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CONDORCET JURY THEOREM IN A SPATIAL MODEL OF ELECTIONS*

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Abstract. We introduce voter uncertainty to the unidimensional spatial model of elections. Strategic voters choose between the status quo and a proposed reform, and there is uncertainty about the location of the reform on the policy space. If each possible location of the reform is on the same side of the status quo, then equilibria are full information equivalent. If there is uncertainty about whether the reform lies to the left or right of the status quo, then we have multiple equilibria. While some equilibria are full information equivalent, in the others the status quo always wins. This work stresses the importance of voter co-ordination and difficulty of aggregating information when there is uncertainty about the winners and losers from a reform. Our finding provides a novel explanation for status quo bias in referenda.

1. Introduction

Elections aggregate both information and preferences. However, most models of elections consider variation across voters either in preference or in information across voters but not in both dimensions. In particular, there is a long tradition dating back to Condorcet (1786) showing that information is efficiently aggregated by elections in absence of (or with very limited) preference heterogeneity (Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997), Myerson (1998), Wit (1998), Duggan and Martinelli (2001), Meirowitz (2002)). In this paper, we examine the extent to which elections can aggregate information in presence of substantial preference variation among voters.

We demonstrate that there are multiple equilibria when there is *distributional uncertainty* in the electorate, and in some of these equilibria the information fails to aggregate. By distributional uncertainty we mean uncertainty regarding the winners and losers from a policy change. More specifically, we think of a situation where there are two groups of voters such that one will be better off and another worse off consequent to a change in policy, but there is uncertainty regarding which one will gain at the expense of the other.

In order to formally develop this idea, we consider a simple Downsian model of elections. Voters have symmetric single peaked preference with "ideal points" varying on a unidimensional policy space $[-1, 1]$. There are two policies $\mathcal{P}$ and $\mathcal{Q}$ in contention. Uncertainty is captured in the simplest way possible: the location of $\mathcal{Q}$ is known to be 0 whereas the location of $\mathcal{P}$ is represented by an uncertain state variable S which can take values L or R, with $-1 < L < R < 1$. We shall use the following metaphor: $\mathcal{Q}$ is the status quo while $\mathcal{P}$ is a proposed reform with uncertain consequences. An alternative interpretation is that Q is the incumbent while $\mathcal{P}$ is the challenger, and voters are uncertain about the challenger's policy preferences. We consider preference distributions such that

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ex-post, a majority prefers the reform in one state and status quo in the other. Voters have noisy, conditionally independent private signals about the state; if in a hypothetical world the signals were common knowledge, then voters could infer the state in a sufficiently large electorate. We ask if the voters voting with their private information alone can produce the same outcome as if they *knew* the state. If so, we say that the electoral environment is *full information equivalent*, or simply that information is efficiently aggregated.

In terms of behavior, we assume that voters vote in order to obtain their preferred policy, and take into account the fact that an individual vote matters only when there is a tie among the other voters. Formally, we look at sequences of Nash equilibria of the voting game (as the number of voters grows unboundedly) and compare the limit outcome with the full information outcome. Using a technique in Bhattacharya (2013), we identify the full set of equilibrium outcomes in the electoral environments under consideration.

Our main insight is that the precise location of the policy alternatives is immaterial for the property of information aggregation. What matters is whether there is uncertainty about the *order* in the location of alternatives. If $L < R < 0$ (or if $0 < L < R$), it is known ex-ante that the reform induces an outcome to the left (respectively, right) of the status quo. In this case, we say that the alternatives are *ordered*. On the other hand, if $L < 0 < R$, there is uncertainty about whether the proposed policy induces an outcome to the right or to the left of the status quo: in this case we say that the alternatives are *unordered*. We demonstrate that elections are guaranteed to aggregate information when the alternatives are ordered. In the unordered alternatives environment, electoral outcomes depend on how each voter expects the others to behave. In other words, elections behave like co-ordination games and have multiple equilibria. In two of these equilibria, the status quo wins in each state. There is also an equilibrium which aggregates information. Therefore, our model predicts a bias in favor of the status quo when there is uncertainty over the spatial order of alternatives.

To fix ideas, consider the (very stylized) example of a vote over trade liberalization. Suppose that a country facing a referendum over whether to allow free trade by joining an economic union with other countries. Because of its isolation, it has developed both an industrial sector and an agricultural sector to suit its own consumption needs. If the country allows free trade, the sector in which it has comparative advantage will grow and the other one will shrink. A situation akin to the unordered alternatives environment arises if there is an ex-ante uncertainty over which sector the comparative advantage lies. In other words, if it is uncertain whether the proposed trade reform will make voters employed in industry better off at the cost of those in agriculture or the other way round, it is possible that such a reform may be blocked even when ex-post it is actually favoured by the majority.

The idea that distributional uncertainty may lead to ex-post inferior electoral outcomes has been noted in other work (e.g., Fernandez and Rodrik (1991), Gersbach (1995), Bhattacharya (2013), Ali, Siga and Mihm (2018)). We identify, in a simple spatial model, conditions under which distributional uncertainty leads to a status quo bias. We discuss the relationship with these papers in detail in section 1.2.

Bhattacharya (2013) provides an abstract analysis showing the existence of minority-preferred outcomes under distributional uncertainty. The spatial structure in the current paper allows us to derive the full set of equilibria for both ordered and unordered environments and stress the role of belief traps in inducing informationally inferior outcomes. More importantly, the current paper tracks the behavior of swing voters and demonstrates why the status quo may win even when it is not preferred by the majority. Finally, the methodology used in the paper may be used as a recipe for finding the entire set of limit outcomes in any two-state, two-alternative large voting game with incomplete information.

In order to see how the uncertainty about spatial order matters, first consider the ordered alternatives environment with the reform known to lie to the left of the status quo ($L < R < 0$). In this environment, voters with ideal points left of $\frac{L}{2}$ prefer the reform in each state, those with ideal points right of $\frac{R}{2}$ prefer the status quo in each state while those with ideal points in the interval $\left[\frac{L}{2}, \frac{R}{2}\right]$ have state sensitive preference: they prefer the reform in state R and the status quo in state L. The fact that all state-sensitive voters have the same preference makes this setting similar to the canonical Condorcet Jury environment. We show that, in equilibrium, a subset of the state-sensitive voters vote as their signal suggests and that this subset always contains the median voter. This ensures that information is aggregated. Moreover, in the limit, all equilibrium sequences lead to the same induced prior belief. The description of moderate swing voters driving election results is consonant with the standard description of electoral behavior by journalists, academics and electoral commentators (see for example, The Swing Voter in the American Politics (2008), ed. William G. Mayer). However, when the alternatives are not ordered, voting behavior may not conform to this description.

In the ordered alternatives environment $(L < 0 < R)$, there are two groups of state-sensitive voters with opposed preference in each state. Those with ideal points left of $\frac{L}{2}$ (henceforth, the L-group) support the reform in state L and oppose it in state R while those with ideal points right of $\frac{R}{2}$ (henceforth, the R-group) support the reform in state R and oppose it in state L. We assume that the R-group is larger and the L-group is smaller than the threshold required for the policy to pass, i.e., reform would win in state R but not in state L if the state were known.

In one class of equilibria, the voters who respond to their private information are the moderate ones within the R-group, and these voters vote for the proposed policy when the signal indicates state R and for the status quo when the signal indicates state L. Thus, the full information outcome obtains almost surely in the limit. Moreover, if the two groups are not too dissimilar in size, the induced prior belief for all information-aggregating equilibrium sequences is uniquely pinned down.

There are two other equilibria in which the status quo is elected in both states. In one of these, each voter believes that almost everyone else is voting uninformatively, dampening her own incentive to use information. Independent of their private information, everyone in the R-group votes for the status quo and almost everyone in the L-group votes for the proposed reform. Since the L-group is smaller, the reform obtains too few votes in either state. With a vanishing fraction of voters in the L-group voting according to their signal, a tie (while itself very rare) is much more likely when these voters vote for $\mathcal{P}$, i.e., when the state is L. Since all voters condition their vote on this very event, almost everyone votes as if the state is *known* to be L. Voter behavior in this equilibrium is akin to what we know as block voting, with opposed groups voting for opposite alternatives.

In the other inefficient equilibrium, only the extremists at either end of the ideological spectrum are responsive to information—but the proposal fails because most of the other voters vote for the status quo uninformatively. Since the status quo receives votes from members of both the L-group and the R-group, pivotality does not strongly indicate one state or the other: this in turn preserves the incentive for moderate voters in both groups to vote for the status quo. We call this the activist voting equilibrium since only those who are likely to care the most about the proposal in either state vote for it (if their signal suggests accordingly).

It must be noted that information aggregation fails in the two "bad" equilibria due to different reasons. In the block voting equilibrium, too few people (a vanishing fraction in a large electorate) utilize information. On the other hand, in the activist voting equilibrium, only the extreme rightists and extreme leftists use information: but since they use information in exactly the opposite ways, one group nullifies the effect of the other. To an outside observer, this voting behavior appears to be one where the proposal is supported by a coalition of some voters from the far right and some from the far left.

As mentioned before, one contribution of this paper is to develop a simple methodology of finding equilibria of large elections where there are two states, two alternatives and voter types are drawn from an i.i.d distribution. We assume that each voter conditions her choice on the event that her vote matters for the outcome. The effect of such conditioning is that she votes as if the prior distribution over states is the one which would produce a tied election given the strategy employed by the other voters. We demonstrate that strategic voting is simply sincere voting with priors determined endogenously. We proceed in two steps: first, we identify the optimal voting behavior for each possible common prior distribution over states, and then we show which of these priors can be supported in equilibrium given the voting environment. While we apply this methodology to the unidimensional spatial model, the same method can be used to find the set of equilibria for a much wider variety of environments.

In the main body of the paper, we consider consequential rules. These are threshold voting rules with the property that if the voters voted with common knowledge of the state, the reform would win in one state and the status quo in the other. With such rules, there is (at least) one equilibrium where the full information outcome is reproduced in each state and two others where the status quo wins in both states. Thus, whether information is aggregated or not depends on which equilibrium obtains. In a later section, we consider all non-unanimous threshold rules and show that for a continuum of voting rules, there is no equilibrium that aggregates information. In particular, these are rules for which the reform should win in both states under full information, but in each equilibrium the status quo wins almost surely in at least one state. We believe that this strong failure of information aggregation is of independent theoretical interest.

1.1. **Applications.** Our leading application is, of course, referenda which are single-issue elections by definition. Any reform creates winners and losers (compared to the status quo), but there is often ex-ante uncertainty about the identity of the winners and losers. There is a large literature in both economics and political science that equates policymaking with experimentation, making the point that choosing or electing a policy is rarely the same as ascertaining an outcome.[1] We show, in a simple framework, when such uncertainty may lead to a status quo bias.

In fact, some proponents of direct democracy invoke the Condorcet Jury Theorem in order to suggest that referenda aggregate information efficiently even if voters may be mistaken about the policy consequences (Matsusaka 2005, Lupia 2001).[2] This paper points out that the argument hinges on the nature of the issue on ballot. If it is an ideological issue like gay marriage or abortion, we are typically in the ordered alternatives case: it is clear whether the proposal is to the right or left of the status quo. However, if the issue on the anvil is distributional, i.e., trade or immigration reform, we are more likely to be in the unordered alternatives environment and the reform is no longer guaranteed to pass even if it is favored by the majority of citizens.

We have already noted that a country voting to enter an economic union fits our set-up directly. Perhaps interestingly, our set-up also captures referenda where the question on the ballot is whether to exit such a union, as was the high profile case of the "Brexit" referendum in the United Kingdom in June 2016. In an environment where trade continues to induce sweeping and unpredictable changes to the economy, prevention of trade retains the current and familiar structure. Therefore, leaving the economic union corresponds to policy $\mathcal{Q}$ and remaining in it corresponds to $\mathcal{P}$. The same idea can be applied to elections where the central issue is about trade and immigration. The American presidential election of November 2016 certainly had that feature. Our framework which predicts an anti-trade bias is consistent with the outcomes in these recent high-profile elections.

[1]See Lindblom (1959) for an early enunciation of the idea of policymakers "muddling through" policies in search of good outcomes. A more recent example is Callander (2011).

[2]For example, Matsusaka (2005, p. 193) claims that "Direct democracy can be effective even when voters have no more or even worse information than legislators....aggregating the opinions of a million voters can be highly accurate by the Law of Large numbers even if each person's chance of being right is small (this is a version of Condorcet Jury Theorem..)"

In fact, there is some evidence that in case of referenda over trade or immigration reforms, the pattern of coalitions are indeed like the activist voting equilibrium that we identify. Johnston et al (1996) (see page 13 and references therein) argues that the in various countries, the referendum to ratify the Maastricht Treaty (i.e., joining the European Union) was opposed by a coalition of the far left and far right. Among these countries, while the measure failed in Switzerland (1992), Norway (1994) and Denmark (1992 and 1994), it passed by a narrow majority in France (1992). However, we cannot explain the recent rise of Euroskepticism (support for status quo in our framework) at both ends of the political spectrum.

Status quo bias in referenda have been well documented in empirical work. In general, the details of the referendum process and the rules for passage might vary, making comparison across countries or aggregation over instances difficult. In Australia, all amendments to the constitution are required to be passed via referenda in which voting is compulsory for everyone on the electoral roll, which makes it the closest approximation to the model we want to study. As of date, of the 44 proposals put forth for referendum in Australia, only 8 have passed. In Switzerland, the "gold standard" for direct democracy, only 36% of all optional referenda have passed in the period from 1991 till 2006, although the proportion was higher earlier (see footnote 7 in Kirchgassner (2007)). In fact, the status quo bias has been well-documented and studied in the case of Switzerland, where authors have held direct democracy responsible for its slow growth during the nineties, delays in reforms and so on (Kirchgassner 2008). In Italy, there have been 71 national referenda since 1974, of which only 25 were approved while others were either rejected or declared invalid due to low turnout.[3] In the United States, among 2360 statewide initiatives to appear on the ballot since the first such initiative in Oregon in 1904 till 2010, only 962 have passed.[4] The success rate of 40% in these initiatives seems a rather low figure, keeping in mind that initiatives appearing on the ballot are already those which are seen by their sponsors to be at least somewhat more likely to pass than fail. We provide a simple theoretical model to explain such a success rate in terms of co-ordination failure among voter groups.

While we use the metaphor of referenda, the current paper applies to political races between two candidates or parties to the extent they can be reducible to a single, possibly ideological dimension. In case of high profile national elections, we often know which candidate is to the right and which one is to the left simply from their party identities. However, in may other situations voters are faced with an uncertainty over the order of the alternatives. In primaries where both candidates are from the same party, the left-right order of candidates may not be clear. In local or municipal elections, candidates often run on the plank of efficiency or local issues, making it difficult for voters to use party affiliation as an informational shortcut for candidate positions.

Even when the candidates do take clearly defined issue positions, there is substantial evidence to the effect that voters often fail to learn the positions or, worse still, fail to even identify the order of the candidates according to their positions. Lenz (2012, table 5.1, page 117-118) presents a survey where he studies several salient issues in US and European national elections (social security in 2000 US elections, EU integration in the British 1997 elections, public works jobs in the 1976 US elections, defense spending in the 1980 US elections, ideology in the 1992 US elections and Chernobyl in the 1986 Dutch elections) and shows that, in each case, less than half of the respondents could start out identifying the order of candidates correctly. These facts suggest that even in electoral races between candidates, there may be uncertainty in voters' minds about the order of candidates.

In case of electoral competitions, our results provide a new explanation for the phenomenon of incumbency advantage. It is well documented that incumbents enjoy a strong and growing advantage in US electorates - both in legislative and executive offices (Ansolabehere, Snyder and

[3]Source: Wikipedia page on Referendums in Italy

[4]Source: Historical database maintained by the Initiative and Referendum Institute at the University of Southern California (http://www.iandrinstitute.org/data.htm)

Stewart 2000, Ansolabehere and Snyder 2002). We hold that if there is incomplete information regarding wether the challenger lies to the left or right of the incumbent, then incumbency advantage may arise due to a co-ordination failure among voters. While the existing set of explanations of incumbency advantage relating to political structure (e.g. decline of the party (Cover 1977), campaign contribution and interest group activities (Jacobson 1980)) apply to legislative offices, our explanation applies to executive offices as well. In fact, our theory is particularly suited to lower offices where information regarding the challenger is harder to come by and party identification plays a smaller role. Another literature (Erikson 1995, Ansolabehere, Snowberg and Snyder 2006) suggests that the advantage of the incumbent arises from being able to corner a larger share of television time both in terms of news coverage as well as campaign advertisements. Our explanation is broadly in line with this position: the incumbent advantage stems from the voters being more informed about the incumbent than about the challenger.

1.2. **Related literature.** Fernandez and Rodrik (1991) was the first to show that welfare improving trade reforms may be blocked due to distributional uncertainty. They have a status quo being voted against a reform which can be majority or minority preferred ex-post. Either alternative can win the first round of elections depending on the parameters. But if the reform wins in the minority preferred state, the information is revealed after implementation and the majority votes it down in the second round. On the other hand, if the status quo wins when the reform is actually majority preferred, the state is never revealed and the population retains the status quo. Thus, unlike our mechanism, their theory of status quo bias crucially hinges on there being multiple rounds of elections.

Following the above paper, there have been a series of papers showing that distributional uncertainty leads to failure of aggregation (e.g., Gersbach (1995), Kim and Fey (2007)). Ali, Mihm and Siga (2018) provide general conditions on the nature of preference variation that leads to the ex-post welfare-maximizing alternative to be voted down in some equilibria. However, their mechanism hinges on private signals being very noisy while ours holds for arbitrarily precise individual information.

Our paper is most closely related to Bhattacharya (2013), which shows that if two voter groups have opposed preference in each state, then *there exists* an equilibrium sequence where the same alternative wins in both states almost surely. The imposition of spatial structure allows us to use the methodology developed in that paper to make clear predictions about the entire set of limit equilibrium outcomes. We show that along with the inefficient equilibria, there is also an informationally efficient equilibrium sequence. Thus, the message here is that whether majoritarian elections lead to the efficient outcome or not depends entirely on voter co-ordination. If the achievement of informational efficiency is the objective of the government (or more generally, the election designer), then our work suggests that policies should be targeted towards co-ordinating on the right equilibrium. Characterization of the entire equilibrium set allows the designer to know precisely which outcomes to avoid while designing such a targeting mechanism.

It is important to mention the formal relationship between conditions on information aggregation in the spatial model (i.e., ordered vs. unordered alternatives) and those in the more general setting in Bhattacharya (2013). According to the Strong Preference Monotonicity (SPM) condition in Bhattacharya (2013), if the distribution of preferences is such that a randomly chosen voter is more likely to prefer $\mathcal{P}$ over $\mathcal{Q}$ for *each* prior belief over states, then information is aggregated in all equilibria. Conversely, if SPM is not satisfied, then there exist signal precisions for which a "wrong" outcome obtains in at least one equilibrium. Bhattacharya (2013) also identifies a joint condition on signal precision and preference distribution called Weak Preference Monotonicity

(WPM) that has the same flavor.[5] In the spatial model, if the alternatives are ordered, both SPM and WPM are satisfied. Hence, it follows directly that information is aggregated efficiently in every equilibrium. On the other hand, when the alternatives are unordered, both SPM and WPM are violated. Moreover, the current paper derives the equilibrium strategies which allows us to track the behavior of responsive (i.e., "swing") voters and provide conditions on responsive sets for the election to achieve the correct outcomes in equilibrium. These conditions throw light on the reasons for why information may or may not be aggregated in certain equilibria.

There is a parallel literature on aggregation failure in common value elections. Mandler (2012) shows that there exist non-aggregating equilibria if there is uncertainty over precision of signals. Our paper shares with Mandler's the idea that voting equilibria are sensitive to local properties of vote share functions (while full information outcomes are not). Aggregation failure due to multiple equilibria can also occur when the number of eligible voters varies across states. While Myerson (1998) shows that there always exists an information aggregating equilibrium, Ekmekci and Lauermann (2016, 2016a) solve the set of equilibria and show that there exist additional non-aggregating equilibria with state-dependent electorate size. In the common value auctions literature, Atakan and Ekmekci (2014) have a similar insight where they show that information aggregation may fail in equilibrium if bidders' expected valuation is non-monotonic in their belief over states.

Persico (2004) and Martinelli (2006) show that in large bodies votes may be uninformativeif information is costly. We demonstrate that uninformative voting may arise (in the block voting equilibrium) due to preference diversity even if signals are free.

In all the above papers. the state space is binary. Barelli et al (2018) shows that with general state and signal spaces there may not exist *any* equilibrium that aggregates information, and generically so if the state space is infinite. While aggregation failure in the papers cited earlier is due to complexity in preferences, the failure in Barelli et al (2018) is due to complexity in the information structure.

The paper is organized as follows. Section 2 discusses the basic model. Section 3 discusses optimal voting behavior as a function of prior beliefs over states. Section 4 characterizes the set of equilibrium outcomes and section 5 identifies information aggregation properties in large elections for different environments. A final section concludes. Most proofs are relegated to the appendix

2. Model

There is an electorate composed of n individuals who vote over a proposed reform $\mathcal{P}$ against a status quo $\mathcal{Q}$. If the reform gets more than a proportion $\theta \in (0,1)$ of the votes, then $\mathcal{P}$ wins; otherwise the status quo $\mathcal{Q}$ wins.[6] Assume that the policy space is $[-1,1]$, and that each policy leads to an outcome that is modelled as a location on the policy space. The location of the status quo $\mathcal{Q}$ is fixed at 0. On the other hand, there is uncertainty about the location of the proposal $\mathcal{P}$: It is equally likely to be located at L or R. The event that $\mathcal{P}$ is located at S, where $S \in \{L, R\}$, is referred to as state S. Additionally, we assume that the policy proposal never coincides with the status quo, i.e., both L and R are non-zero and $-1 < L < R < 1$.

[5]WPM is said to be satisfied if a change in the signal from a to b makes a randomly chosen voter more likely to switch from $\mathcal{P}$ to $\mathcal{Q}$ than from $\mathcal{Q}$ to $\mathcal{P}$ for *each* belief over states. For a given distribution of preference, SPM holds if and only if WPM holds for every possible distribution over signals.

[6]To simplify the analysis, assume the tie breaking rule that if the policy receives exactly θ proportion of votes, the status quo wins.

Voters have single peaked preferences defined on the policy space. Every individual has a privately known bliss point x that is drawn independently from a commonly known non-atomic distribution $F(\cdot)$ with support $[-1,1]$ and a positive, bounded and continuous density $f(\cdot)$ on the entire support.

For a voter with ideal point x, the utility from a policy with location a is assumed to be $-(x-a)^2$. The assumption of quadratic utilities is not important but made for the sake of tractability. Denote by $v(x,S) \equiv x^2 - (x-S)^2$ the difference in utility between $\mathcal{P}$ and $\mathcal{Q}$ when $\mathcal{P}$ is located at $S \in \{L,R\}$. Each voter receives a noisy private signal $\sigma \in \{l,r\}$ drawn independently from the following distribution conditional on the state

$$\Pr(l|L) = \Pr(r|R) = q \in \left(\frac{1}{2}, 1\right)$$

We denote q as the signal precision. Both the ideal point x and signal σ are private information.

We define a voting environment by the tuple (F,q,L,R), and a voting game by an environment together with voting rule $\theta \in (0,1)$ and a finite number n of voters. Given a voting rule θ, we obtain the full information outcome in a given environment: This is the outcome that would have prevailed if the number of voters were infinitely large and the state of the world were known. Our objective is to compare the limit outcome in a sequence of games as n becomes large with the full information outcome for the same environment and voting rule. We say that information is aggregated in an equilibrium sequence if the limit outcome matches the full information outcome. In particular, we contrast the information aggregation properties for environments with ordered and unordered alternatives.

With the assumptions made above, there are three possible configurations: $-1 < L < R < 0$, $0 < L < R < 1$ and $-1 < L < 0 < R < 1$. In the first (second) case, alternative $\mathcal{P}$ lies to the left (right) of $\mathcal{Q}$ in both states. While both these cases are instances of environments with *ordered alternatives,* we shall henceforth consider only the configuration $-1 < L < R < 0$ under this classification.

The third configuration above fails the ordering property since the reform is to the left of the status quo in state L and to the right in state R. We shall make a few more assumptions when we analyze this configuration. First, the locations of the reform in the two states are symmetric, i.e., $L = -b$ and $R = b$ for some $b \in (0,1)$. This assumption is innocuous since we allow fairly general distributions over voter ideal points. Second, the signals are sufficiently precise, i.e., $q > \frac{1}{2} + \frac{b}{4}$. This assumption is necessary to guarantee that voters are pivotal with positive probability for all beliefs over states. Notice that since we are trying to demonstrate failure of aggregation, this assumption is stacked against our conclusion. Finally, we assume with little loss of generality that $F\left(-\frac{b}{2}\right) < 1 - F\left(\frac{b}{2}\right)$.[7] When we refer to an environment with *unordered alternatives,* we shall consider the case $-1 < L < 0 < R < 1$ satisfying these three additional assumptions.

The equilibrium concept we employ is Bayesian Nash equilibrium in weakly undominated strategies, with the added restriction that individuals with the same private information use the same strategy. A strategy is a probability $\pi(x,\sigma)$ of voting for $\mathcal{P}$. Since F is nonatomic, we can concentrate on pure strategies, i.e., $\pi(x,\sigma) \in \{0,1\}$. We assume that the voter votes for $\mathcal{P}$ when indifferent.

As a convention in this paper, whenever we use the term belief over states, we mean the probability of the state being L.

In equilibrium, each voter conditions her decision on the event of being pivotal. The criterion that voters use weakly undominated strategies boils down to the requirement that in equilibrium, a voter has positive probability of being pivotal in each state. Pivotal inference involves a twofold updation in belief over states. First, given the others' strategy π_{-i}, a voter i updates her prior belief to condition on the event that the vote tally for $\mathcal{P}$ is exactly θ. Notice that in a given equilibrium,

[7] The only loss of generality here is in ruling out the knife-edge case where the two quantities are equal.

all voters will hold the same belief over states conditioning on a tie: we can this belief the *induced prior* and denote it by β. In addition, each voter updates her posterior to β_σ using her signal σ. By Bayes Rule we have

$$\left.\begin{array}{l} \beta_l = \frac{q\beta}{q\beta+(1-q)(1-\beta)} \\ \beta_r = \frac{(1-q)\beta}{(1-q)\beta+q(1-\beta)} \end{array}\right\} \tag{1}$$

Equilibrium requires that each voter's strategy $\pi(x,\sigma)$ be optimal given her belief β_σ, and in turn this strategy leads to the belief β conditioning on a tie.

We proceed with our analysis in two steps. First, in section 3 we consider voting behavior *as if* voters were voting sincerely as a function of any common prior β. In the second step (section 4), we consider which of these values of β can arise as induced priors in equilibrium.

3. Voting Behavior

For a voter with ideal point x, signal σ and prior probability β, the condition for voting for $\mathcal{P}$ is $Ev(x,\sigma) \geq 0$, where expectation is taken with respect to the posterior β_σ. We assume that when a vote is indifferent, she votes $\mathcal{P}$, but this assumption is innocuous since F is nonatomic.

For quadratic preferences, we have

$$Ev(x,\sigma) = 2x(\beta_\sigma L + (1-\beta_\sigma)R) - (\beta_\sigma L^2 + (1-\beta_\sigma)R^2), \tag{2}$$

Since $Ev(x,S)$ is monotonic in x, the optimal strategy $\pi(x,\sigma)(\beta)$ is given by a pair of cut-offs x_σ, $\sigma \in \{l,r\}$, which solves $E(v(x,S)) = 0$. Types x above x_σ vote for one alternative while types below vote for the other.

Given the optimal strategy $\pi(x,\sigma)(\beta)$, define $z_\sigma(\beta)$ as the probability that a randomly chosen voter with signal σ votes for the reform. We have

$$z_\sigma = \int_{-1}^{1} \pi(x,l)(\beta) f(x)dx$$

The expected vote share for the reform $\mathcal{P}$ conditional on state $S \in \{L,R\}$ is given by

$$t(L,\beta) = qz_l(\beta) + (1-q)z_r(\beta) \tag{3}$$

$$t(R,\beta) = (1-q)z_l(\beta) + qz_r(\beta) \tag{4}$$

At this stage, we introduce an important definition. Given a strategy $\pi(\cdot,\sigma)$, we call a voter with ideal point x a *responsive type* if $\pi(x,l) \neq \pi(x,r)$.[8] For such a voter, the voting behavior depends on the information (signal) she obtains.

We now separately describe the optimal strategy, responsive sets and vote share function for environments with ordered and unordered alternatives.

3.1. **Ordered Alternatives.** Recall that in the case of ordered alternatives, we consider only the case $-1 < L < R < 0$. Here, types $x < \frac{L}{2}$ prefer the proposal $\mathcal{P}$ in both states, and those with $x > \frac{R}{2}$ prefer the status quo $\mathcal{Q}$ in both states. While the preference of the extreme left and extreme right is insensitive to the location of the proposed reform, the types in the range $\left(\frac{L}{2}, \frac{R}{2}\right)$ are "independent", i.e., they prefer the proposed policy in state R (the "moderate" state) and the status quo in state L (the "extreme" state). It is only these types that may potentially take into account information about the state in their voting behavior.

The following proposition that describes the optimal strategy $\pi(x,\sigma)(\beta)$ in the ordered alternatives case.

[8] While x and σ are both private information, we refer to ideal point x alone as the "type" of the voter.

Proposition 1. *Consider an environment with ordered alternatives. Given a prior belief* $\beta \in [0,1]$ *and signal* $\sigma \in \{l, r\}$ *there exists a cut-off* $x_\sigma(\beta) \in \left[\frac{L}{2}, \frac{R}{2}\right]$ *such that the optimal strategy is to vote for* $\mathcal{P}$ *if* $x \leq x_\sigma(\beta)$ *and* $\mathcal{Q}$ *if* $x > x_\sigma(\beta)$*. Moreover,* $x_\sigma(\cdot)$ *is a decreasing function with (i)* $x_r(\beta) > x_l(\beta)$ *for* $\beta \in (0,1)$*, (ii)* $x_r(0) = x_l(0) = \frac{R}{2}$*, and (iii)* $x_r(1) = x_l(1) = \frac{L}{2}$*.*

In other words, types left of x_l always vote for $\mathcal{P}$ and those right of x_r vote for $\mathcal{Q}$, while types in the responsive set $[x_l, x_r]$ vote $\mathcal{P}$ if they get signal r and $\mathcal{Q}$ if they get signal l. As β increases, the responsive set moves towards the left.

From Proposition 1, we have that $z_\sigma(\beta) = F(x_\sigma(\beta))$. Using this in expressions (3) and (4), we obtain the vote share functions for the ordered alternative case. The following proposition demonstrates how the vote share in each state changes as a function of the prior belief. Since the

Proposition 2. *Consider an environment with ordered alternatives. In each state S,the expected vote share* $t(S,\beta)$ *for the reform* $\mathcal{P}$ *strictly decreases with* β*. Moreover, (i)* $t(L,\beta) < t(R,\beta)$ *for all* $\beta \in (0,1)$*, (ii)* $t(L,0) = t(R,0) = F(\frac{R}{2})$*, and (iii)* $t(L,1) = t(R,1) = F(\frac{L}{2})$*.*

Proposition 2 states that as the prior belief over the "extreme" state L increases, the expected share of votes for the proposal decreases in both states. Informative voting by the responsive set of voters ensures that the proposal receives more votes in the "moderate" state (R). The expected vote shares in the two states are plotted against the prior in figure 1.

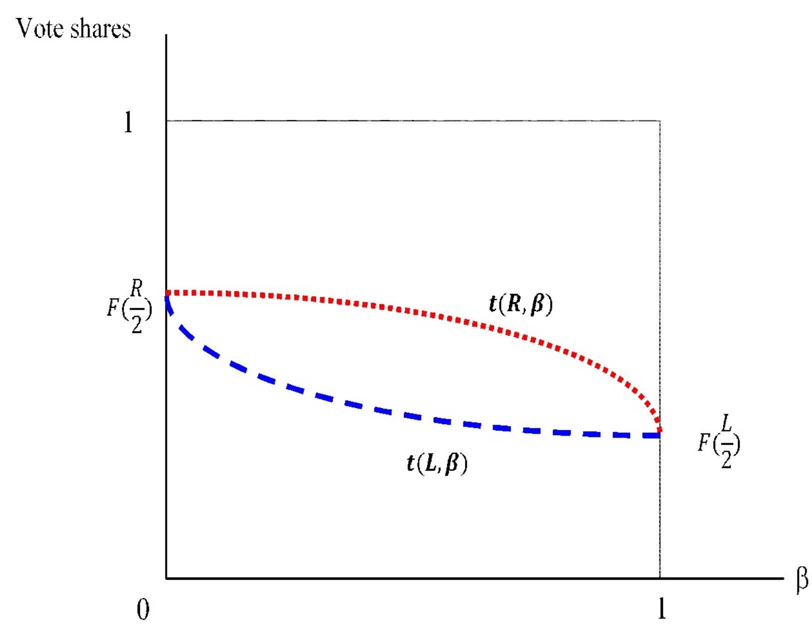

Figure 1: Vote shares in each state under ordered alternatives

The ordered alternatives case serves as a benchmark. Before we go to the unordered alternatives case, we observe two important implications of Proposition 2. First, since $F(\frac{L}{2}) < t(L,\beta) < t(R,\beta) < F(\frac{R}{2})$, the ordered alternatives environment satisfies the Strong Preference Monotonicity condition in Bhattacharya (2013). Second, since the cut-offs lie in $\left[\frac{L}{2}, \frac{R}{2}\right]$, $t(S,\beta) \in (0,1)$ for all β by the assumption of full support of F.

3.2. **Unordered Alternatives.** Next, we study the optimal strategies for the case where $-b = L < 0 < R = b$. In this case, voters with ideal points close to the status quo, i.e., those with

$x \in \left(\frac{-b}{2}, \frac{b}{2}\right)$ prefer $\mathcal{Q}$ in each state. Those with $x < \frac{-b}{2}$ prefer $\mathcal{P}$ in state L and $\mathcal{Q}$ in state R, while those with $x > \frac{b}{2}$ prefer $\mathcal{P}$ in state R and $\mathcal{Q}$ in state L. We shall informally call voters with $x \leq \frac{-b}{2}$ the L-group and those with $x \geq \frac{b}{2}$ the R-group. The fact that the state-sensitive types are split into two groups with opposed preferences in each state is a special feature of the unordered alternatives environment. By our assumption that $F\left(-\frac{b}{2}\right) < 1 - F\left(\frac{b}{2}\right)$, the R-group is larger than the L-group.

With unordered alternatives, the condition for type x voting for the policy $\mathcal{P}$ after having received σ is:

$$Ev(x,\sigma) \geq 0 \Rightarrow 2x(1-2\beta_\sigma) \geq b,$$

which gives us the following optimal strategies in terms of β_σ

$$\pi(x,\sigma)(\beta_\sigma) = \begin{cases} \left.\begin{array}{l} 1 \; for \; x \leq \frac{b}{2(1-2\beta_\sigma)} \\ 0 \; for \; x > \frac{b}{2(1-2\beta_\sigma)} \end{array}\right\} if \; \beta_\sigma \geq \frac{1}{2} + \frac{b}{4} \\ 0 \; for \; all \; x \quad if \; \beta_\sigma \in \left(\frac{1}{2} - \frac{b}{4}, \frac{1}{2} + \frac{b}{4}\right) \\ \left.\begin{array}{l} 1 \; for \; x \geq \frac{b}{2(1-2\beta_\sigma)} \\ 0 \; for \; x < \frac{b}{2(1-2\beta_\sigma)} \end{array}\right\} if \; \beta_\sigma \leq \frac{1}{2} - \frac{b}{4} \end{cases} \tag{5}$$

leading to the following cut-offs

$$x_\sigma(\beta) = \begin{cases} \min\left\{1, \frac{b}{2(1-2\beta_\sigma)}\right\}, & 0 \leq \beta_\sigma < \frac{1}{2} \\ \max\left\{-1, \frac{b}{2(1-2\beta_\sigma)}\right\}, & \frac{1}{2} \leq \beta_\sigma \leq 1 \end{cases} \tag{6}$$

Define, for each signal $\sigma \in \{l, r\}$, $\underline{\beta^\sigma}$ and $\overline{\beta^\sigma}$ as the values of the prior β that generate posteriors $\frac{1}{2} - \frac{b}{4}$ and $\frac{1}{2} + \frac{b}{4}$ respectively. It is easy to see from Bayes Rule that (i) $\underline{\beta^\sigma} < \overline{\beta^\sigma}$, (ii) $\underline{\beta^l} < \underline{\beta^r}$ and (iii) $\overline{\beta^l} < \overline{\beta^r}$. In addition, the informativeness assumption $q > \frac{1}{2} + \frac{b}{4}$ implies that $\overline{\beta^l} < \frac{1}{2} < \underline{\beta^r}$. Since $\overline{\beta^l} < \underline{\beta^r}$, there is no value of β such that both β_l and β_r are simultaneously in the range $\left[\frac{1}{2} - \frac{b}{4}, \frac{1}{2} + \frac{b}{4}\right]$, ensuring that $t(S,\beta) > 0$ for all β.[9]

The following proposition that describes the behavior of the cut-offs in the unordered alternatives environment.

Proposition 3. *Consider an environment with unordered alternatives. For each signal* $\sigma \in \{l, r\}$, *there exist thresholds* $\underline{\beta^\sigma}$ *and* $\overline{\beta^\sigma}$ *with the feature that: for* $\beta \in [0, \underline{\beta^\sigma}]$, *there is a cut-off* $x_\sigma(\beta) \in \left[\frac{b}{2}, 1\right]$ *and increasing in* β *such that the optimal strategy is to vote for* $\mathcal{P}$ *if* $x \geq x_\sigma(\beta)$ *and for* $\mathcal{Q}$ *if* $x < x_\sigma(\beta)$; *for* $\beta \in (\underline{\beta^\sigma}, \overline{\beta^\sigma})$ *it is optimal for all* $x \in [-1, 1]$ *to vote for* $\mathcal{Q}$; *and for* $\beta \in [\overline{\beta^\sigma}, 1]$, *there is a cut-off* $x_\sigma(\beta) \in \left[-1, -\frac{b}{2}\right]$ *and decreasing in* β *such that the optimal strategy is to vote for* $\mathcal{P}$ *if* $x \leq x_\sigma(\beta)$ *and for* $\mathcal{Q}$ *if* $x > x_\sigma(\beta)$. *For all* $\beta \in (0,1)$ *and* $\sigma \in \{l, r\}$ *it must be the case that* $x_r(\beta) < x_l(\beta)$. *Moreover,* $x_\sigma(0) = \frac{b}{2}$ *and* $x_\sigma(1) = -\frac{b}{2}$.

Notice that it unlike the ordered alternative case, types to the left of the cut-off vote for the reform when the cut-off is in the L-group and those to the right of the cut-off vote for the reform when the cut-off is in the R-group. Therefore, in this case we have

$$z_\sigma(\beta) = \begin{cases} F(x_\sigma(\beta)) \; if \; x_\sigma(\beta) \leq -\frac{b}{2} \\ 1 - F(x_\sigma(\beta)) \; if \; x_\sigma(\beta) \geq \frac{b}{2} \end{cases} \tag{7}$$

We can now describe the responsive sets. When $\beta \in \left(0, \underline{\beta^l}\right)$, the responsive set $[x_r, x_l]$ is entirely in the R-group, and the types in this set vote for $\mathcal{P}$ when they get the signal r and $\mathcal{Q}$ when they

[9] Also, since the types $[\frac{-b}{2}, \frac{b}{2}]$ always vote for the status quo, $t(S,\beta) < 1$.

get the signal l. Those to the right of the responsive set vote for $\mathcal{P}$ and those to the left vote for $\mathcal{Q}$ irrespective of the signal. On the other hand, when $\beta \in \left(\overline{\beta^r}, 1\right)$, the responsive set $[x_r, x_l]$ is entirely in the L-group. The responsive types vote for $\mathcal{P}$ when they get the signal l and $\mathcal{Q}$ when they get the signal r, which is the exact opposite of what the responsive voting strategy would be if the responsive set were in the R-group.

Perhaps interestingly, the responsive set may itself be split. For $\beta \in (\overline{\beta^l}, \underline{\beta^r})$, we have x_l in the L-group and x_r in the R-group. In this case, $x \leq x_l$ vote for $\mathcal{P}$ if and only if they obtain the signal l, and those $x \geq x_r$ vote for $\mathcal{P}$ if and only if they obtain the signal r. The extreme types from both ends vote based on information while those in the middle (i.e. $x \in (x_l, x_r)$) vote for the status quo uninformatively.

The following proposition describes the expected vote share $t(S, \beta)$ for the reform as a fuction of the prior β.

Proposition 4. *Consider an environment with unordered alternatives. Both* $t(L, \beta)$ *and* $t(R, \beta)$ *are decreasing in* $(0, \overline{\beta^l})$ *and increasing in* $(\underline{\beta^r}, 1)$*. There exists some number* $\beta^* \in (\overline{\beta^l}, \underline{\beta^r})$ *satisfying* $0 < \beta^* < 1$ *such that for* $\beta < \beta^*$, $t(R, \beta) > t(L, \beta)$, *for* $\beta > \beta^*$, $t(R, \beta) < t(L, \beta)$ *and for* $\beta = \beta^*$, $t(R, \beta) = t(L, \beta)$. *Moreover,* $t(R, 0) = t(L, 0) = 1 - F\left(\frac{b}{2}\right)$ *and* $t(R, 1) = t(L, 1) = F\left(\frac{-b}{2}\right)$.

Proof. See Appendix. ■

This proposition says that if the prior falls below a critical value β^*, then the expected vote share for the reform in state R is higher than that in state L. If, on the contrary, the belief is higher than β^*, then the alternative $\mathcal{P}$ is expected to get a higher vote share in state L than in state R. Moreover, in both states, the expected share of votes for the reform is higher for beliefs closer to 0 or 1. As the voters get more unsure about the state, only the very extreme types vote for the reform.

The expected vote share functions are shown in figure 2.

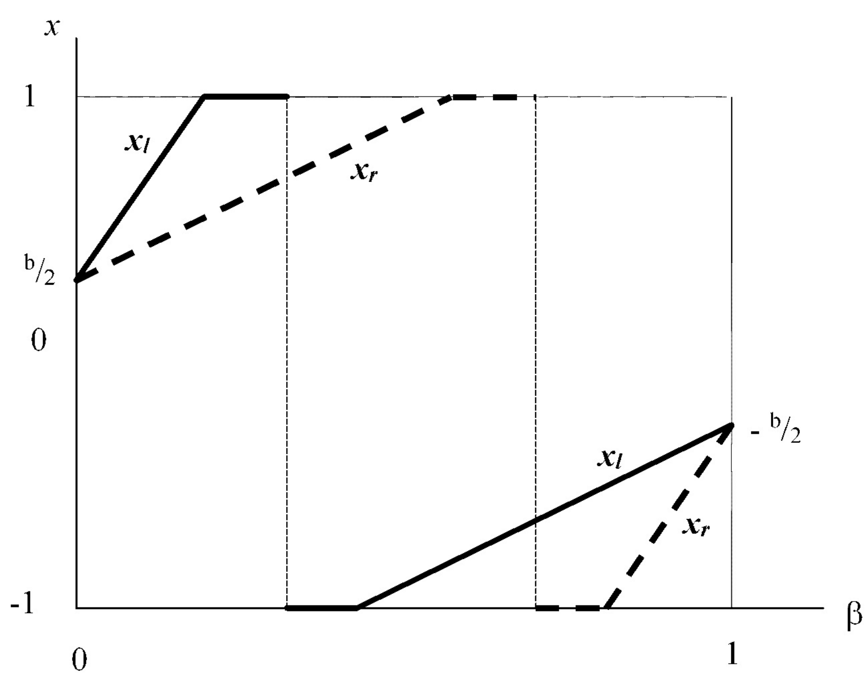

Figure 2: Construction of vote share functions for unordered alternatives

4. Equilibrium Analysis

In this section, we borrow the technique developed in Bhattacharya (2013) to solve for equilibria in a game (F, q, L, R, θ, n). We identify those prior beliefs, which when commonly shared by all voters, induce strategies that lead to the same beliefs conditioning on being pivotal. We call such beliefs the equilibrium *induced prior* beliefs. A strategy that is optimal given one of these beliefs is indeed an equilibrium strategy. We first show existence for finite population and then provide a characterization of limit values of equilibrium induced priors for large electorates.

4.1. **Finite population.** Consider a candidate induced prior belief β. When all other voters use the strategy $\pi(x,\sigma)(\beta)$, it leads to the vote share $t(S,\beta)$ in state S. Under a threshold rule θ a voter is pivotal if $\lfloor n\theta \rfloor$ votes are cast for the policy $\mathcal{P}$ from among the remaining n voters, where $\lfloor t \rfloor$ denotes the largest integer weakly less than t. So, the probability of being pivotal in state S is given by:

$$\Pr(piv|\beta, S) = \binom{n-1}{\lfloor n\theta \rfloor} (t(S,\beta))^{\lfloor n\theta \rfloor} (1 - t(S,\beta))^{n-1-\lfloor n\theta \rfloor} \tag{8}$$

Note that for both environments, $0 < t(S,\beta) < 1$, implying that $\Pr(piv|\beta, S) > 0$. Since the two states are equally likely, the belief on the state S conditional on being pivotal is given by

$$\Pr(S|piv, \beta) = \frac{\Pr(piv|\beta, S)}{\Pr(piv|\beta, L) + \Pr(piv|\beta, R)} \tag{9}$$

In equilibrium, we must have $\beta = \Pr(L|piv, \beta)$. This consistency condition pins down the belief β held in equilibrium. Formally,

$$\frac{\beta}{1-\beta} = \frac{\beta(L|piv, \beta)}{\beta(R|piv, \beta)} = \frac{\Pr(piv|\beta, L)}{\Pr(piv|\beta, R)},$$

which, using the pivot equations (8), gives us

$$\begin{aligned} \beta &= \frac{H(\beta, n, \theta)}{1 + H(\beta, n, \theta)} \text{ where} \\ H(\beta, n, \theta) &= \frac{(t(L,\beta))^{\lfloor n\theta \rfloor} (1 - t(L,\beta))^{n-1-\lfloor n\theta \rfloor}}{(t(R,\beta))^{\lfloor n\theta \rfloor} (1 - t(R,\beta))^{n-1-\lfloor n\theta \rfloor}} \end{aligned} \tag{10}$$

Equation (10) is called the equilibrium condition. Any solution to this equation is an equilibrium belief denoted by β_θ^n (indexing by the voting rule and number of voters), and the equilibrium symmetric strategy profile is given by $\pi(x,\sigma)(\beta_\theta^n)$.

To see that equation (10) admits a solution, first note that the right hand side is continuous in β.[10] Since $t(S,\beta)$ is bounded above 0 and below 1 for all β, the right hand side of the equilibrium condition is also bounded the same way. However, the left hand side continuously changes from 0 to 1. Therefore, an equilibrium exists for any given voting game (F, q, L, R, θ, n). It is important to note that since $0 < t(S,\beta) < 1$, we must have $\beta_\theta^n \in (0,1)$ from the equation (10). Next we characterize the limit of equilibrium beliefs in large elections.

4.2. **Large Electorates.** Fix an environment (F, q, L, R). Given a voting rule θ, consider a sequence of games noted by by letting the number of voters grow unboundedly, and denote the limit of the equilibrium sequence $\langle \beta_\theta^n \rangle$ by β_θ^0.[11] Since condition (10) must be satisfied along the sequence,

[10] Vote share functions $t(S,\beta)$ are continuous in β since cut-offs $x_\sigma(\beta)$ are continuous and F is non-atomic.

[11] The existence of such a limit point (more formally, accumulation point of a subsequence) is guaranteed by the fact that the space of beliefs is compact, i.e., $\beta \in [0,1]$.

we obtain the following *limit equilibrium condition.*

$$\beta^0_\theta = \lim_{n\to\infty}\left[\frac{H(\beta^n_\theta, n, \theta)}{1+H(\beta^n_\theta, n, \theta)}\right] \tag{11}$$

The vote share functions $t(L,\cdot)$ and $t(R,\cdot)$ are determined entirely by the environment. Given these functions, for each value of $\beta \in [0,1]$, we shall determine the set $\Theta(\beta)$ of voting rules θ for which there is a sequence of equilibria with the induced prior β^n_θ converging to β. Once we have the set $\Theta(\beta)$, we can invert it and describe the limit outcomes for each voting rule in a given environment.

For values of β that produce different vote shares in the different states, define the function

$$\theta^*(\beta) = \frac{\log\frac{1-t(R,\beta)}{1-t(L,\beta)}}{\log\frac{t(L,\beta)(1-t(R,\beta))}{t(R,\beta)(1-t(L,\beta))}} \text{for } \beta \text{ such that } t(L,\beta) \neq t(R,\beta) \tag{12}$$

The following remark establishes a few important properties of the function $\theta^*(\beta)$.

Remark 1. *$\theta^*(\beta)$ is continuous and lies strictly between $t(L,\beta)$ and $t(R,\beta)$. Moreover, if both $t(L,\beta)$ and $t(R,\beta)$ are strictly increasing, then so is $\theta^*(\beta)$.*

The second part of the remark ensures that $\theta^*(\beta)$ is decreasing in the ordered alternatives environment (from Proposition 2). In the unordered alternatives case, $\theta^*(\beta)$ is decreasing in the range $(0, \overline{\beta^l})$ and increasing in $(\underline{\beta^r}, 1)$ (from Proposition 4). $\theta^*(\beta)$ can, however, be non-monotonic in the range $(\overline{\beta^l}, \underline{\beta^r})$.

In order to state the (partial) characterization result in a concise way, we need another definition. We say that $\beta \in [0,1]$ is *regular* if either $\theta^*(\beta)$ is well-defined and $\frac{d\theta^*(\beta)}{d\beta} \neq 0$, or $t(A,\beta) = t(B,\beta) \neq \theta$

Proposition 5. *For an ordered alternatives environment, define the correspondence $\Theta(\beta)$ as follows:*

$$\Theta(\beta) = \Theta^O(\beta) \equiv \begin{cases} (i)\ \textit{For } \beta \in (0,1), \Theta(\beta) = \theta^*(\beta) \\ (ii)\ \Theta(0) = \left\{\theta : \theta \geq F\left(\frac{R}{2}\right)\right\} \\ (iii)\ \Theta(1) = \left\{\theta : \theta \leq F\left(\frac{L}{2}\right)\right\} \end{cases}$$

For an unordered alternatives environment, define the correspondence $\Theta(\beta)$ as follows:

$$\Theta(\beta) = \Theta^U(\beta) \equiv \begin{cases} (i)\ \textit{For } \beta \in (0,\beta^*) \cup (\beta^*,1),\ \Theta(\beta) = \theta^*(\beta) \\ (ii)\ \Theta(\beta^*) = \{\theta : \theta \in (0,1)\} \\ (iii)\ \Theta(0) = \left\{\theta : \theta \geq 1 - F\left(\frac{b}{2}\right)\right\} \\ (iv)\ \Theta(1) = \left\{\theta : \theta \geq F\left(-\frac{b}{2}\right)\right\} \end{cases}$$

For both classes of environments, given any $\beta \in [0,1]$, there exists a sequence of equilibria with induced prior beliefs β^n_θ converging to β only if $\theta \in \Theta(\beta)$. Conversely, there is a sequence of equilibria with induced prior β^n_θ converging to β if $\theta \in \Theta(\beta)$ provided β is regular.

The above proposition characterizes the limits of all equilibrium sequences for each voting rule and each environment under consideration. It says that the equilibrium beliefs in the limit are determined by the shape of the vote share functions $t(L,\beta)$ and $t(R,\beta)$. If β produces unequal vote shares in the two states, then there is at most one voting rule $\theta^*(\beta)$ (and exactly one as long as $\frac{d\theta^*(\beta)}{d\beta} \neq 0$) that supports an equilibrium sequence converging to β. If β produces equal vote shares in the two states, there exists a continuum of voting rules that support an equilibrium sequence converging to β. For $\beta = 0$ or $\beta = 1$, such voting rules have to be high enough or low enough depending on the environment. In the unordered alternatives environment, for *every* non-unanimous voting rule there is an equilibrium with beliefs converging to β^* except possibly for the case $\theta = t(S,\beta^*)$. However, the indeterminacy at $\theta = t(S,\beta^*)$ is not going to matter for the voting rules we shall consider in the main body of this paper.

Figure 3 shows the correspondence $\Theta(\beta)$ for the ordered alternatives case $(\Theta^O(\beta))$ and figure 4 shows the same for the unordered alternatives case $(\Theta^U(\beta))$. In both cases, the correspondence $\Theta(\beta)$ is marked with the thick black line.

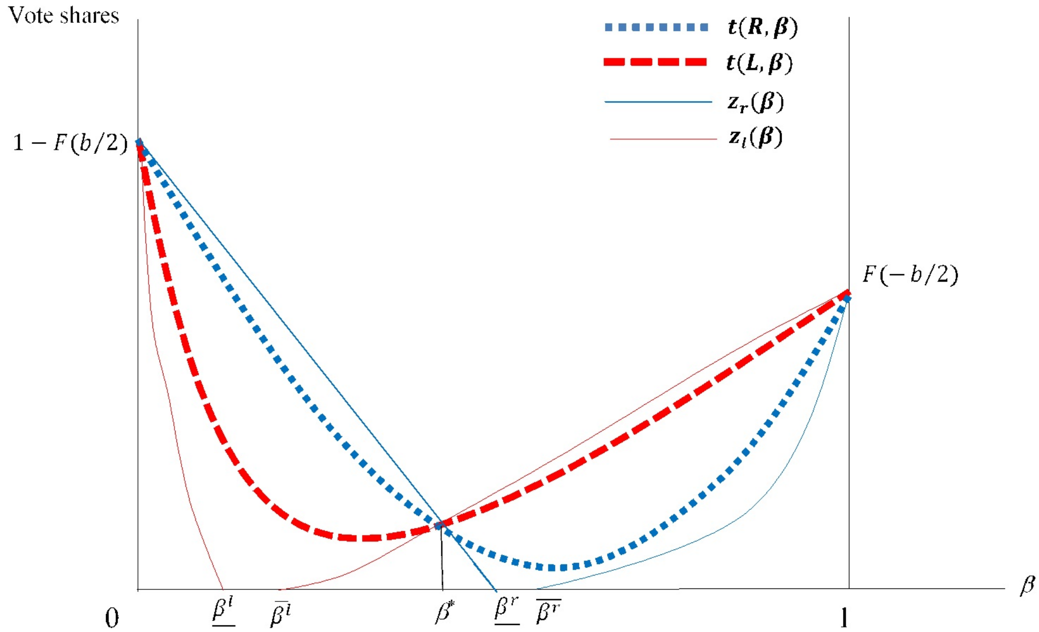

Figure 3: The correspondence $\Theta(\beta)$ in the ordered alternatives environment

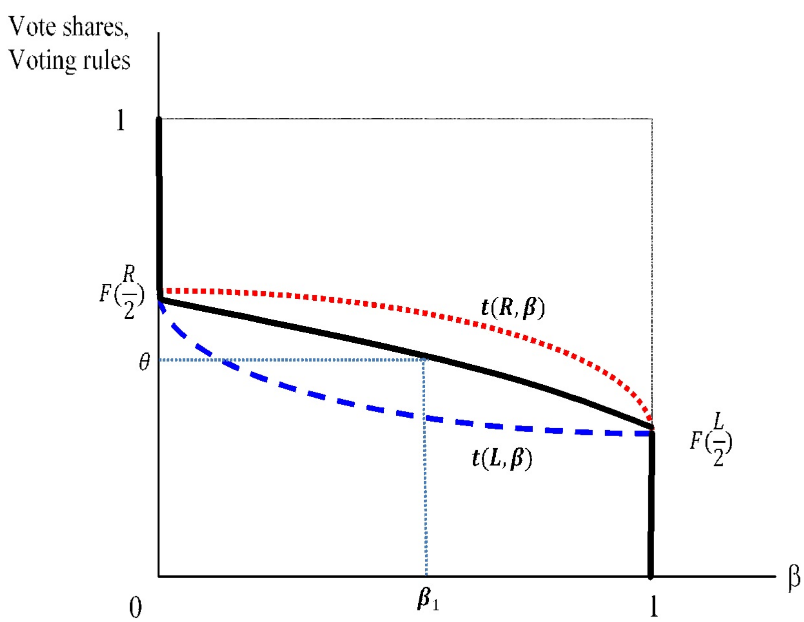

Figure 4: The correspondence $\Theta(\beta)$ in the unordered alternatives environment

Now we illustrate the intuition behind the "only if" part in Proposition 5 (the proof of the "if" part is constructive). Suppose that there exists a sequence of equilibria with $\beta_\theta^n \to \beta^0 \in (0,1)$ and that $t(L,\beta^0) < t(R,\beta^0)$ for some $\beta^0 \in (0,1)$. Ignoring the integer issue and considering $n\theta$ as the threshold rule, the limit equilibrium condition boils down to

$$\frac{\beta^0}{1-\beta^0} = \lim_{n\to\infty}\left[\frac{(t(L,\beta_\theta^n))^\theta\,(1-t(L,\beta_\theta^n))^{1-\theta}}{(t(R,\beta_\theta^n))^\theta\,(1-t(R,\beta_\theta^n))^{1-\theta}}\right]^n$$

Now, since the left hand side is a positive finite number, the expression inside square brackets on the right hand side must converge to 1. Alternatively, in order to have a non-degenerate limit distribution conditioning on pivotality, the pivot probabilities in the two states must be about equal in the limit, i.e.,

$$\left(t(L,\beta^0)\right)^\theta\left(1-t(L,\beta^0)\right)^{1-\theta} = \left(t(R,\beta^0)\right)^\theta\left(1-t(R,\beta^0)\right)^{1-\theta} \tag{13}$$

The unique voting rule θ that satisfies equation 13 is given by $\theta^*(\beta^0)$ as in equation 12. To see why it must be the case that $t(L,\beta^0) < \theta^*(\beta^0) < t(R,\beta^0)$, notice that voting rules less than $t(L,\beta^0)$ produce strictly higher pivot probability in state L and rules greater than $t(R,\beta^0)$ produce strictly higher pivot probability in state R.

Now, consider beliefs that produce equal vote shares. For extreme beliefs ($\beta = 0$ or $\beta = 1$), certain voting rules are ruled out as candidates for supporting equilibria at those beliefs. For illustration, consider $\beta = 0$ for the unordered alternatives case. To support an equilibrium sequence with beliefs converging to $\beta = 0$, the pivot probability must be strictly higher in state R than in state L for sufficiently large n. For any sequence $\beta^n \to 0$, while the vote shares in both states approach $F\left(\frac{R}{2}\right)$, we always have $t(L,\beta^n) < t(R,\beta^n) < F\left(\frac{R}{2}\right)$. Now, for any $\theta < F\left(\frac{R}{2}\right)$, we must have $\theta < t(L,\beta^n) < t(R,\beta^n)$ for large enough n. In this case, the likelihood of there being exactly θ share of votes for $\mathcal{P}$ is strictly higher in state L than in state R for large electorates, leading to a contradiction. On the other hand, if $\theta > F\left(\frac{R}{2}\right)$, then we always have $\theta > t(R,\beta^n) > t(L,\beta^n)$, which is consistent with the pivot probability being strictly higher in state R than in state L.

The formal proof of Proposition 5 follows Lemma 1, 2 and 3 in Bhattacharya (2013) with minor variations and is discussed in the appendix. It consists of three main steps. First, we formally prove condition (13). Then we prove that for any sequence $\beta^n \to \beta$, if $\theta \notin \Theta(\beta)$, then $\frac{H(\beta^n,n,\theta)}{1+H(\beta^n,n,\theta)}$ is bounded away from β as $n \to \infty$. Finally, we show that, given a regular β and any $\theta \in \Theta(\beta)$, the function $\frac{H(\beta',n,\theta)}{1+H(\beta',n,\theta)}$ has a sequence of fixed points β_θ^n converging to β.

Given the induced priors arising in the limit, we can identify the electoral outcomes. For example, suppose we have a sequence $\beta_\theta^n \to \beta^0$ and $t(L,\beta^0) < \theta < t(R,\beta^0)$. By the Strong Law of Large Numbers, the actual vote share for the reform converges in probability to $t(S,\beta^0)$ in state S. Clearly then, the reform passes in state R but fails in state L with an arbitrarily high probability. In the next section, we compare these limit outcomes with the full information outcome in the two environments under consideration.

5. Election Outcomes and Information Aggregation

In this section, we ask whether election outcomes under incomplete information converge to the complete information outcome. Alternatively, do electoral outcomes embody all the dispersed private information in the electorate? We demonstrate that while dispersed information is efficiently aggregated in ordered alternative environments, information aggregation is not guaranteed when the alternatives are unordered.

For purposes of judging informational efficiency, we concentrate on *consequential* voting rules, i.e., rules under which the proposed reform would be elected in one state and the status quo in the other if the state were common knowledge. Under ordered alternatives, we assume that $F\left(\frac{L}{2}\right) < \theta < F\left(\frac{R}{2}\right)$, i.e., a θ-majority of the population prefers $\mathcal{P}$ in state R and $\mathcal{Q}$ in state L.

In case of unordered alternatives, the consequential rules are defined by $F\left(-\frac{b}{2}\right) < \theta < 1 - F\left(\frac{b}{2}\right)$: these are the rules for which a θ-majority prefers $\mathcal{P}$ in state R and $\mathcal{Q}$ in state L.[12]

Given a consequential rule and the environment under consideration, we say that an equilibrium sequence is *Full Information Equivalent* if under the sequence, the probability that $\mathcal{P}$ wins in state R and $\mathcal{Q}$ wins in state L converges to 1.[13]

The next proposition establishes that information aggregation is guaranteed when the alternatives are ordered.

Proposition 6. *Consider an environment with ordered alternatives and a consequential voting rule. All equilibrium sequences are full information equivalent and the induced prior belief in all such sequences converge to the a unique limit.*

The proof follows from remark 1 and Proposition 5, and the idea can be readily seen from figure 3. There is a unique value β_1 for which $\theta^*(\cdot) = \theta$. Since $t(R, \beta_1) < \theta < t(L, \beta_1)$, by the strong Law of Large numbers, $\mathcal{P}$ wins almost surely in state L and $\mathcal{Q}$ in state R.

To see why the equilibria satisfy FIE, we need to examine the behavior of the responsive set in the limit of the sequence. Denote the type x with $F(x) = \theta$ as the θ-median. There are two properties of the responsive set that drives information aggregation. First, since θ-median is contained in the responsive set and the types on two sides of the set vote for opposite alternatives, the responsive voters are *influential*: they can affect the outcome of the election. Second, the preferences of the responsive voters are *aligned* with the complete information outcome: they prefer $\mathcal{P}$ in state R and $\mathcal{Q}$ in state L. Thus, by voting for $\mathcal{P}$ on receiving signal r and for $\mathcal{Q}$ on receiving signal l, they "swing" the election in favor of the ex-post θ-majority preferred alternative.

Proposition 6 can be treated as the two-state alternative to the main theorem in Feddersen and Pesendorfer (1997). It is worthmentioning that while the ordered alternatives environment satisfies the Strong Preference Monotonicity condition, the above proposition is stronger than Theorem 1 in Bhattacharya (2013) in that we establish uniqueness of the limit.

The next proposition describes the outcomes in the unordered alternatives case.

Proposition 7. *Consider an environment with unordered alternatives and a consequential voting rule. Each equilibrium sequence satisfies one of the following limit properties: (i) The induced prior converges to* 0 *and the status quo wins almost surely in each state, (ii) the induced prior converges to* β^* *and the status quo wins almost surely in each state; and (iii) the induced prior converges to some value in* $(0, \beta^*)$ *and the outcome is full information equivalent.*

The proposition says that only two electoral results are possible in equilibrium: either the status quo wins in both states or the full information outcome obtains. Observe the limit of β_θ^n in the information-aggregating sequence is not uniquely determined. In Remark 2, we show that if the L-group and the R-group is not too dissimilar in size, then the limit of the sequence satisfying FIE is also unique.

Figure 4 illustrates the three different kinds of equilibria in this environment. The three possible limit values of the induced prior are β_1, $\beta_2 (= \beta^*)$ and $\beta_3 (= 1)$. As is clear from the picture, information is aggregated in the sequence with beliefs converging to β_1 since $t(L, \beta_1) < \theta < t(R, \beta_1)$. In the two other sequences, we have $t(L, \beta) = t(S, \beta) < \theta$, and the status quo almost surely wins in both states.

The existence of the information-aggregating equilibrium sequence follows from continuity of $\theta^*(\cdot)$ in $(0, \beta^*)$ and the fact that $\theta^*(\beta)$ is larger than θ at the left end and smaller than θ at the right end of the interval. Recall that the reform wins under complete information when it is supported

[12] Our framework allows us to discuss the information aggregation property for non-consequential rules too. We do so in a later section.

[13] We have chosen our parameters in such a way that the full information outcome is the same for both ordered and unordered environments.

by the larger independent group, i.e., the R-group. In the equilibrium sequence that satisfies FIE, the limit belief β_1 places large enough (but not the entire) weight on state R such that the L-group votes for the status quo while the responsive set lies in the R-group. Thus, the alignment condition for responsive voters is satisfied. Those to the left (right) of the responsive set vote for the status quo (proposed reform), and the θ-median from the right (the type x satisfying $\theta = 1 - F(x)$) is contained in the responsive set. Thus, the responsive voters are influential in affecting the voting outcome, and we obtain the complete information outcome in each state.

5.0.1. *Block Voting Equilibrium.* Next, consider the equilibrium sequence with $\beta_\theta^n \to 1$. In this sequence, since both cutoffs x_l^n and x_r^n converge to $\frac{-b}{2}$, the responsive set constitutes a vanishing fraction of types close to $\frac{-b}{2}$. Since the θ-median is not contained in this set, the responsive voters are not influential. Moreover, the alignment condition also fails since the responsive voters are entirely in the L-group. Therefore, FIE fails for this sequence.

Observationally, behavior in this equilibrium resembles block voting: the majority independent group mobilizes in favor of the status quo while the minority group votes for the proposed reform, and voting behavior (and outcome) is independent of the merits of the reform. Almost all voters vote as if the state is L irrespective of their private signals. Such uninformative voting behavior is fuelled by the belief that (almost) everyone else is also going to vote uninformatively.

As an aside, notice that while there is an uninformative equilibrium with beliefs converging to 1, there is no equilibrium sequence with beliefs converging to 0. In the former case, $t(R, \beta^n) < t(L, \beta^n) < \theta$ for large enough n, implying that a vote share of exactly θ for $\mathcal{P}$ is much more likely in sate L than in state R which is consistent with $\beta = 1$.On the other hand, suppose voters believe $\beta = 0$. In this case, $t(R, \beta^n) > t(L, \beta^n) > \theta$ for large enough n, making the pivotal event much more likely in state L, which is inconsistent with $\beta = 0$. Therefore, the belief $\beta = 0$ cannot be sustained in equilibrium.

5.0.2. *Activist Voting Equilibrium.* Finally, consider the equilibrium sequence with $\beta_\theta^n \to \beta^*$. In this case, the belief does not put large enough weight on either state, leading to the following behavior: the extreme types in either group vote for the proposed reform if they get a signal that favors voting for the reform, while everybody else votes for the status quo. Formally, we have $x_l(\beta^*) < \frac{-b}{2}$ and $x_r(\beta^*) > \frac{b}{2}$: those to the left of $x_l(\beta^*)$ vote for $\mathcal{P}$ if they get the signal l, and those to the right of $x_r(\beta^*)$ vote for $\mathcal{P}$ if they get the signal r.[14] The responsive voters fail to influence the outcome as one group's votes against the proposed reform cancels the other group's votes for it: in fact, the reform gets less than θ share of votes in each state.

The extreme types to the left have the highest net utility from the reform in state L while the extreme types to the right have the highest net utility from reform in state R. Since these are the only types that vote for the reform, we call this an activist voting equilibrium. Observationally, the extreme types in the two groups appear to be voting for the same reform for opposite reasons, but the majority of the voters vote against the reform. Such behavior is quite different from the conventional wisdom that the centrist voters are the ones that swing the election one way or the other depending on the information they receive.

With a general distribution of voter preferences, we cannot rule out the possibility of multiple information-aggregating equilibrium sequences with different limits. The problem arises from that fact that the function $\theta^*(\cdot)$ can be non-monotonic in the range $(\overline{\beta^l}, \beta^*)$, and if the threshold θ is low enough, we may have multiple solutions to $\theta^*(\beta) = \theta$ in this range. However, if the L-group is sufficiently large, θ cannot be "too low" in this sense since we must have $\theta > F(\frac{b}{2})$.

The following remark shows that if the L-group and R-group are not too dissimilar in size, then all information aggregating equilibrium sequences have a unique limit.

[14] Clearly, the alignment condition also fails.

Remark 2. *Consider an environment with unordered alternatives and a consequential voting rule. If* $F(\frac{b}{2}) \geq q\left(1 - F(x_r\left(\overline{\beta^l}\right)\right)$, *then the induced prior belief in each equilibrium sequence converges to one of three possible values* 0, β^* *and some* $\beta_1 \in (0, \overline{\beta^l})$. *In sequences with beliefs converging to* 0 *and* β^*, *the status quo wins almost surely in each state. Sequences with beliefs converging to* β_1 *are full information equivalent.*

6. Discussion

In the main body of the paper, we have shown that information may not be aggregated due to a co-ordination failure among voters and provided conditions under which a status quo bias exists in certain equilibria. We conclude by providing some comments on robustness of results and multiplicity of equilibria.

6.1. **Multiplicity.** In the unordered alternatives environment, there are at least three different equilibrium sequences. One difficulty in our model is that there is no simple way to refine away any of these equilibria. In this sense, our message is that uncertainty about outcomes due to co-ordination problems is central to elections with diverse preferences. However, both the non-aggregating equilibria have one focality property that the aggregating equilibria do not have. In these equilibria, the induced prior belief and strategies used in the limit are independent of the particular voting rule in use. On the other hand, the information aggregating equilibria involve strategies and beliefs that are very sensitive to the particular value of θ.

6.2. **Robustness.** One stark feature of the model is that the outcomes (in particular, the non-aggregating equilibria) are remarkably robust to variation in the parameters. For example, in both the block voting equilibria and activist voting equilibrium, the status quo is elected in both states with arbitrarily high probability even for very small noise in the signals as long as the electorate is large enough. Similarly, these outcomes do not depend on the relative sizes of the opposing groups. In this sense, the aggregation failure arises from the *existence of* and not from the extent of state-contingent conflict in preferences.

We have assumed that a priori, the two states are equally likely. While this simplifies the calculations, all that matters for the characterization is the vote share functions $t(S, \cdot)$ which depend only on F and q.[15] The only place where we use the value of the prior is to find the lower cut-off $\frac{1}{2} + \frac{b}{4}$ for signal precision. With general priors, we can still have the same result with a different lower bound for q.

In the unordered alternatives case, we provide the characterization of equilibria for high signal precisions. While this assumption stacks the deck against non-informative equilibria and therefore makes our results sharper, we are forced to make this assumption in order to ensure that pivot probabilities are positive in each state for all prior beliefs. In absence of such an assumption, there might be additional equilibria where voters use weakly undominated strategies.[16]

6.3. **Other voting rules.** For all practical purposes, we are interested in consequential rules. But our theoretical structure allows us to determine the outcomes for all voting rules $\theta \in (0, 1)$. Now, we briefly mention the aggregation properties of the non-consequential rules. We skip the proof as the idea is clear from figures 3 and 4.

Non-consequential rules implement the same alternative in both states under full information. We define a threshold rule θ as $\mathcal{P}$-trivial (respectively, $\mathcal{Q}$-trivial) if more than θ share of the population

[15] Bhattacharya (2013) provides the entire analysis with general priors.

[16] An alternative way to ensure positive pivot probabilities for all β would be to assume the existence of committed voters for each alternative. While this has been the standard assumption in the existing literature (Feddersen and Pesendorfer (1997), Bhattacharya 2013), this does not sit well with the idea of voter ideal points being distributed on the Downsian space.

prefers the reform $\mathcal{P}$ (respectively, $\mathcal{Q}$) in each state. In an ordered alternatives environment, threshold rules $\theta < F\left(\frac{L}{2}\right)$ are $\mathcal{P}$-trivial and rules $\theta > F\left(\frac{R}{2}\right)$ are $\mathcal{Q}$-trivial. From figure 3, it is clear that in this environment given a $\mathcal{P}$-trivial rule, $\beta_\theta^n \to 1$. Since $t(L,1) = t(R,1) = F\left(\frac{L}{2}\right) > \theta$, the reform wins in both states almost surely. Similarly, for any $\mathcal{Q}$-trivial rule, $\beta_\theta^n \to 0$ and the status quo wins in both states almost surely. Therefore, information is aggregated for all consequential as well as non-consequential rules in the ordered alternatives environment.

In an unordered alternatives environment, $\theta < F\left(-\frac{b}{2}\right)$ define $\mathcal{P}$-trivial rules and $\theta > 1 - F\left(\frac{b}{2}\right)$ define $\mathcal{Q}$-trivial rules. Information aggregation is guaranteed for a $\mathcal{Q}$-trivial rule simply because $t(S,\beta) < \theta$ for every β.[17] To see that the similar result does not hold for all $\mathcal{P}$-trivial rules, first define $t(L,\beta^*) = t(R,\beta^*) = t$, and note that $0 < t < F\left(-\frac{b}{2}\right)$.[18] For $\mathcal{P}$-trivial rules $\theta < t$, $\beta_\theta^n \to \beta^*$ and information is aggregated. Now consider $\mathcal{P}$-trivial rules $t < \theta < F\left(-\frac{b}{2}\right)$. For these rules, *there is no equilibrium that aggregates information.* In one equilibrium sequence, $\beta_\theta^n \to \beta^*$ and the policy loses in both states. There can be additional equilibria with beliefs in $(0,\beta^*) \cup (\beta^*,1)$. In these equilibria, the status quo wins almost surely in one state. Figure 5 demonstrates equilibria for $\mathcal{P}$-trivial rules $t < \theta < F\left(-\frac{b}{2}\right)$.

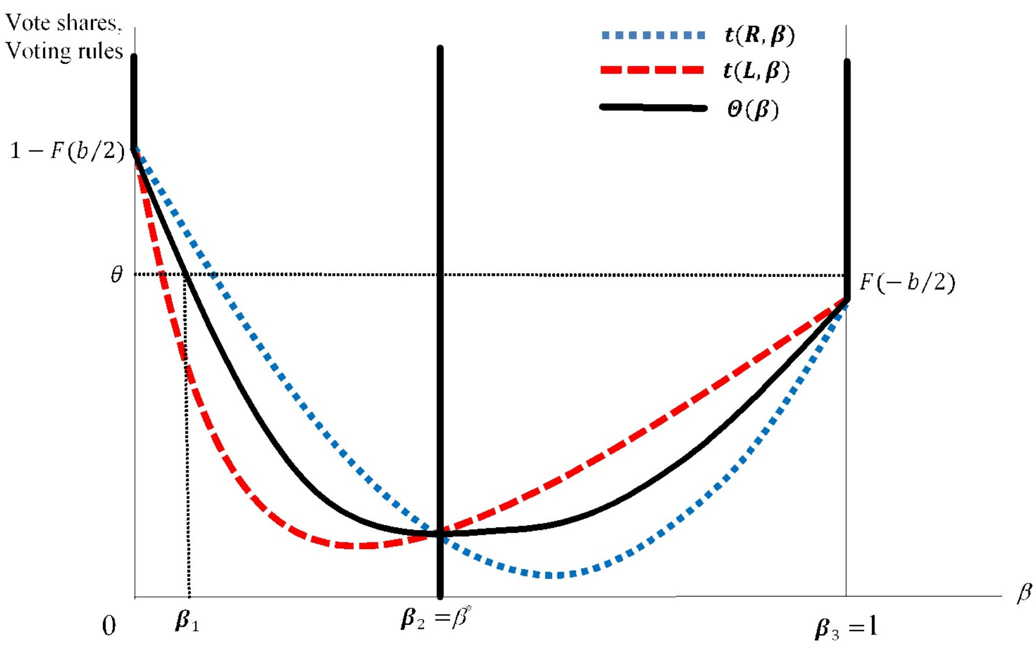

Figure 5: Equilibria in the unordered alternatives environment for P-trivial rules with $\theta > t$

There are two important takeaways from the above discussion. First, for an unordered alternatives environment, the only voting rules that are aggregate information in all equilibrium sequences are the very high or very low thresholds which implement the same outcome in both states under full information. Moreover, there is a continuum of voting rules for which the information-aggregating equilibrium fails to exist. This is in contrast with the ordered alternatives environment where all non-unanimous voting rules aggregate information in every equilibrium. Second, the status quo bias shows up in a different form when we consider non-consequential rules in the unordered alternatives environment: while information is aggregated in every equilibrium for all $\mathcal{Q}$-trivial rules,

[17] There are three equilibrium sequences with induced priors converging to 0, β^* and 1 respectively.

[18] This is shown in the Proof of Proposition 7

there exist $\mathcal{P}$-trivial rules for which the status quo wins in at least one state in all equilibria. This again underscores how preference conflict among voters undermines aggregation of information.

7. Appendix: Proofs from the main paper

7.1. **Proof of Propositions 1 and 2.** From equation (2), for any β_σ, we obtain x_σ as the unique solution to $E(v(x,S)=0$.

$$x_\sigma(\beta)=\frac{1}{2}\left(\frac{(L)^2\beta_\sigma+(R)^2(1-\beta_\sigma)}{L\beta_\sigma+R(1-\beta_\sigma)}\right)\in\left[\frac{L}{2},\frac{R}{2}\right]$$

Now $\frac{dEv(x,s)}{dx}=2(\beta_\sigma L+(1-\beta_\sigma)R)<0$ since $L<R<0$. Hence it must be the case that $Ev(x,s)>0$ for $x<x_\sigma$ and $Ev(x,s)<0$ for $x>x_\sigma$. The rest of proposition 1 can be easily verified from the expression for $x_\sigma(\beta)$ and expression (1) for posterior β_σ.

In Proposition 2, the vote share functions $t(L,\beta)$ and $t(R,\beta)$ are decreasing in β since $x_\sigma(\beta)$ is strictly decreasing in β. Since $z_\sigma(\beta)=F(x_\sigma(\beta))$, we have from (3) and (4)that $t(R,\beta)-t(L,\beta)=(2q-1)\left(F\left(x_r(\beta)\right)-F\left(x_l(\beta)\right)\right)>0$ since $x_r(\beta)>x_l\left(\beta\right)$ from Proposition 1. The rest follows from inspection.

7.2. **Proof of Propositions 3 and 4.** We first establish the fact that $q>\frac{1}{2}+\frac{b}{4}$ implies that $\overline{\beta^l}<\frac{1}{2}<\underline{\beta^r}$. It suffices to show that there is no β such that both β_l and β_r lie in the interval $[\frac{1}{2}-\frac{b}{4},\frac{1}{2}+\frac{b}{4}]$. Notice that when $\beta_l=q$, we must have $\beta_r=1-q$. Since β_σ is monotonic in β, $\beta_l<q$ implies $\beta_r<1-q$ and $\beta_r>1-q$ implies $\beta_l>q$. By the informativeness assumption, $q>\frac{1}{2}+\frac{b}{4}$ and $1-q<\frac{1}{2}-\frac{b}{4}$. Now,

$$\begin{aligned}\beta_l &\in [\frac{1}{2}-\frac{b}{4},\frac{1}{2}+\frac{b}{4}]\Rightarrow\beta_l<q\Rightarrow\beta_r<1-q<\frac{1}{2}-\frac{b}{4},\text{ and}\\ \beta_r &\in [\frac{1}{2}-\frac{b}{4},\frac{1}{2}+\frac{b}{4}]\Rightarrow\beta_r>1-q\Rightarrow\beta_l>q>\frac{1}{2}+\frac{b}{4}\end{aligned}$$

Proposition 3 can be verified by inspecting the expressions (5) and (6).

For proposition 4, first notice that $0<\underline{\beta^l}<\overline{\beta^l}<\underline{\beta^r}<\overline{\beta^r}<1$. At $\beta=0$, $z_l=z_r=t(L,\beta)=t(R,\beta)=1-F\left(\frac{b}{2}\right)$ and at $\beta=1$, $z_l=z_r=t(L,\beta)=t(R,\beta)=F\left(\frac{-b}{2}\right)$. In the range $\left(0,\underline{\beta^l}\right)$, both z_l and z_r are decreasing. In the range $\left(\underline{\beta^l},\overline{\beta^l}\right)$, $z_l=0$ while z_r is decreasing. Therefore, both $t(L,\beta)$ and $t(R,\beta)$ are decreasing in $\left(0,\overline{\beta^l}\right)$. Similarly, In the range $\left(\overline{\beta^r},1\right)$, both z_l and z_r are increasing. In the range $\left(\underline{\beta^r},\overline{\beta^r}\right)$, $z_r=0$ while z_l is increasing. Therefore, both $t(L,\beta)$ and $t(R,\beta)$ are increasing in $\left(\underline{\beta^r},1\right)$.

Recall that $x_l(\beta)>x_r(\beta)$ for all β from Proposition 3. In the range $\left(0,\underline{\beta^l}\right)$, $z_\sigma(\beta)=1-F(x_\sigma(\beta)$, which implies that $z_r(\beta)>z_l(\beta)$ by the full support assumption. In the range $\left(\underline{\beta^l},\overline{\beta^l}\right)$, $z_r(\beta)>z_l(\beta)=0$. Therefore, $z_r(\beta)-z_l(\beta)>0$ in the range $\left(0,\overline{\beta^l}\right)$. By similar logic, $z_r(\beta)-z_l(\beta)<0$ in the range $\left(\underline{\beta^r},1\right)$. In the range $\left[\overline{\beta^l},\underline{\beta^r}\right]$, z_r decreases from a positive value to 0 and z_l increases from 0 to a positive value. Therefore, there must be a unique intersection at $\beta^*\in(\overline{\beta^l},\underline{\beta^r})$ where $z_l=z_r$. In other words, we must have $z_r(\beta)-z_l(\beta)>0$ at $(0,\beta^*)$ and negative at $(\beta^*,1)$, with an intersection at β^*. We are done proving Proposition 4, since $t(R,\beta)-t(L,\beta)=(2q-1)\left(z_r(\beta)-z_l(\beta)\right)$.

7.3. **Proof of Remark 1.** Continuity of $\theta^*(\beta)$ follows from continuity of vote share functions. Now, denote $t(L,\beta)=x$ and $t(R,\beta)=y$, and assume WLOG $0<y<x<1$. For any $\theta\in(0,1)$, the function $f(z)=z^\theta(1-z)^{1-\theta}$ is single-peaked in $[0,1]$ and achieves maximum at $z=\theta$. Therefore, the equation $x^\theta(1-x)^{1-\theta}=y^\theta(1-y)^{1-\theta}$ has a unique solution θ^* and $y<\theta^*<x$.

Consider now the expression for θ^*.

$$\frac{\theta^*}{1-\theta^*}=\frac{\log(1-y)-\log(1-x)}{\log x-\log y}$$

Denote the right hand side by $h(x,y)$. We have

$$\frac{\partial h}{\partial x}=\frac{1}{(\log x-\log y)^2}\left[\frac{1}{1-x}(\log x-\log y)-\frac{1}{x}\log(1-y)-\log(1-x)\right]$$

Thus $\frac{\partial h}{\partial x}>0$ if $\frac{x}{1-x}>\frac{\theta^*}{1-\theta^*}$, i.e., $x>\theta^*$. Similarly, $\frac{\partial h}{\partial y}>0$ if $y<\theta^*$. Since we have $y<\theta^*<x$, $h(x,y)$ is increasing in both x and y. Since $\frac{\theta^*}{1-\theta^*}$ is itself increasing in θ^*, it must be increasing in both x and y.

7.4. **Proof of Proposition 5.** We prove this proposition in several steps. We start with Lemma 1 in Bhattacharya (2013)

Lemma 1. *For any given environment, if the limit belief in an equilibrium sequence is β_θ^0, then*

$$\left(t(L,\beta_\theta^0)\right)^\theta\left(1-t(L,\beta_\theta^0)\right)^{1-\theta}=\left(t(R,\beta_\theta^0)\right)^\theta\left(1-t(R,\beta_\theta^0)\right)^{1-\theta}$$

Proof. Note first, that by the usual continuity arguments, if $\beta_\theta^n\to\beta_\theta^0$, then $t(S,\beta_\theta^n)\to t(S,\beta_\theta^0)$. Whenever $t(L,\beta_\theta^0)=t(R,\beta_\theta^0)$, the Lemma holds trivially. Notice that this covers the cases $\beta_\theta^0\in\{0,1\}$ both for ordered and unordered alternatives. Now, consider $\beta_\theta^0\in(0,1)$ and rewrite $H(\beta_\theta^n,n,\theta)$ as $\left[\frac{1-t(L,\beta_\theta^n)}{1-t(R,\beta_\theta^n)}\right]^{n-m}\left[\frac{t(L,\beta_\theta^n)^\theta\left(1-t(L,\beta_\theta^n)\right)^{1-\theta}}{t(R,\beta_\theta^n)^\theta\left(1-t(R,\beta_\theta^n)\right)^{1-\theta}}\right]^m$ where $m=\frac{\lfloor n\theta\rfloor}{\theta}$. Since $m\geq n-\frac{1}{\theta}$, we have $m\to\infty$ as $n\to\infty$. Also, since $0<t(S,\beta)<1$ and $m-n\in\left[0,\frac{1}{\theta}\right]$, there is some $0<\underline{t}<\bar{t}$ such that $\underline{t}\leq\left[\frac{1-t(L,\beta_\theta^n)}{1-t(R,\beta_\theta^n)}\right]^{n-m}\leq\bar{t}$ for all m and n. If there is some $\epsilon>0$ such that $\frac{\left(t(L,\beta_\theta^n)\right)^\theta\left(1-t(L,\beta_\theta^n)\right)^{1-\theta}}{\left(t(R,\beta_\theta^n)\right)^\theta\left(1-t(R,\beta_\theta^n)\right)^{1-\theta}}>1+\epsilon$ for all n large enough, then $\lim_{n\to\infty}H(\beta_\theta^n,n,\theta)>\lim_{n\to\infty}\underline{t}\left[\frac{t(L,\beta_\theta^n)^\theta\left(1-t(L,\beta_\theta^n)\right)^{1-\theta}}{t(R,\beta_\theta^n)^\theta\left(1-t(R,\beta_\theta^n)\right)^{1-\theta}}\right]^{\frac{\lfloor n\theta\rfloor}{\theta}}>\underline{t}\left[\lim_{m\to\infty}(1+\epsilon)^m\right]\to\infty$. Hence the RHS of equation (11) is not bounded away from 1, which is a contradiction. Similarly, if there is some $\epsilon>0$ such that $\frac{t(L,\beta_\theta^n)^\theta\left(1-t(L,\beta_\theta^n)\right)^{1-\theta}}{t(R,\beta_\theta^n)^\theta\left(1-t(R,\beta_\theta^n)\right)^{1-\theta}}<1-\epsilon$ for all n large enough, then $\lim_{n\to\infty}H(\beta_\theta^n,n,\theta)=0$, we have a contraction since the RHS of equation (11) is not bounded away from 0. ■

Next, we state a version of Lemma 2 in Bhattacharya (2013) adapted to our setting.

Lemma 2 (NECESSITY)**.** *Define by $\Theta(\beta)$ for each environment as given in Proposition 5. For a given $\beta\in[0,1]$, consider any sequence $\beta^n\to\beta$. Now, if $\theta\notin\Theta(\beta)$, then $H(\beta^n,n,\theta)$ is bounded away from $\frac{\beta}{1-\beta}$ as $n\to\infty$.*

Proof. Consider the environment with ordered alternatives ($-1<L<R<0$) first.

The necessity of case (i) follows directly from Lemma 1. Next, consider case (ii), i.e., $\Theta(0)=\left\{\theta:\theta\geq F\left(\frac{R}{2}\right)\right\}$. From Proposition 2, by continuity of vote share functions, we know that as $\beta^n\to 0$, (i) $t(L,\beta^n)\to F\left(\frac{R}{2}\right)$, (ii) $t(R,\beta^n)\to F\left(\frac{R}{2}\right)$, and (iii) $t(L,\beta^n)<t(R,\beta^n)<F\left(\frac{R}{2}\right)$. Notice now that the the function $z^\theta(1-z)^{1-\theta}$ is single peaked in z and attains its maximum at $z=\theta$. Now, if $\theta<F\left(\frac{R}{2}\right)$, then for all large enough n, we have $\theta<t(L,\beta^n)<t(R,\beta^n)$, which would imply that $\left(t(L,\beta_\theta^n)\right)^\theta\left(1-t(L,\beta^n)\right)^{1-\theta}>\left(t(R,\beta^n)\right)^\theta\left(1-t(R,\beta^n)\right)^{1-\theta}$. Therefore, we must have

$H(\beta^n, n, \theta) > \underline{t}\left[\frac{t(L,\beta^n)^\theta(1-t(L,\beta^n))^{1-\theta}}{t(R,\beta^n)^\theta(1-t(R,\beta^n))^{1-\theta}}\right]^{\frac{\lfloor n\theta \rfloor}{\theta}} > \underline{t}$, which is bounded away from 0 in the limit. The proof of case (iii) is analogous.

Next, consider the environment with unordered alternatives. Case (i) follows from Lemma 1. In case (ii), there is nothing to prove. Cases (iii) and (iv) are analogous to case (ii) of the ordered alternatives case. ■

Our sufficiency result follows Lemma 3 in Bhattacharya (2013). We need a slightly more relaxed concept of regularity in the present paper.

Lemma 3 (SUFFICIENCY)**.** *There is a sequence of equilibria with induced prior β_θ^n converging to β if $\theta \in \Theta(\beta)$ and β is regular.*

Proof. Define the function $G_n(\beta, \theta) = \frac{H(\beta,n,\theta)}{1+H(\beta,n,\theta)}$. We show that provided that $\widehat{\beta}$ is regular, if $\theta \in \Theta(\widehat{\beta})$, then there is a sequence of fixed points β_θ^n of $G_n(\beta, \theta)$ such that $\beta_n^\theta \to \widehat{\beta}$. We prove this separately for different values of $\widehat{\beta}$.

We use the following result repeatedly in the proof: Suppose $g(x, y, \theta) = \frac{x^\theta(1-x)^{1-\theta}}{y^\theta(1-y)^{1-\theta}}$ for some $1 > x > y > 0$. The, we must have $\frac{\partial g(x,y,\theta)}{\partial \theta} > 0$.

First consider some regular $\widehat{\beta}$ such that $t(L, \widehat{\beta}) \neq t(R, \widehat{\beta})$. WLOG, assume $t(L, \widehat{\beta}) > t(R, \widehat{\beta})$. For such a $\widehat{\beta}$, $\Theta(\widehat{\beta}) = \theta^*(\widehat{\beta})$ by Lemma 2 and notice that $\theta^*(\cdot)$ has continuous and bounded derivatives since f is continuous and bounded. Since $\widehat{\beta}$ is regular, there must be a neighborhood $(\widehat{\beta} - \epsilon, \widehat{\beta} + \epsilon)$ where $\theta^*(\beta)$ is either only increasing or only decreasing, and because f is bounded, $t(L, \beta) > t(R, \beta)$. Suppose first that $\theta^*(\cdot)$ is decreasing in $(\widehat{\beta} - \epsilon, \widehat{\beta} + \epsilon)$. Write $H(\beta, n, \theta)$ as $B(\beta)\left[g(x, y, \theta)\right]^m$ where $x = t(L, \beta)$, $y = t(R, \beta)$, $B(\beta) = \left[\frac{1-x}{1-y}\right]^{n-m}$ and $m = \frac{\lfloor n\theta \rfloor}{\theta}$. $B(\beta)$ is bounded above and below. Now, for $\beta \in (\widehat{\beta}, \widehat{\beta} + \epsilon)$, we must have $g(x, y, \theta^*(\widehat{\beta})) > 1$, since $\theta^*(\widehat{\beta}) > \theta^*(\beta)$ as $\theta^*(\cdot)$ is decreasing. Moreover, $g(x, y, \theta^*(\beta)) = 1$ by definition. As $n \to \infty$, m must also go to ∞, and then, $\left[g(x, y, \theta^*(\widehat{\beta}))\right]^m \to \infty$, implying that $H(\beta, n, \theta^*(\widehat{\beta})) \to \infty$, i.e., $G_n(\beta, \theta^*(\widehat{\beta})) \to 1$.

We have just shown that for $\beta \in (\widehat{\beta}, \widehat{\beta} + \epsilon)$, we must have $G_n(\beta, \theta^*(\widehat{\beta})) \to 1$ as $n \to \infty$. On the other hand, for $\beta \in (\widehat{\beta} - \epsilon, \widehat{\beta})$, we must have $G_n(\beta, \theta^*(\widehat{\beta})) \to 0$ as $n \to \infty$. Consider the (continuous) function $G_n(\beta, \theta^*(\widehat{\beta})) - \beta$ in the range $\beta \in (\widehat{\beta} - \epsilon, \widehat{\beta} + \epsilon)$. Given ϵ, for large enough n, it is positive for $\beta = \widehat{\beta} + \epsilon$, and negative for $\beta = \widehat{\beta} - \epsilon$. Thus, there must exist some $\beta_n^{\theta^*(\widehat{\beta})} \in (\widehat{\beta} - \epsilon, \widehat{\beta} + \epsilon)$ such that $G_n\left(\beta_n^{\theta^*(\widehat{\beta})}, \theta^*(\widehat{\beta})\right) - \beta_n^{\theta^*(\widehat{\beta})} = 0$ for all n large enough. Thus, there exists a sequence $\beta_n^{\theta^*(\widehat{\beta})}$ such that for any $\epsilon > 0$ small enough, there is some M such that for all $n > M$, $G_n\left(\beta_n^{\theta^*(\widehat{\beta})}, \theta^*(\widehat{\beta})\right) = \beta_n^{\theta^*(\widehat{\beta})}$ and $\left|\beta_n^{\theta^*(\widehat{\beta})} - \widehat{\beta}\right| < \epsilon$. If $\theta^*(\beta)$ is increasing in $(\widehat{\beta} - \epsilon, \widehat{\beta} + \epsilon)$, then we can prove the result in an analogous way.

Next, consider $\widehat{\beta} = \beta^* \neq \frac{1}{2}$, and denote $t(L, \beta^*) = t(R, \beta^*) = t$. WLOG, suppose first that $\beta^* > \frac{1}{2}$. Since $t(L, \widehat{\beta}) = t(R, \widehat{\beta})$, $G_n(\widehat{\beta}, \theta) = \frac{1}{2}$ for all (n, θ). Then, consider any $\theta > t$. Since by proposition 4, $t(L, \beta) > \theta^*(\beta) > t(R, \beta)$ for all $\beta \in (\widehat{\beta}, \widehat{\beta} + \epsilon)$, given θ we can choose ϵ small enough such that $\theta > \theta^*(\beta)$ for all $\beta \in (\widehat{\beta}, \widehat{\beta} + \epsilon)$. Therefore $G_n(\beta, \theta) \to 1$ in this interval. Now, consider the continuous function $G_n(\beta, \theta) - \beta$ in this interval. For large enough n, it is positive at $\widehat{\beta} + \epsilon$ and negative at $\widehat{\beta}$. Therefore, $G_n(\beta, \theta)$ must have a fixed point β_n in this interval. Thus, there exists a sequence β_n such that for any $\epsilon > 0$ small enough, there is some m such that for all $n > m$, $G_n(\beta_n, \theta) = \beta_n$ and $\left|\beta_n - \widehat{\beta}\right| < \epsilon$ for any $\theta > t$. To show the existence of a sequence of beliefs converging to $\widehat{\beta}$ for voting rules $\theta < t$, follow an analogous method.

If $\widehat{\beta} = \beta^* = \frac{1}{2}$, consider a sequence $\beta_n = \widehat{\beta}$ for all n. We are done, since $G_n(\widehat{\beta}, \theta) = \frac{1}{2} = \widehat{\beta}$ for all n.

Finally, consider the cases with $\widehat{\beta} \in \{0, 1\}$. Suppose first that $\widehat{\beta} = 0$. For the ordered alternatives case, consider any $\theta > F\left(\frac{R}{2}\right)$. Now, by proposition 2 we must have $\theta > t(R, \beta) > \theta^*(\beta) > t(L, \beta)$ for small enough $(0, \epsilon)$. Notice that $G_n(\beta, \theta) \to 0$ in this interval. The function $G_n(\beta, \theta) - \beta$ is equal to $\frac{1}{2}$ at $\beta = 0$ and negative at $\beta = \epsilon$ for large enough n. Therefore, there must be a fixed point of $G_n(\beta, \theta)$ in $(0, \epsilon)$. The other cases with $\widehat{\beta} \in \{0, 1\}$ are similar. ■

7.5. **Proof of Proposition 6.** Consider $\theta \in \left(F(\frac{L}{2}), F(\frac{R}{2})\right)$. By Proposition 5, the only way θ can belong to $\Theta^U(\beta)$ would be if $\beta \in (0, 1)$. By Proposition 2 and remark 1, $\theta^*(\beta)$ is continuous and monotonically decreasing in the range $(0, 1)$. Moreover, $\theta^*(\beta) \to F(\frac{R}{2}) > \theta$ as $\beta \to 0$ and $\theta^*(\beta) \to F(\frac{L}{2}) < \theta$ as $\beta \to 1$. Thus, there is a uniuqe value β_1 such that $\theta^*(\beta_1) = \theta$. Since $\theta^*(\cdot)$ is decreasing, β_1 is regular. Therefore, for all equilibrium sequences, $\beta_\theta^n \to \beta_1$. Notice that $t(L, \beta_1) < \theta < t(R, \beta_1)$ by Proposition 2. By the Strong Law of Large numbers, $\mathcal{P}$ wins in state R and $\mathcal{Q}$ in state L almost surely.

7.6. **Proof of Proposition 7.** Consider a voting rule $\theta \in \left(F(\frac{-b}{2}), 1 - F(\frac{b}{2})\right)$. We identify the three classes of equilibrium sequences in (i), (ii) and (iii) we establish the existence of equilibrium sequences converging to 1, β^* and some $\beta \in (0, \beta^*)$ respectively. In (iv) we establish that there is no equilibrium sequence that converges to $(\beta^*, 1)$.

(i) From Proposition 5, $\theta \in \Theta^U(0)$. Since $t(L, 1) = t(R, 1) = F(\frac{-b}{2}) < \theta$, $\beta = 1$ is regular. Therefore there is an equilibrium sequence with $\beta_n^\theta \to 0$, and in the limit of this sequence, $\mathcal{Q}$ wins almost surely in both states.

(ii) From Proposition 5, we also have $\theta \in \Theta^U(\beta^*)$. Observe that $t(L, \beta^*) = t(R, \beta^*) < F(\frac{-b}{2}) < \theta$, since $t(S, \beta^*) = z_l(\beta^*)$ and $F(\frac{-b}{2}) = z_l(1)$, and z_l increases in the range $(\overline{\beta^l}, 1)$. Therefore, β^* is regular and there is an equilibrium sequence $\beta_n^\theta \to \beta^*$ and in the limit of this sequence, $\mathcal{Q}$ wins almost surely in both states.

(iii) To show that there some $\beta \in (0, \beta^*)$ such that there is an equilibrium sequence that converges to β, we proceed in two steps. First, we show that there is some $\beta \in (0, \beta^*)$ such that the necessary condition $\theta = \Theta^U(\beta)$. Then we show that among the different values of β satisfying the necessary condition, there must exist some β that satisfies the sufficiency condition too.

For necessity, observe that (a) $\theta^*(\beta)$ is continuous in the range $(0, \beta^*)$, (b) $\theta^*(\beta) \to 1 - F(\frac{b}{2}) > \theta$ as $\beta \to 0$, and (c) $\theta^*(\beta) \to t < F(\frac{-b}{2}) < \theta$ as $\beta \to \beta^*$. Therefore, there must be some β_1 in the range $(0, \beta^*)$ such that $\theta^*(\beta_1) = \theta$.

To check for sufficiency, note that conditions (a), (b) and (c) above imply that some solution to $\theta^*(\beta_1) = \theta$ must satisfy at least one of the following two properties: (I) $\theta^*(\cdot)$ is locally decreasing at β_1, or (II) there is some interval $[\beta_1', \beta_1''] \subset (\overline{\beta^l}, \beta^*)$ containing β_1 such that $\theta^*(\cdot) = \theta$ in $[\beta_1', \beta_1'']$, and for some $\epsilon > 0$, $\theta^*(\cdot)$ is decreasing in $(\beta_1' - \epsilon, \beta_1')$ as well as in $(\beta_1'', \beta_1'' + \epsilon)$.

If property (I) holds, then β_1 is regular and by Proposition 5, there must exist an equilibrium sequence $\beta_n^\theta \to \beta_1$. It is, however, possible that the only values of β for which $\theta^*(\cdot) = \theta$ are not regular because $\theta^*(\cdot)$ is constant over some interval $[\beta_1', \beta_1'']$. In this case, property (II) must hold. We now show that if property (II) holds, then there must exist an equilibrium sequence with β_n^θ converging to some value in $[\beta_1', \beta_1'']$.

Denote $t(L, \beta) = x$, $t(R, \beta) = y$, $g(x, y, \theta) = \frac{x^\theta (1-x)^{1-\theta}}{y^\theta (1-y)^{1-\theta}}$ and $B(\beta) = \left[\frac{1-x}{1-y}\right]^{n-m}$ where $m = \frac{\lfloor n\theta \rfloor}{\theta}$. We can now write $H(\beta, n, \theta)$ as $B(\beta)\left[g(x, y, \theta)\right]^m$. Notice that in the interval $[\beta_1' - \epsilon, \beta_1'' + \epsilon]$, $0 < x < y < 1$. In $[\beta_1', \beta_1'']$, $g(x, y, \theta) = 1$ since $\theta = \theta^*$ for this interval. Therefore, $H(\beta, n, \theta) = B(\beta)$. We have $G_n(\beta, \theta) = \frac{B(\beta)}{1 + B(\beta)}$, which is bounded above 0 and below 1 for each n. By an argument employed before in the the Sufficiency Lemma in the proof of Proposition 5, we have $G_n(\beta, \theta) \to 1$

for all $\beta \in [\beta_1' - \epsilon', \beta_1')$, and $G_n(\beta,\theta) \to 0$ for all $\beta \in (\beta_1'', \beta_1'' + \epsilon']$ for $\epsilon' > 0$ small enough. Notice also that $G_n(\beta,\theta)$ is continuous in β for all θ in $[\beta_1', \beta_1'']$. For large enough n, $G_n(\beta,\theta) - \beta > 0$ at β_1' and $G_n(\beta,\theta) - \beta < 0$ at β_1''. Therefore, there must exist a fixed point $\beta_n \in (\beta_1', \beta_1'')$ for all n large enough. Since (β_1', β_1'') is contained in a closed and bounded interval $[\beta_1', \beta_1'']$, a limit point must also exist and lie in $[\beta_1', \beta_1'']$.

We thus establish the existence of an equilibrium sequence with induced beliefs converging to $\beta_1 \in (0, \beta^*)$. By Proposition 4, $t(L, \beta_1) < \theta < t(R, \beta_1)$ in $(0, \beta^*)$ and $\mathcal{P}$ wins in state R and $\mathcal{Q}$ in state L almost surely.

It remains to show that we cannot have any equlibrium sequence with induced prior converging to values in $(\beta^*, 1]$. For all β in $(\beta^*, 1)$, $\theta^*(\beta) < t(L, \beta) < z_l(\beta) < F(\frac{-b}{2})$, where the last inequality follows from the fact that $z_l(\beta)$ is increasing in that range and $z_l(1) = F(\frac{-b}{2})$. Since $\theta > F(\frac{-b}{2})$, there is no β in $(\beta^*, 1)$ for which $\theta = \theta^*(\beta)$. Finally, observe that $\theta \notin \Theta^U(0)$, and we are done.

7.7. **Proof of Remark 2.** For a consequential rule, it must be the case that $\theta > F(\frac{b}{2})$. At $\beta = \overline{\beta^l}$, $\theta^*(\beta) < t(R, \beta) = q z_r(\overline{\beta^l}) = q\left(1 - F(x_r\left(\overline{\beta^l}\right)\right)$. By the condition that $F(\frac{b}{2}) \geq q\left(1 - F(x_r\left(\overline{\beta^l}\right)\right)$, it must be the case that $\theta > \theta^*(\overline{\beta^l})$. On the other hand, we have $\theta < \theta^*(\beta)$ for $\beta \to 0$ since $\theta < 1 - F(\frac{-b}{2})$. By Remark 1 $\theta^*(\beta)$ is strictly decreasing in $(0, \overline{\beta^l})$. Therefore, there is a unique solution β_1 to $\theta = \theta^*(\beta)$ in the range $(0, \beta^*)$. Moreover, there is no solution in the range $[\overline{\beta^l}, \beta^*)$. At such a β_1, property (I) holds, and the rest follows from the proof of Proposition 7.

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