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ABSTRACT

In the recent article, Darwish and Odah [1] develop a scheme that allows for identical replenishment cycles for all the retailers, in the context of a single vendor supplying a group of retailers under VMI partnership. This paper proposes an alternative replenishment scheme allowing for different replenishment cycles for each retailer. An example has been shown to illustrate the cost savings under the proposed model.

KEYWORDS

(D) Supply chain management, Multi-Item Joint Replenishment, Vendor Managed Inventory, Inventory Management

1. INTRODUCTION

The benefits of Vendor managed Inventory in terms of reduced cost and improved service are clearly stated by Waller, Johnson and Davis[5]. Since then many authors have tried to come up with single vendor multi retailer replenishment schemes. Viswanathan and Piplani [4] propose a scheme where the replenishment cycle for supplier is fixed and retailers may order at those intervals only. Zhanga et al. [6] present a model wherein the vendor has constant production cycle and retailers can have different ordering cycles. Hariga et al. [2] take unequal reorder intervals for vendor and retailers can receive more than one shipment in each vendor cycle. Recently in their paper entitled "Vendor managed inventory model for single-vendor multi-retailer supply chains", Darwish & Odah [1] present a mathematical model for retailer replenishment and provide an optimal solution for the same. In developing the model, they consider a policy of replenishing all retailers at the same time. They assume that each retailer gets replenished every T periods and the supplier sets up every nT period, where n is an integer greater than equal to 1. In this note, we show that changing the replenishment policy may lead to cost savings. Specifically, we assume that the retailers are not replenished simultaneously, instead, a retailer i is replenished every m_iT period, where, T is the base replenishment cycle, and

m_i is an integer. The solution methodology ensures that at least one retailer is replenished every T period and the others may be replenished every T, $2T$, $3T$ etc. periods. We further assume that the supplier sets up every T periods and unlike Darwish & Odah [1] supplier does not carry any inventory. Thus n is assumed to be 1 in our case. This replenishment policy does not force every retailer to get replenished every cycle. Our proposed policy can be seen as a generalization of the policy adopted by Darwish & Odah [1], to the extent that, for $m_i = 1$, for all i, all retailers would get replenished every T period. Our proposed policy can be found in Silver [3], who has used it in the context of joint replenishment of items.

2. MATHEMATICAL MODEL

The relevant factors involved in the model are given below:

T Base replenishment cycle

m_i Integer variable for i^{th} retailer

A Supplier setup cost ($)

h_s Inventory carrying cost of supplier ($/unit/unit time)

a_i Order cost for i^{th} retailer($)

h_i Inventory carrying cost of i^{th} retailer($/unit/unit time)

D_i Annual demand for the i^{th} retailer.

D Annual demand for supplier. ($\sum D_i$)

U_i Upper limit set by the retailer.

P_i Penalty for i^{th} retailer for exceeding the upper limit U_i ($/unit)

X_i Quantity by which the upper limit is exceeded

The Total Relevant Cost (TRC) can be expressed as follows:

Minimize :-

$$\frac{A}{T}+\sum_{i=1}^{n}\frac{a_i}{m_iT}+\frac{1}{2}\sum_{i=1}^{n}D_im_iTh_i+\frac{1}{2}\sum_{i=1}^{n}\frac{P_i}{m_iTD_i}X_i^2$$

Subject to $D_im_iT - U_i <= X_i$

$X_i \geq 0$, $m_i \in I$ (Set of all integers)

The factor that controls the cycle time of a retailer is a_i/D_ih_i. The higher this ratio, the higher is the value of m_i, and vice-versa. We identify the retailer with the lowest a_i/D_ih_i ratio and set the value of mi of the retailer equal to 1. This ensures that at least one retailer is replenished every cycle. We have used the software 'Lingo 13.0' to solve for values of other m_i's and T. The replenishment quantities for retailers are calculated as $Q_i = D_i\ m_i\ T$.

The example considered by Darwish and Odah [1] is such that the (a_i/D_ih_i) ratio for all the retailers are very close, suggesting that the replenishment cycle for the items should be similar and in turn all the m_i's come out to be 1. This is shown in the table given below:

Retailer	D_i	a_i	h_i	a_i/D_ih_i
R1	2300	45	7.5	.0026
R2	1200	30	8.5	.0029
R3	3000	60	7	.0028
R4	1800	35	8	.0024
R5	800	25	9	.0034
Supplier	D	A	h_s	A/Dh_s
	$\sum D_i = 9100$	300	.75	.044

As mentioned, the retailer with the lowest value of a_i/D_ih_i (say R_{lowest}) will get replenished every cycle. In a similar way the supplier's A/Dh_s ratio controls its replenishment cycle. A lower value of A/Dh_s ratio should lead to a lower value of n. Moreover, n will take a value of more than 1, only in the case where A/Dh_s ratio of supplier is relatively much higher as compared to a_i/D_ih_i ratio of R_{lowest}. Such a case arises in the context of single supplier – single retailer situation, as the setup cost of the supplier is relatively higher compared to the ordering cost of the retailer.

However in the context of multiple retailers, the supplier's A/Dh_s ratio is either close to or less than a_i/D_ih_i ratio of R_{lowest} (since D is now the cumulative demand of all retailers taken together) and hence the assumption of n =1 is justifiable. In the example considered by Darwish and Odah [1] A/Dh_s has been taken as 0.044, which is more than a_i/D_ih_i of any of the retailer suggesting lower frequency of setups and hence supporting n greater than 1.

The example and the resulting cost savings are shown in the next section.

3. EXAMPLE

Consider a case involving four retailers and one supplier with the input data shown in Table 1:

Table 1: Data corresponding to the case of Multiple Retailers with different values of a_i/D_ih_i

RETAILER	ANNUAL DEMAND	a_i	h_i	a_i/D_ih_i	EOQ	Upper limit (U_i)	Penalty P_i (per extra unit)
R1	400	40	.8	.125	200	250	2
R2	1000	45	.8	.0563	336	400	1.5
R3	8000	50	1	.0063	895	1050	2
R4	18000	60	1	.0033	1470	1750	1
Supplier	*D*	*A*	h_s	A/Dh_s	-	-	-
	$\sum D_i = 27400$	120	.75	.0058	-	-	-

The value for *m4* (Retailer 4 has lowest value for a_i/D_ih_i) is set to 1 and then the other values are obtained using LINGO 13.0, as shown in Table 2.

Table 2: Results corresponding to the proposed model.

Retailer i get replenished for every m_iT period with T=.12=6.24 Weeks			
RETAILERS	m_i	REPLENISHMENT QTY	TRC
R1	4	192	3944.64

R2	3	360	
R3	1	960	
R4	1	2160	

For the same example, using the model and algorithm suggested by Darwish & Odah [1], the value of n came out to be 1; the other results are given in the table below:

Table 3: Results corresponding to the model by Darwish & Odah [1]

All the retailers gets replenished at the same time		
RETAILERS	REPLENISHMENT QUANTITIES	TRC
R1	55.6	4267.38
R2	139	
R3	1112	
R4	2503	

We can clearly see the reduction in the total relevant cost.

4. CONCLUSION

The factor a_i/D_ih_i plays a important role in realizing the benefit of different replenishment cycle for retailers. We show that allowing different replenishment cycles for the retailers results in the reduction of total relevant cost as compared to the scenario where all the retailers are restricted to have the same replenishment cycle. In the context where the retailers are heterogeneous with respect to the factors such as demand, ordering cost, and inventory carrying cost the proposed model will perform better as compared to the model suggested by Darwish & Odah [1].

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