac{2(\alpha+s)+\gamma(1-s)}{2\theta(2-\beta\gamma)} > \frac{(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)}{4\theta(2-\beta\gamma)}$$

It can be easily shown that the above inequality holds, and hence the proof.

e) Since Proposition (13) shows that online sales increase and Proposition (14) shows that online sales decrease, the proof is straightforward.

f) Propositions (13) and (14) show that the profit of the online retailer increases under sequential moves. Hence, Π_2^{Sim} will be the lowest. To show $\Pi_2^{Seq(TR=L)} > \Pi_2^{Seq(OR=L)}$, we have to prove the following inequality, with reference to Eqs. (30) and (36):

$$\frac{[(4-\beta\gamma)(\alpha+s)+2\gamma(1-s)]^2}{\theta[4(2-\beta\gamma)]^2} > \frac{[2(\alpha+s)+\gamma(1-s)]^2}{8\theta(2-\beta\gamma)}$$

Supposing the above inequality holds, it gives the following upon simplification:

$2\beta\gamma^3(1-s)^2 + 4\beta\gamma^2(\alpha+s)(1-s) + \beta^2\gamma^2(\alpha+s)^2 > 0$ which is true.

Therefore, the inequality holds, and hence the proof.

g) Since Propositions (13) and (14) show that total offline and online sales decrease under sequential moves, it is clear that $(q_1+q_2)^{Sim}$ will be the highest. To prove $(q_1+q_2)^{Seq(OR=L)} > (q_1+q_2)^{Seq(TR=L)}$, we have to compare the expressions for total offline and online sales derived in part (d) of Propositions (13) and (14), and show the following:

$$\frac{[2\beta+2(2-\beta\gamma)](\alpha+s)+[4-\beta\gamma+\gamma(2-\beta\gamma)](1-s)}{4(2-\beta\gamma)} >$$
$$\frac{[\beta(2-\beta\gamma)+4-\beta\gamma](\alpha+s)+[2(2-\beta\gamma)+2\gamma](1-s)}{4(2-\beta\gamma)}$$

which upon simplification gives

$$\beta\gamma(1-\gamma)(1-s) > \beta\gamma(1-\beta)(\alpha+s)$$

However, the above inequality is true given $0<\beta,\gamma<1$, $\beta>\gamma$ and $1-s>\alpha+s$. Hence the proof.